PREPARATORY MATH SKILLS LAB

Lab PMS 6

MATH ACTIVITY

Area and Volume Measurement

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. ***Calculate an area or volume. Give the answer in correct English or SI units.***
2. ***Convert an area or volume in English units to the equivalent area or volume in SI units.***

MATERIALS

For this activity, you'll need a pencil, a straightedge and graph paper.

WHAT IS AREA?

Area is the measure of a flat surface. Suppose you want to find the area of a geometric figure, such as a rectangle. First, you find the number of squares required to cover the surface of the rectangle. The squares are special in that they are 1 unit on a side. The unit may be an inch on a side (English system) or a centimeter on a side (SI). For example, Figure 1 shows a rectangle. The surface has been covered by a number of squares, 1 inch to a side. The rectangle is 5 inches long. It's 4 inches wide. You'll find that it takes 20 squares, each 1 inch on a side, to cover the surface. So you can say that the area of the rectangle is 20 square inches. We write the area as 20 in^2.

The area of a rectangle in square units can be determined without counting squares. All you have to do is multiply the number of units along one edge by the number of the same units along the other edge. This means you multiply LENGTH times WIDTH. The same unit of measure must be used to describe both "length" and "width." Therefore, the surface area for any rectangle is given by the following equation.

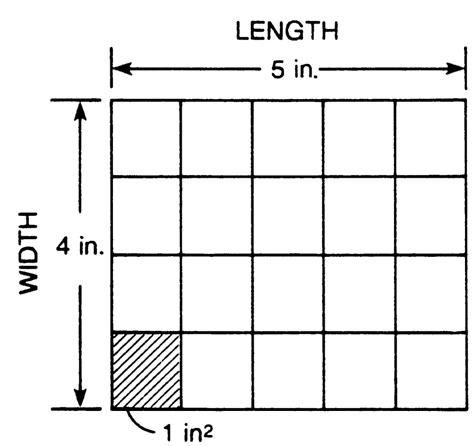


Fig. 1 Finding the area of a rectangle.

$$A = \ell \times w$$

where: A = area in squared units of appropriate measure (in^2, cm^2, ft^2, etc.)
ℓ = length (one side) in units of appropriate measure (in., cm, ft, etc.)
w = width (one side) in same units of appropriate measure (in., cm, ft, etc.)

If we use this formula for the rectangle shown in Figure 1, we get:

$A = \ell \times w$ where: $\ell = 5$ in.
$w = 4$ in.

$A = 5 \text{ in.} \times 4 \text{ in.}$

$A = (5 \times 4)(\text{in.} \times \text{in.})$

$A = 20 \text{ in}^2$ (in. × in. is written as in^2)

So the equation, $A = \ell \times w$, gives the same answer as counting squares: 20 in^2. But notice how much easier and faster it is to multiply 5×4 and get 20 than it is to draw and count 20 squares.

Another common area we often need to know is that of the circle. The area of a circle is given in terms of π and the radius (r). This is shown in Figure 2. The area of a circle is equal to ***π times the radius squared.*** In equation form this is:

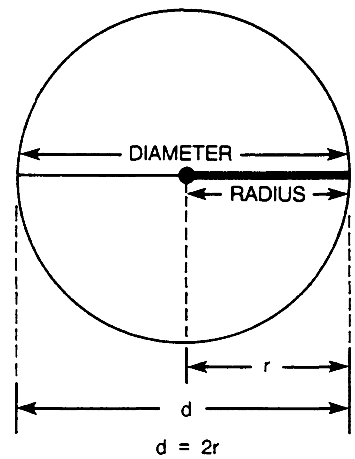


Fig. 2 **Dimensions for finding the area of a circle.**

$$A = \pi \times r^2$$

where: A = the area in squared units of appropriate measure
π = a number equal to 3.1416 (or 3.14 for short)
r = the radius in an appropriate unit of length

The area of a circle, given in terms of diameter (d)—rather than the radius (r)—is:

$$A = \frac{\pi}{4} \times d^2 = \frac{3.1416}{4} \times d^2$$

$$\text{or, } A = 0.7854\, d^2$$

where: d = the diameter in appropriate units of length
$\frac{\pi}{4}$ = 0.7854 (a number)

If the radius in Figure 2 is 2 inches (r = 2 in.), the area is:

$A = \pi r^2 = 3.14 \times (2 \text{ in.})^2 = 3.14 \times 2 \text{ in.} \times 2 \text{ in.} = (3.14 \times 2 \times 2)(\text{in.} \times \text{in.})$

$A = 12.56 \text{ in}^2$

If we use the formula $A = 0.7854\, d^2$, we get:

$A = 0.7854\, d^2$, where $d = 2r$ (twice the radius), or 4 inches

$A = 0.7854 (4 \text{ in.})^2 = 0.7854 \times 4 \text{ in.} \times 4 \text{ in.} = (0.7854 \times 4 \times 4)(\text{in.} \times \text{in.})$

$A = 12.56 \text{ in}^2$ (same answer)

LET'S SUM UP WHAT YOU'VE LEARNED SO FAR

Area of rectangle = $\ell \times w$	where: ℓ = length w = width	
Area of circle = πr^2	where: r = radius of circle π = constant (3.14)	
Area of circle = $0.7854\ d^2$	where: d = diameter of circle	

WHAT IS VOLUME?

Volume is the space a body occupies. Volume is measured in **cubic units**. To find area, as we just learned, we covered a rectangle with special squares which measured 1 unit to a side. This gave us a length (ℓ) and width (w). Then we used ℓ and w in a formula, $A = \ell \times w$, to find the area in squared units.

