Math Lab 4 MS 4

Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

Solving Thermal Resistance Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There are Preparatory Math Skills Labs, located in a separate book entitled *PT Resource Manual*, that contain concepts your students should have mastered before they begin this lab. These labs are coded and titled:

PMS 8: "Substituting in Formulas"
PMS11: "Conducting Mathematical Operations with Numbers Expressed in Power-of-ten Form"
PMS13: "Checking Dimensions in Equations"
PMS16: "Multiplying and Dividing Units Written as Fractions"

Encourage students to refer to these preparatory labs if they need help.

RESOURCE MATERIALS

Student Text: Math Skills Lab
PT Resource Manual

CLASS GOALS

1. Teach students how to rearrange symbols in resistance equations to isolate certain unknowns.
2. Teach students how to solve thermal resistance problems.

CLASS ACTIVITIES

1. Take five or 10 minutes to go through the Student Exercises. Make sure that students understand the correct answers
2. Complete as many activities as time permits. Students should already have read the discussion material and looked at the examples for each activity before coming to this class. (How much is accomplished depends on the math skills that your students already have.)
   a. Summarize the explanatory material for Activity 1: "Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns." Then have students complete the Practice Exercises given at the end of Activity 1:
   b. Summarize in explanatory material for Activity 2: "Solving Thermal-resistance Problems." Then have students complete the Practice Exercises given at the end of Activity 2.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 4T1, "Measuring Resistance of Thermal Insulation," as homework.

Math Skills Laboratory

Lab 4 MS 4

MATH ACTIVITIES

Activity 1: ***Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns***

Activity 2: ***Solving Thermal-resistance Problems***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Rearrange the basic equation for thermal resistance,*** $R_T = \Delta T / \dot{Q}_H$, ***to isolate the temperature difference*** (ΔT) ***or the heat-flow rate*** $\dot{Q}_H$.
2. ***Isolate the material thickness*** (ℓ), ***thermal conductivity*** (k) ***or the area*** (A), ***by rearranging the equation for thermal resistance,*** $R_T = \ell / kA$.
3. ***Substitute correct numerical values and units in resistance equations. Solve the equations for a numerical value with the proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Work the problems for Activities 1 and 2.***

ACTIVITY 1

Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

MATERIALS

For this activity, you'll need pencil and paper.

You know equations and formulas are used to express a relationship between several physical quantities. Equation 1 relates the concept of thermal resistance, temperature difference and heat-flow rate. Equation 1 is usually stated as follows:

$$\text{Thermal Conductivity} = \frac{\text{Temperature Difference}}{\text{Heat-flow Rate}}$$

This relationship helps you find the value of one physical quantity if you know the value and units of the other two physical quantities. The relationship often is written with symbols rather than words.

$$R_T = \frac{\Delta T}{Q_H} \qquad \textit{Equation 1}$$

where: R_T = resistance measured in typical units, such as $\frac{C°}{cal/hr}$ or $\frac{F°}{Btu/hr}$
ΔT = temperature measured in C° or F°
Q_H = heat-flow rate measured in cal/hr or Btu/hr

Equation 1 describes ***thermal resistance*** in terms of temperature difference and heat-flow rate. You also can describe thermal resistance in terms of the properties of the material. These include the thermal conductivity, thickness, and area—as follows:

$$\text{Thermal Conductivity} = \frac{\text{Thickness of Material}}{\left(\begin{matrix}\text{Thermal}\\ \text{Conductivity}\end{matrix}\right) \times \left(\begin{matrix}\text{Area of}\\ \text{Material}\end{matrix}\right)}$$

This relationship lets you find the value of one physical quantity if you know the value and units of the other physical quantities. The symbol form of this relationship is:

$$R_T = \frac{\ell}{kA} \qquad \textit{Equation 2}$$

where: R_T = thermal resistance in typical units, like $\frac{C°}{cal/hr}$ or $\frac{F°}{Btu/hr}$
ℓ = length in cm or in.
k = thermal conductivity in $\frac{cal \cdot cm}{hr \cdot cm^2 \cdot C°}$ or $\frac{Btu \cdot in.}{hr \cdot ft^2 \cdot F°}$
A = area in cm^2 or ft^2

Table 1 sums up the units used with each physical quantity in Equations 1 and 2. It is meant to help you learn the units. So study Table 1 on the following page.

