Lesson 4.3A Intervals of Increase and Decrease and End Behavior

Objectives

- Identify intervals on which a function is increasing, decreasing, or constant.
- Identify end behavior of a function.

In mathematics, a function is identified as increasing if the values of $f(x)$ increase as the values of x increase. A function is identified as decreasing if the values of $f(x)$ decrease as the values of x increase. A function is identified as constant if the values of $f(x)$ stay the same as the values of x increase.

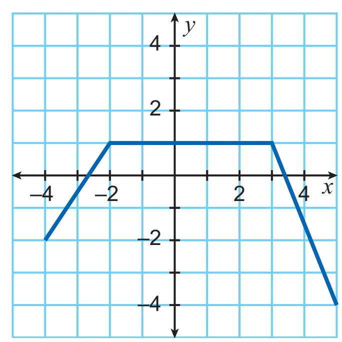
Increasing, Decreasing, and Constant Intervals

A given function can be always increasing, always decreasing, always constant, or any combination of increasing, decreasing, and constant.

A function's behavior is identified for a certain interval. An interval is any consecutive group of x-values. Interval notation is used to describe the x-values. For example, $(-\infty, 2]$ is notation for the interval negative infinity to positive 2 inclusive. A parentheses, (or), indicates the number is not part of the interval. A bracket, [or], indicates the number is included as part of the interval.

Example 1 Piecewise Function

Identify the intervals for which the function graphed is increasing, decreasing, or constant.



Solution

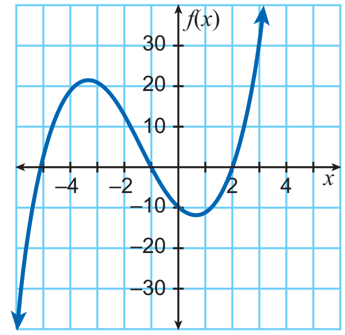
When the x-values are between –4 and –2, the y-values are increasing. In the interval $[-4, -2)$, the function is increasing.

When the x-values are between –2 and 3, the y-values are always 1. In the interval $(-2, 3)$, the function is constant.

When the x-values are between 3 and 5, the y-values are decreasing. In the interval $(3, 5]$, the function is decreasing.

Example 2 Cubic Function

Identify the intervals for which the function $f(x) = x^3 + 4x^2 - 7x - 10$ is increasing, decreasing, or constant.



Solution

Use the maximum and minimum features on your graphing calculator to determine the x-values where the graph changes from increasing to decreasing and changes from decreasing to increasing.

The graph changes from increasing to decreasing at $x \approx -3.4$ and from decreasing to increasing at $x \approx 0.7$. In interval notation, the graph of $f(x)$ is increasing from $(-\infty, -3.4)$ and from $(0.7, +\infty)$ and decreasing from $(-3.4, 0.7)$.

End Behavior

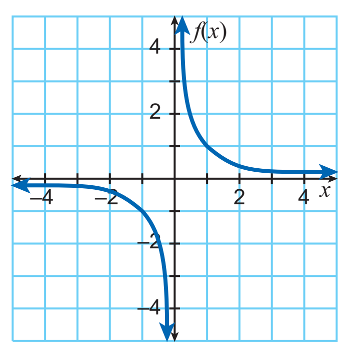
The end behavior of a function $f(x)$ refers to the $f(x)$-values of the function as x approaches positive and negative infinity. The $f(x)$-values of the function may get increasingly more positive, more negative, or approach a given value of $f(x)$.

Example 3 End Behavior

Identify the end behavior of the function $f(x) = \frac{1}{x}$ as x approaches $-\infty$ and as x approaches $+\infty$.

Solution

Use your graphing calculator to graph $f(x) = \frac{1}{x}$.



From the graph, as x approaches $-\infty$ the graph gets closer and closer to the x-axis meaning the values of $f(x)$ approach 0. As x approaches $+\infty$ the graph gets closer and closer to the x-axis meaning the values of $f(x)$ approach 0. In mathematical notation:

as $x \to -\infty$, $f(x) \to 0$ and as $x \to +\infty$, $f(x) \to 0$.

Lesson Assessment

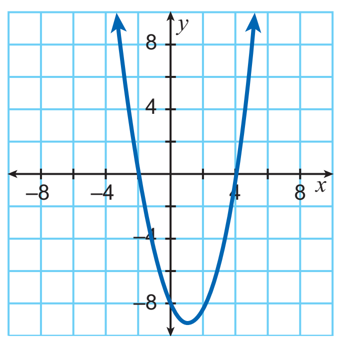
Think and Discuss

1. Describe what it means for a function to be increasing on a given interval.

2. Describe a strategy for identifying the end behavior of a function.

Practice and Problem Solving

For each function, identify the intervals where the function is increasing, decreasing, and constant. If necessary, round answers to the nearest tenth.

3.


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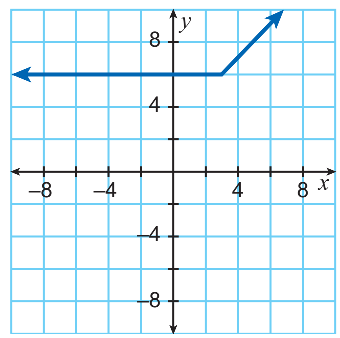


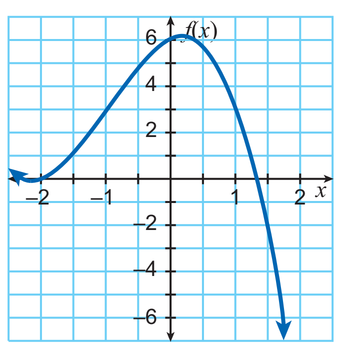
5. $f(x) = x^2 - 1$

6. $f(x) = x^3$

7. $f(x) = x^3 + 2x^2 - x - 2$

8. $f(x) = 2$

For each function, identify the end behavior as x approaches $-\infty$ and as x approaches $+\infty$.

9.


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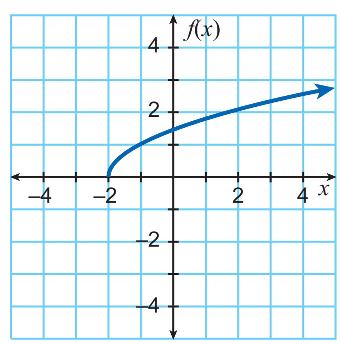

11. $f(x) = |2x - 3|$

12. $f(x) = -4x^2 + 1$

13. $f(x) = \frac{1}{x^2}$

14. $f(x) = \frac{1}{x^2} + 2$