To find volume, we add a third side, height (h), which changes our ***unit square*** to a ***unit cube,*** one unit to an edge. Figure 3 shows a cube where the unit is 1 inch. Many such unit cubes make up the volume. Suppose the length (ℓ) of our box is 5 inches, the width (w) is 4 inches, and the height (h) is 2 inches. Then what the box shows is an area of 20 in^2 stacked to a height of 2 inches. The volume may be found by counting the cubes or (more easily) by using the formula:

Volume = Length × Width × Height

In symbols, this is written:

$$V = \ell \times w \times h$$

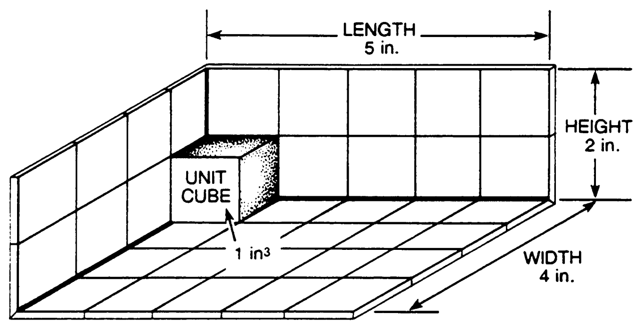


Fig.3 Volume of a rectangular box.

But $A = \ell \times w$. So another way to write this is the following.

Volume = Area × Height or $V = A \times h$	
where: $A = \ell \times w$ = area in units of length squared	
h = height in units of same length	
V = volume in units of length cubed	

Now let's use these formulas to calculate the volume of the box in Figure 3.

$$V = \ell \times w \times h$$
$$V = 5\text{ in.} \times 4\text{ in.} \times 2\text{ in.}$$
$$V = (5 \times 4 \times 2)\ (\text{in.} \times \text{in.} \times \text{in.})$$
$$V = 40\ \text{in}^3$$

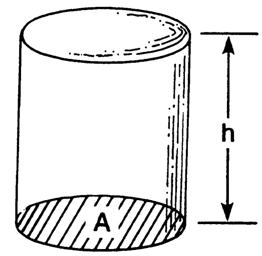
(The measurement "in. × in. × in." is written as "in^3," and is read as "inch cubed" or "cubic inches.")

Using the other form, $V = A \times h$, where $A = 20\ in^2$ and $h = 2$ in., we find:

$$V = 20\ \text{in}^2 \times 2\text{ in.}$$
$$V = (20 \times 2) \times (\text{in}^2 \times \text{in.})$$
$$V = 40\ \text{in}^3 \quad \text{(same answer)}$$

$$\boxed{(\text{in}^2 \times \text{in.}) = (\text{in.} \times \text{in.}) \times \text{in.} = \text{in}^3}$$

When working with cylinders, the volume formula, $V = A \times h$, is very useful. This is because A is the area of the base of the cylinder, a circle. We know that the area of a circle is $A = \pi r^2$. So if we multiply the area of the cylinder base (a circle) by the height of the cylinder, we get the volume of the cylinder.



CYLINDER

Let's calculate the volume of a metal rod that's 10 inches long (height) and 0.80 inch in diameter (0.40-inch radius). First, find A.

$$A = 0.7854\ d^2$$
$$A = 0.7854\ (0.80\ \text{in.})^2$$
$$A = 0.7854 \times 0.80\ \text{in.} \times 0.80\ \text{in.}$$
$$A = (0.7854 \times 0.80 \times 0.80)\ (\text{in.} \times \text{in.})$$
$$A = 0.50\ \text{in}^2$$

So,

$$V = A \times h \qquad \text{where } h = 10\ \text{in. and } A = 0.50\ \text{in}^2$$
$$V = (0.50\ \text{in}^2) \times (10\ \text{in.})$$
$$V = (0.50 \times 10)\ (\text{in}^2 \times \text{in.})$$
$$V = 5\ \text{in}^3$$

The volume of the rod is 5 cubic inches.

HOW DO YOU CONVERT FROM ENGLISH TO SI UNITS?

You worked the previous problems in English units. And just as 1 foot can be converted to 12 inches, you can also change inches to centimeters or meters. In your study of *Principles of Technology*, you'll learn many relations like 1 inch = 2.54 cm or 1 foot = 30.48 cm. You already know you can change back and forth between angles in degrees and angles in radians. You can also change back and forth between in^2 and cm^2 or between in^3 and cm^3. After you learn—or look up—the equation that converts the two units, the rest is easy. Conversion just takes practice.

One thing that's important to remember is that you must always work in ONE system of units. Don't mix SI and English units when you multiply a length times a width. You should be able to look at an area in inches squared and recognize that in SI units that same area could be changed to area units in centimeters squared or meters squared. You could also change a volume in cubic centimeters in SI units to one in cubic inches in English units, if necessary.

1 ft^2	=	144 in^2	1 ft^3	=	1728 in^3
1 in^2	=	6.45 cm^2	1 in^3	=	16.4 cm^3
1 m^2	=	10,000 cm^2	1 m^3	=	1,000,000 cm^3
			1 liter	=	1000 cm^3
			1 U.S. gallon	=	231 in^3
			1 British Imperial gallon	=	277 in^3

Here are some area and volume conversions you may find useful.

PRACTICE EXERCISES

1. Find the ***area*** of a flat piece of aluminum plate, 4 in. by 8 in.
2. Find the ***volume*** of a jewelry box 4 in. by 8 in. by 3 in. deep.
3. You have a cylindrical container that has a radius of 4 inches and a height of 16 inches. How many cubic inches of water can it hold? How many U.S. gallons? How many cm^3?
4. A cylinder in an automobile engine is 3 inches in diameter. What's the area of the piston face in in^2? In cm^2?