SOLUTIONS TO REVIEW OF UNITS

a . Celsius

b. Fahrenheit degrees

c. cal/hr

d. $\dfrac{\text{Btu} \cdot \text{in}}{\text{hr} \cdot \text{ft}^2 \cdot \text{F}^\circ}$

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1

Problem 1: Solved in text.

TABLE 1. THERMAL RESISTANCE UNITS

		System of Units: English	System of Units: SI
Equation 1: $R_T = \frac{\Delta T}{Q_H}$	R_T	$\frac{F^\circ}{Btu/hr}$	$\frac{C^\circ}{cal/hr}$
	ΔT	F°	C°
	Q_H	$\frac{Btu}{hr}$	$\frac{cal}{hr}$
Equation 2: $R_T = \frac{\ell}{kA}$	R_T	$\frac{F^\circ}{Btu/hr}$	$\frac{C^\circ}{cal/hr}$
	ℓ	in.	cm
	k	$\frac{Btu \cdot in.}{hr \cdot ft^2 \cdot F^\circ}$	$\frac{cal \cdot cm}{hr \cdot cm^2 \cdot C^\circ}$
	A	ft^2	cm^2

LET'S REVIEW UNITS!

Before you begin the Practice Exercises, answer these questions on units for thermal resistance. Fill in the blanks with the correct word or words.

a. A temperature difference in SI units is measured in _____ (Fahrenheit, Celsius) degrees.

b. A temperature difference in English units is measured in _____ _____ (degrees Fahrenheit, Fahrenheit degrees).

c. A thermal-flow rate in SI units usually is measured in _____ (Btu/hr, cal/hr).

d. The English units for thermal conductivity are _____ $\left(\frac{cal \cdot cm}{hr \cdot cm^2 \cdot C^\circ}, \frac{Btu \cdot in.}{hr \cdot ft^2 \cdot F^\circ} \right)$.

PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: Given: $R_T = {}^{\Delta T}/_{Q_H}$ (Equation 1).

Find: ΔT.

Solution: Rearrange the equation. Isolate ΔT.

Step 1: First write down the equation.

$$R_T = \frac{\Delta T}{Q_H}$$

Step 2: Multiply both sides by Q_H.

$$R_T \times Q_H = \frac{\Delta T}{\cancel{Q_H}} \times \cancel{Q_H} \quad \text{(Cancel } Q_H \text{ on the right side.)}$$

Step 3: Rewrite the equation without the canceled Q_H terms.

$$R_T \times Q_H = \Delta T \quad (\Delta T \text{ is isolated.})$$

Step 4: Reverse the order of the equation. Put ΔT on the left side.

$$\Delta T = R_T \times Q_H.$$

The problem has been solved. The equation, $R_T = {}^{\Delta T}/_{Q_H}$, has been rearranged. ΔT has been isolated.

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1, Continued

Problem 2: Solved in text.

Problem 3: Step 1: Isolate "Q_H" by rearranging Equation 1. First write down Equation 1.

$$R_{TOT} = \frac{(\Delta T)}{Q_H}$$

Step 2: Multiply both sides by "Q_H." Then divide both sides by R_{TOT}. This is the same as multiplying both sides by **(Q_H/R_{TOT})**.

$$R_{TOT} \times \left(\frac{Q_H}{R_{TOT}}\right) = \frac{(\Delta T)}{Q_H} \times \left(\frac{Q_H}{R_{TOT}}\right)$$ (Cancel R_{TOT}'s on left side and Q_H's on right.)

Step 3: Rewrite the equation without "R_{TOT}'s" and "Q_H's" that canceled out.

$$Q_H = \frac{(\Delta T)}{R_{TOT}}$$ (Q_H is isolated.)

The problem has been solved. Equation 1 has been rearranged, and "Q_H" has been isolated.

Problem 4: Step 1: Isolate "ℓ" by rearranging Equation 2. First write down Equation 2.

$$R_{TOT} = \frac{\ell}{k \times A}$$

Step 2: Multiply both sides by (k x A).

$$R_{TOT} \times (k \times A) = \frac{\ell}{k \times A} \times (k \times A)$$ (Cancel k's and A's on right side.)

Step 3: Rewrite the equation without the "k's" and "A's" that canceled out.

$$R_{TOT} \times k \times A = \ell$$

Step 4: Reverse the order of the equation with ℓ on the left.

$$\ell = R_{TOT} \times k \times A$$

The problem has been solved. Equation 2 has been rearranged, and "ℓ" has been isolated.

