Lesson 4.3A Intervals of Increase and Decrease and End Behavior

In mathematics, a function is identified as increasing if the values of \( f(x) \) increase as the values of \( x \) increase. A function is identified as decreasing if the values of \( f(x) \) decrease as the values of \( x \) increase. A function is identified as constant if the values of \( f(x) \) stay the same as the values of \( x \) increase.

**Increasing, Decreasing, and Constant Intervals**

A given function can be always increasing, always decreasing, always constant, or any combination of increasing, decreasing, and constant.

A function’s behavior is identified for a certain interval. An interval is any consecutive group of \( x \)-values. Interval notation is used to describe the \( x \)-values. For example, \((–\infty, 2]\) is notation for the interval negative infinity to positive 2 inclusive. A parentheses, ( or ), indicates the number is not part of the interval. A bracket, [ or ], indicates the number is included as part of the interval.

**Example 1 Piecewise Function**

Identify the intervals for which the function graphed is increasing, decreasing, or constant.

**Solution**

When the \( x \)-values are between –4 and –2, the \( y \)-values are increasing. In the interval \([-4, -2)\), the function is increasing.

When the \( x \)-values are between –2 and 3, the \( y \)-values are always 1. In the interval \((-2, 3)\), the function is constant.

When the \( x \)-values are between 3 and 5, the \( y \)-values are decreasing. In the interval \((3, 5]\), the function is decreasing.
Example 2 Cubic Function

Identify the intervals for which the function \( f(x) = x^3 + 4x^2 - 7x - 10 \) is increasing, decreasing, or constant.

Solution

Use the maximum and minimum features on your graphing calculator to determine the \( x \)-values where the graph changes from increasing to decreasing and changes from decreasing to increasing.

The graph changes from increasing to decreasing at \( x \approx -3.4 \) and from decreasing to increasing at \( x \approx 0.7 \). In interval notation, the graph of \( f(x) \) is increasing from \(( -\infty, -3.4)\) and from \((0.7, +\infty)\) and decreasing from \((-3.4, 0.7)\).

End Behavior

The end behavior of a function \( f(x) \) refers to the \( f(x) \)-values of the function as \( x \) approaches positive and negative infinity. The \( f(x) \)-values of the function may get increasingly more positive, more negative, or approach a given value of \( f(x) \).

Example 3 End Behavior

Identify the end behavior of the function \( f(x) = \frac{1}{x} \) as \( x \) approaches \(-\infty\) and as \( x \) approaches \(+\infty\).

Solution

Use your graphing calculator to graph \( f(x) = \frac{1}{x} \).

From the graph, as \( x \) approaches \(-\infty\) the graph gets closer and closer to the \( x \)-axis meaning the values of \( f(x) \) approach 0. As \( x \) approaches \(+\infty\) the graph gets closer and closer to the \( x \)-axis meaning the values of \( f(x) \) approach 0. In mathematical notation:

\[
\text{as } x \to -\infty, f(x) \to 0 \text{ and as } x \to +\infty, f(x) \to 0.
\]
Lesson Assessment

Think and Discuss

1. Describe what it means for a function to be increasing on a given interval.

2. Describe a strategy for identifying the end behavior of a function.

Practice and Problem Solving

For each function, identify the intervals where the function is increasing, decreasing, and constant. If necessary, round answers to the nearest tenth.

3. \[ x \]
   \[ y \]
   \[ \begin{array}{c|c|c|c}
   x & -8 & -4 & 0 \\
   y & -8 & -4 & 0 \\
   \end{array} \]

4. \[ x \]
   \[ y \]
   \[ \begin{array}{c|c|c|c}
   x & -8 & -4 & 0 \\
   y & -8 & -4 & 0 \\
   \end{array} \]

5. \( f(x) = x^2 - 1 \)

6. \( f(x) = x^3 \)

7. \( f(x) = x^3 + 2x^2 - x - 2 \)

8. \( f(x) = 2 \)

For each function, identify the end behavior as \( x \) approaches \(-\infty \) and as \( x \) approaches \(+\infty \).

9. \[ f(x) \]

10. \[ f(x) \]

11. \( f(x) = |2x - 3| \)

12. \( f(x) = -4x^2 + 1 \)

13. \( f(x) = \frac{1}{x^2} \)

14. \( f(x) = \frac{1}{x^2} + 2 \)