

Lesson 7.6A More Measures of Dispersion

Objectives

- Calculate the mean absolute deviation (MAD).
- Calculate the z-scores of a data set.

In Lesson 7-1, you learned that measures of central tendency are used to describe a data set by identifying a “normal” or average value in the set.

In Lesson 7-6, you learned about measures of dispersion. Measures of dispersions describe the spread of data in a data set. Some measures of dispersion are calculated in regard to a measure of central tendency, usually the median or mean.

Mean Absolute Deviation

The **mean absolute deviation** is a statistical measure of dispersion that is based on the *absolute deviation*. The absolute deviation of any data value is the absolute value of the difference between the data value and a predetermined value (or measure). That predetermined value is usually a measure of central tendency; in this case the mean.

Absolute Deviation

$$D = |x_1 - M|$$

where D is the absolute deviation, x_1 is a data element, and M is the mean of the data set.

Mean Absolute Deviation (MAD)

is the mean of the absolute deviations of a data set.

$$\text{MAD} = \frac{D_1 + D_2 + \dots + D_n}{n}$$

where D_n is the absolute deviation of element n and n is the number of elements in the data set.

The MAD equals a value that is less than or equal to the standard deviation of the data set.

Example 1 Calculate Mean Absolute Deviation

While working out, Craig completed five repetitions of bench presses. With each repetition he lifted a different weight. The weights lifted were 155 lb, 162 lb, 170 lb, 148 lb, and 135 lb. What was the mean absolute deviation of the weight that Craig bench-pressed during his workout?

Solution

Find the mean of the weights.

$$M = \frac{155 + 162 + 170 + 148 + 135}{5} = \frac{770}{5} = 154$$

Find the absolute deviation of each weight from the mean weight.

$$|155 - 154| = 1 \quad |162 - 154| = 8 \quad |170 - 154| = 16$$

$$|148 - 154| = 6 \quad |135 - 154| = 19$$

Find the mean of the absolute deviations.

$$\text{MAD} = \frac{1 + 8 + 16 + 6 + 19}{5} = \frac{50}{5} = 10$$

The mean absolute deviation is 10, which means that the average deviation from the mean weight is 10 lb.

Ongoing Assessment

Joelle eats frozen boxed meals for lunch four days a week. Last week the meals had following calories: 145, 160, 165, and 170. What was the mean absolute deviation of the calories in the meals that Joelle ate for lunch?

Z-score

Actual data from a data set are known as *raw scores*, and in the real world data are measured in units, such as dollars, pounds, days, etc. You can standardize a raw score by converting it to a *z-score*. A *z-score* is a relative score, not an absolute score, that tells the number of standard deviations a raw score differs from the mean.

A *z-score* can be positive, negative, or zero. A positive *z-score* means that the actual data element is greater than the mean. A negative *z-score* means that the actual data element is less than the mean. When a *z-score* is 0, the actual data element equals the mean.

Z-score

$$z = \frac{x - M}{s}$$

where x is the raw score being converted, M is the mean of the data set, and s is the standard deviation of the data set.

Example 2 Calculate a z-score

Kennedy took a make-up test and scored 78 points. The mean score of his classmates, who took the same test the week prior, was 75 points with a standard deviation of 8. What is the the z -score of Kennedy's make-up test?

Solution

$$z = \frac{78 - 75}{8} = \frac{3}{8} = 0.375$$

The z -score of Kennedy's test points is 0.375.

Ongoing Assessment

Lita scored 70 poines on her test. What is the z -score of Lita's test?

Lesson Assessment

Think and Discuss

see margin

1. The mean of a data set that has values x_1 to x_9 is 25. The z -score of x_3 is 0. Explain how you know the value of x_3 . What is the value of x_3 ?
2. Explain the difference between the absolute deviation and the mean absolute deviation.

Practice and Problem Solving

Calculate the mean absolute deviation of each data set. Round to the nearest hundredth.

3. 14, 33, 87, 55, 50
4. 2,250; 3,805; 2,680
5. 144, 186, 198, 102
6. 10, 22, 30, 33, 22, 12, 10
7. 6, 11, 23, 17, 15, 19
8. 390, 288, 304, 500
9. Lamia read six books in one month. The books had page counts of 512, 880, 288, 752, 330, and 1,280. What is the mean absolute deviation of the page count of the books Lamia read?

Calculate the z -score if the value given is from a data set that has a mean of 100 and a standard deviation of 12. Round to the nearest hundredth.

10. 78
11. 100
12. 134
13. 99
14. 89
15. 156