Problem 5: Use Problem 2 as a pattern to isolate "A" by the same pattern. The answer is:

$$A = \frac{\ell}{kR_{TOT}}.$$

. . .solutions continued on page T-111a

Problem 2: Given: $R_T = \dfrac{\ell}{kA}$ (Equation 2).

Find: k. (Rearrange Equation 2 and isolate k.)

Solution: (***Hint:*** Multiply both sides by k. Then divide both sides by R_T. This will isolate k when the terms cancel out.)

Step 1: Isolate k by rearranging Equation 2. First write down Equation 2.

$$R_T = \frac{\ell}{kA}$$

Step 2: Multiply both sides by k. Then divide by R_T. This is the same as multiplying both sides by $^{k}/_{R_T}$.

$$\cancel{R_T} \times \left(\frac{k}{\cancel{R_T}}\right) = \frac{\ell}{\cancel{k}A} \times \left(\frac{\cancel{k}}{R_T}\right)$$ (Cancel k's on the right and R_T's on the left.)

Step 3: Rewrite the equation without canceled k and R_T terms.

$$k = \frac{\ell}{AR_T}$$ (The k is isolated.)

The problem has been solved. Equation 2 has been rearranged, and k has been isolated.

Problem 3: Given: $R_T = {}^{\Delta T}/_{Q_H}$ (Equation 1).

Find: Q_H.

Solution:

Problem 4: Given: $R_T = {}^{\ell}/_{kA}$ (Equation 2).

Find: ℓ.

Solution:

Problem 5: Given: $R_T = {}^{\ell}/_{kA}$ (Equation 2).

Find: A.

Solution:

ACTIVITY 2

Solving Thermal-resistance Problems

You'll use Equations 1 and 2 from Activity 1 to solve thermal-resistance problems in this section. Suppose that Equation 1 or 2 isn't in a "useful" form. If not, first rearrange the equation to isolate the symbol that you want to find. Then solve the rearranged equation for the unknown value. Your final answer should include a correct numerical answer with the proper units. Use Table 1 to help check units in the answer.

PRACTICE EXERCISES FOR ACTIVITY 2

Problem 6: Given: The temperature difference (ΔT) between the inside and outside of an electric oven is 300 F°. The heating element must supply 1200 Btu/hr of heat energy to the oven to overcome the heat flow out of the oven.

Find: The thermal resistance of the oven walls.

Solution: (***Hint:*** Use equation $R_T = {}^{\Delta T}/_{Q_H}$.)

SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 2

Problem 6: $R_{TOT} = \dfrac{(\Delta T)}{Q_H}$ where: $(\Delta T) = 300\ F°$, $Q_H = 1200\ Btu/hr$

$$R_{TOT} = \frac{300\ F°}{1200\ Btu/hr}$$

$$= \frac{300}{1200}\ \frac{F°}{Btu/hr}$$

$$R_{TOT} = 0.25\ \frac{F°}{Btu/hr}.$$

Problem 7: From Problem 1, Activity 1--

$(\Delta T) = R_{TOT}Q_H$ where: $R_{TOT} = 0.006\ \dfrac{F°}{Btu/hr}$, $Q_H = 9600\ Btu/hr$

$$(\Delta T) = 0.006\ \frac{F°}{(Btu/hr)} \times 9600\ (Btu/hr)$$

$$(\Delta T) = (0.006 \times 9600)\ \frac{F° \times (Btu/hr)}{(Btu/hr)} \quad \text{(Cancel Btu/hr.)}$$

$$(\Delta T) = 57.6\ F°.$$

Problem 8: From Problem 2, Activity 1--

$k = \dfrac{\ell}{R_{TOT}A}$ where: $\ell = 0.5\ in$, $A = 16\ ft^2$, $R_{TOT} = 19.1 \times 10^{-6}\ \dfrac{F°}{Btu/hr}$

$$k = \frac{0.5\ in}{19.1 \times 10^{-6}\ \dfrac{F°}{(Btu/hr)} \times 16\ ft^2}$$

$$k = \left(\frac{0.5}{19.1 \times 16 \times 10^{-6}}\right) \frac{in}{1} \times \frac{1}{\dfrac{F°}{(Btu/hr)}} \times \frac{1}{16\ ft^2}$$

$$k = (16.36 \times 10^2)\left(\frac{in}{1} \times \frac{(Btu/hr)}{F°} \times \frac{1}{ft^2}\right)$$

(Use rule for handling fractions divided by fractions.)

$$k = 1636\ \frac{Btu \cdot in}{hr \cdot ft^2 \cdot F°}.$$

. . .solutions continued on page T-111c

(See page T-111c for Problems 9 and 10.)

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2, Continued

Problem 9: From Problem 3, Activity 1--

$$Q_H = \frac{\Delta T}{R_{TOT}} \qquad \text{where:} \quad \Delta T = 1000\ C° \quad R_{TOT} = \frac{C°}{\text{kcal/hr}}$$

$$Q_H = \frac{1000\ C°}{50\ \frac{C°}{\text{kcal/hr}}} = \left(\frac{1000}{50}\right)\left(\frac{C°}{1} \times \frac{(\text{Btu/hr})}{C°}\right)$$

$Q_H = 20$ kcal/hr.

SOLUTIONS TO STUDENT CHALLENGE

Problem 10: a. $Q_H = kA\,\frac{\Delta T}{\ell}$ where: $k = 1636\ \frac{\text{Btu}\cdot\text{in}}{\text{hr}\cdot\text{ft}^2\cdot F°}$

$A = 16\ \text{ft}^2$

$\Delta T = 50\ F°$

$\ell = 0.5$ in

$$Q_H = \frac{1636\ \frac{\text{Btu}\cdot\text{in}}{\text{hr}\cdot\text{ft}^2\cdot F°} \times 16\ \text{ft}^2 \times 50\ F°}{0.5\ \text{in}}$$

$$Q_H = \frac{1636 \times 16 \times 50}{0.5}\ \frac{\text{Btu}\cdot\text{in}\cdot\text{ft}^2\cdot F°}{\text{hr}\cdot\text{ft}^2\cdot F°}\ \frac{1}{\text{in}} \quad \text{(Cancel like terms.)}$$

$Q_H = 2.6176 \times 10^6$ Btu/hr.

b. Yes, there is an easier way! Use--

$$Q_H = \frac{\Delta T}{R_{TOT}} \qquad \text{where:} \quad \Delta T = 50\ F° \quad R_{TOT} = 19.1 \times 10^{-6}\ \frac{F°}{\text{Btu/hr}}$$

$$Q_H = \frac{50\ F°}{19.1 \times 10^{-6}\ \frac{F°}{\text{Btu/hr}}} = \left(\frac{50}{19.1 \times 10^{-6}}\right)\left(\frac{F°}{1} \times \frac{(\text{Btu/hr})}{F°}\right)$$

$$Q_H = \frac{50 \times 10^6}{19.1}\ \text{Btu/hr}$$

$Q_H = 2.62 \times 10^6$ Btu/hr.

Problem 7: Given: The thermal resistance (R_T) of a plate of glass is 0.006 $\frac{F^\circ}{Btu/hr}$, and 9600 Btu of heat energy flow through the plate glass every hour.

Find: The temperature difference across the glass that's causing this flow of heat energy.

Solution: (***Hint:*** Rearrange equation, $R_T = {}^{\Delta T}\!/_{Q_H}$, to isolate ΔT. Then solve.)

Problem 8: Given: The thermal resistance of a 16-ft^2 piece of ½-inch thick aluminum is 19.1×10^{-6} $\frac{F^\circ}{Btu/hr}$.

Find: The thermal conductivity constant (k) for the aluminum plate.

Solution: (***Hint:*** Rearrange the equation, $R_T = {}^{\ell}\!/_{kA}$, to isolate k. Then solve.)

Problem 9: Given: A firefighter's suit has a thermal resistance of 50 $\frac{C^\circ}{kcal/hr}$. When exposed to a fire, the temperature difference from outside to inside the suit may be as high as 1000 C°.

Find: The heat-flow rate (Q_H) through the suit at 1000 C°.

Solution: (***Hint:*** Rearrange the equation, $R_T = {}^{\Delta T}\!/_{Q_H}$. Isolate Q_H. Then solve.)

Student Challenge

Problem 10: Given: The conditions of Problem 8 and a constant temperature difference of 50 F° across the material.

Find:

a. The heat-flow rate (Q_H), using the equation $Q_H = \frac{kA\Delta T}{\ell}$.

b. Is there an easier way? If so, solve for Q_H the easiest way.

Solution: