Supplemental Experiment 3
Projectile Motion

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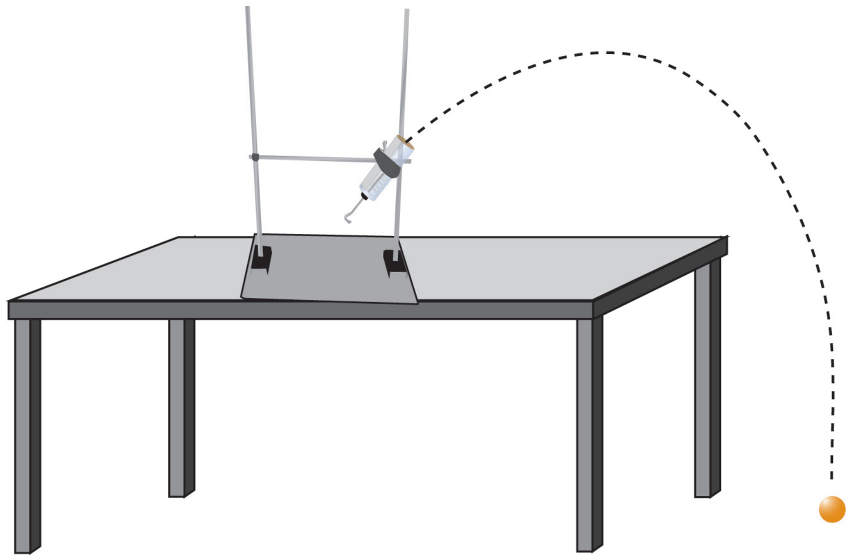


Figure 1
Setup for Supplemental Experiment 3

Supplemental
Experiment 3

Projectile Motion

Experiment Objectives

- Explain how gravity works in projectile motion.
- Apply the concept of vector components to solve projectile motion problems.
- Understand how air resistance affects projectile motion.

Laboratory Proficiencies

- Set up and perform an experiment that will demonstrate projectile motion.
- Measure the weight of the projectile using a triple beam balance.

Discussion

A projectile is any object that is projected by something and continues in motion by its own **inertia**. Projectiles follow curved paths that may seem complicated at first, but are surprisingly simple when the horizontal and vertical components of motion are looked at separately.

The horizontal component of projectile motion is the same as a ball rolling at a constant rate horizontally on a table without friction. The ball covers equal distances in equal intervals of time by its own inertia. It rolls without accelerating because there are no components of force acting in its direction of motion.

Inertia A property of an object by which it resists change in motion.

The vertical component of projectile motion is the applied force acting as it goes up and is the same as a free falling object. Like a free falling object, projectile motion accelerates downward towards the direction of earth's gravity. Gravity increases the speed in the vertical direction which causes greater distances to be covered in equal time intervals. When the gravity vector is in the same direction as the vertical component of the velocity, the velocity increases; however, if the gravity vector is against the velocity vector, then the velocity decreases. The horizontal and vertical components of projectile motion are completely independent of each other. The curved motion is produced by the combination of the two components.

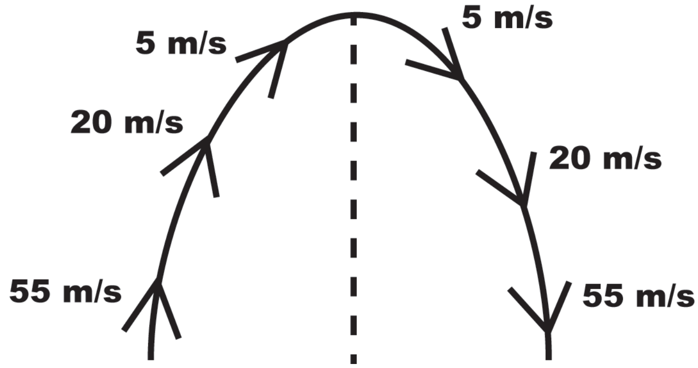
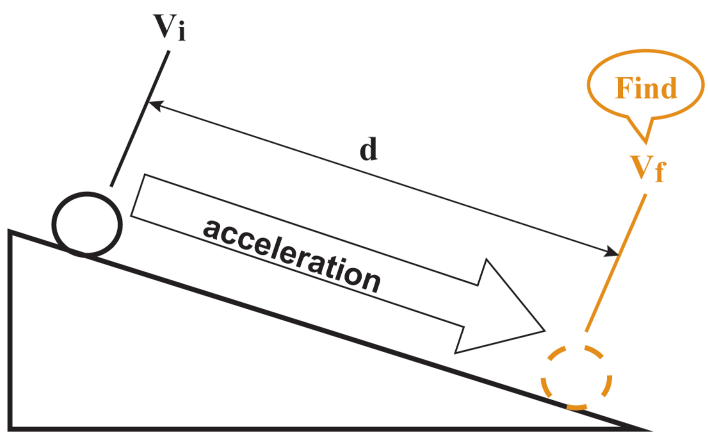


Figure 2
Without air resistance the speed gained while going down equals the speed lost while going up.

The horizontal velocity vector is consistent throughout the motion, only the vertical component changes. At the maximum height, the vertical component reduces to zero. For ideal projectile motions, the path of an object draws a perfect parabola, but in the presence of air resistance, the projectile motion falls short of a parabola. Figure 2 shows the curved path with no air resistance. The velocity lost while going up is equivalent to velocity gained while coming down. This also means that the time up equals the time down.

Basic Formulas of Kinematics

The basic formulas of **kinematics** are given below. If you know the initial velocity, acceleration, and distance, use the formula in Figure 2 to find the final velocity.



$$v_f^2 = v_i^2 + 2ad$$

Where

v_f = Final velocity in meters/second

v_i = Initial velocity in meters/second

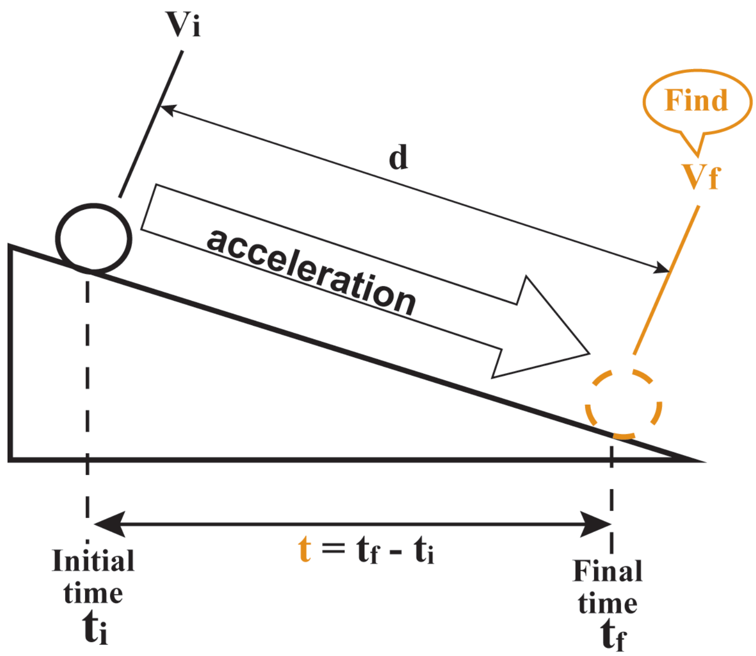
a = Acceleration in meters/second^2

d = Displacement in meters

Figure 3
Finding the final velocity, given the initial velocity, acceleration, and displacement

Kinematics The branch of mechanics that describes the motion of objects without considering the forces that create it.

See Figure 4 to find the final velocity if the initial velocity, acceleration, and time are known.



$$v_f = v_i + at$$

Where

v_f = Final velocity in meters/second

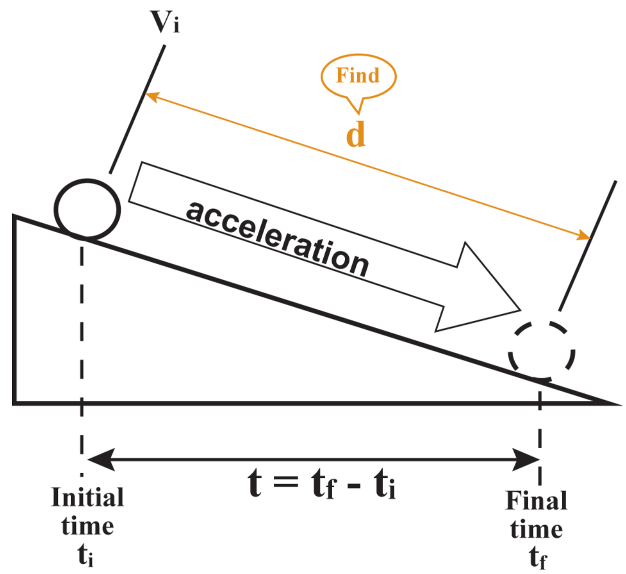
v_i = Initial velocity in meters/second

a = Acceleration in meters/second^2

t = Time in seconds

Figure 4
Finding the final velocity, given the initial velocity, acceleration, and the time in which the acceleration is acting

See Figure 5 to the find the displacement, given the initial velocity, acceleration, and the time of the displacement.



$$d = v_i\, t + 0.5\, at^2$$

Where

d = Displacement in meters

v_i = Initial velocity in meters/second

a = Acceleration in meters/second^2

t = Time in seconds

Figure 5
Finding the displacement, given the initial velocity, acceleration, and the time the acceleration is acting

Equipment and Materials Required

- Balls (2)
- Long Crossbar
- Mechanical Breadboard
- Meter Stick
- Projectile Apparatus
- Protractor
- Rod Connectors (3)
- Stopwatch
- Supports Rods (2)
- Triple Beam Balance

Procedure
Part 1

The students should work in groups of three.

- ❑ 1. Have one person be in charge of rolling a ball off a table. See Figure 6.

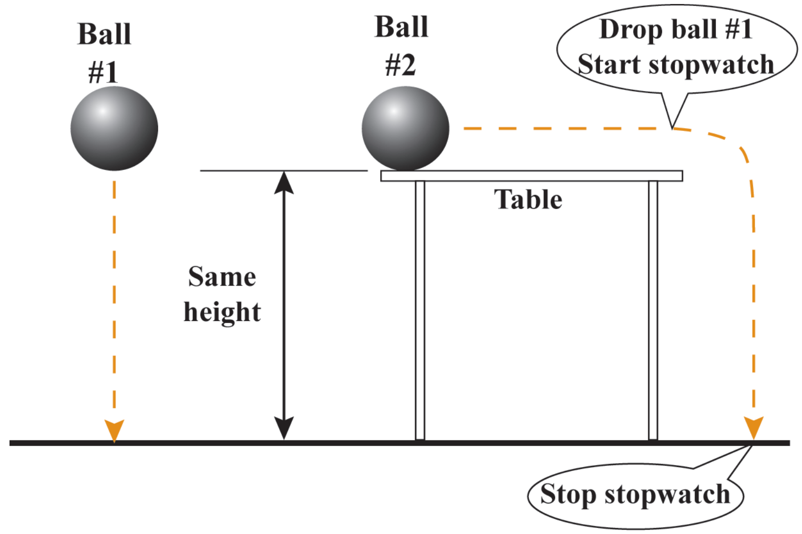


Figure 6
A ball being dropped from a hand verses a ball rolling off a table at the same height

- ❑ 2. Have a second person be in charge of dropping the other ball from the same height as the table. This person should drop the ball as ball #2 reaches the edge of the table.
- ❑ 3. The last person will be responsible for starting and stopping the stopwatch. This person should start the stopwatch as both balls start to fall to the ground and stop it once they both hit the ground.
- ❑ 4. Roll the ball at different speeds and note the time for each try. Record these answers in Data Table 1 of your Student Journal.
- ❑ 5. Did the speed of the rolling ball matter? If so, how? Enter you answer in your Student Journal.

Procedure Part 2

- ❑ 1. Have one person throw a ball straight up and catch the ball where it was released. Refer to Figure 7. Try this a few times before proceeding to the next step.

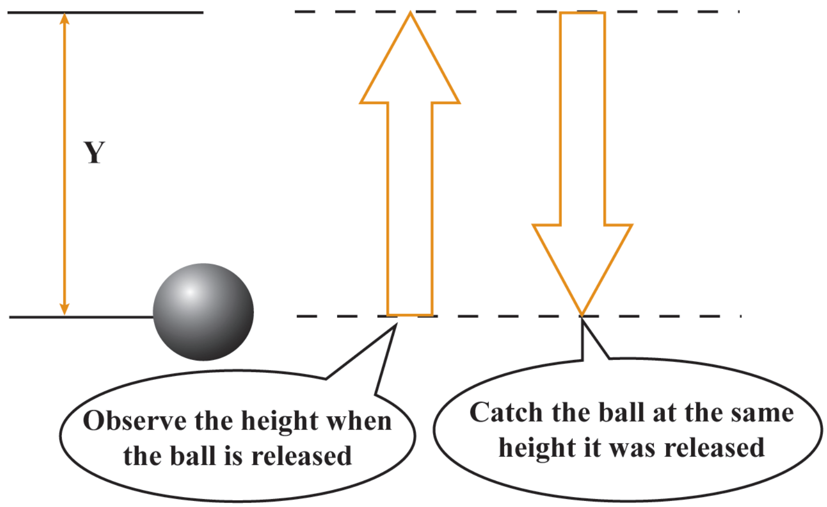


Figure 7
Throwing a ball upwards and catching it at the same spot

- ❑ 2. Have a second person start the stopwatch as the ball is released, and stop the stopwatch as soon as the ball is caught. See Figure 8. Enter the time in Data Table 2.

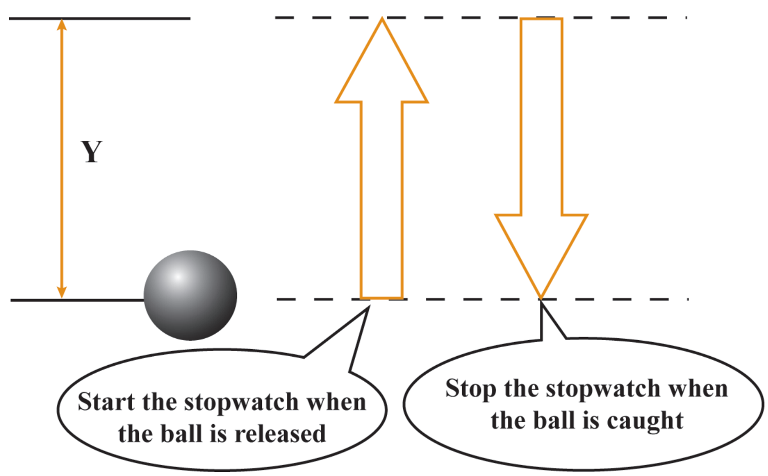


Figure 8
Starting and stopping the stopwatch

❑ 3. To find the height of the ball this problem needs to be divided into two parts. First, the ball is thrown up, and second the ball comes down. In the first or going up part we do not know what the initial velocity is; however, the second or coming down part has more information. See Figure 9.

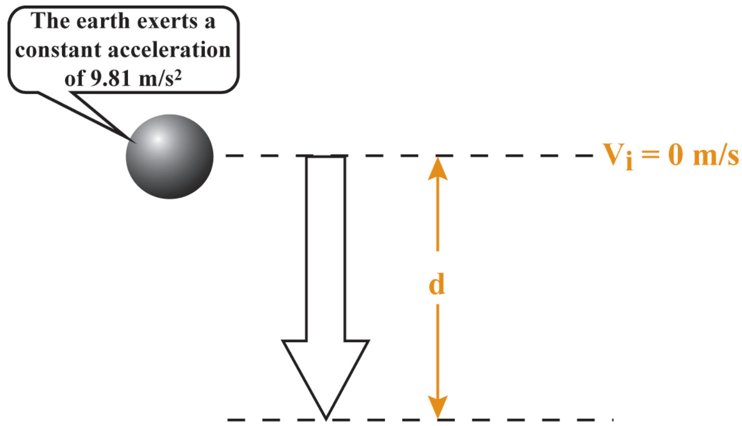


Figure 9
The known information about the ball toss

❑ 4. Of the formulas we have seen, the following formula uses initial velocity and acceleration to find the distance. Enter "d" in Data Table 2.

$$d = v_i\, t + 0.5\, at^2$$

❑ 5. To find the final velocity we have a choice of two formulas:

$$v_f = v_i + at \quad \text{or} \quad v_f^2 = v_i^2 + 2ad$$

Use one of these formulas to find the velocity just before the ball is caught. Enter your answer in Data Table 2.

What is the velocity of ball when it is released from the hand? Enter you answer in Data Table 2.

Procedure
Part 3

❑ 1. Set up the lab equipment as shown in Figure 10. Position the black bases along the second column of holes as Figure 10 shows.

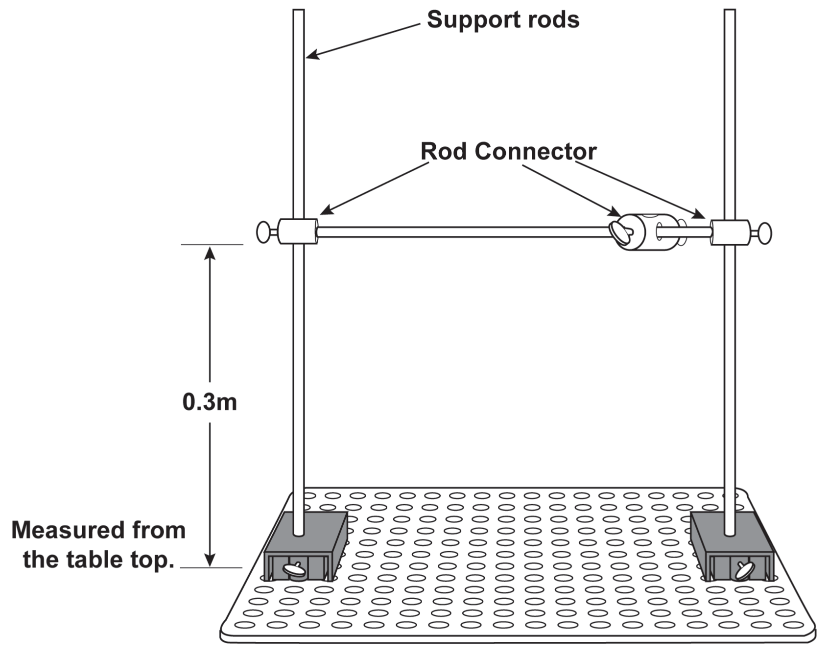


Figure 10
Setup for Part 3

❑ 2. Mount the rod connector 30 cm from the table, not the breadboard.

❑ 3. Insert the projectile apparatus into the middle rod connector.

❑ 4. Adjust the projectile apparatus for a 70° angle to the long crossbar. See Figure 11.

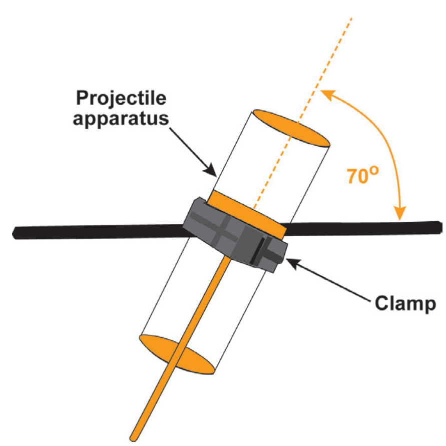


Figure 11
Setting the projection angle

❑ 5. Clamp the projectile apparatus at the 60 mm mark of the scale as shown in Figure 12.

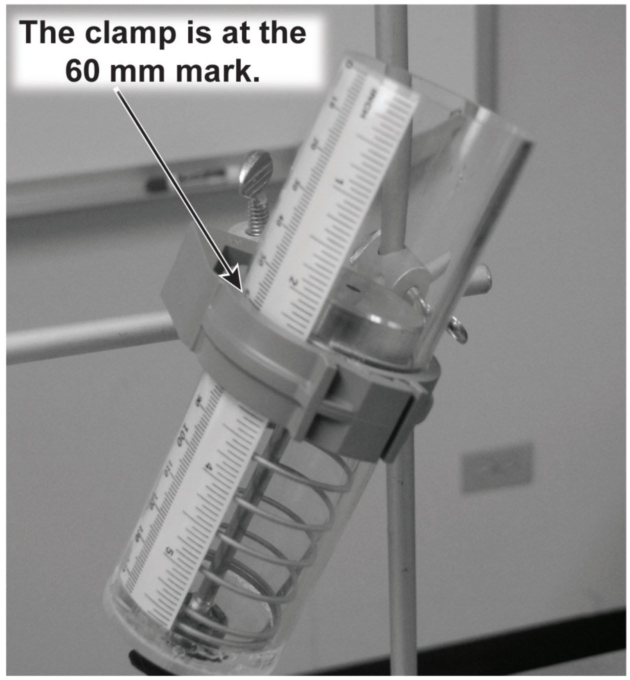


Figure 12
The projectile apparatus clamped at 60 mm

❑ 6. Adjust the rubber stopper until the top of the spring is positioned at 50 mm. See Figure 13.

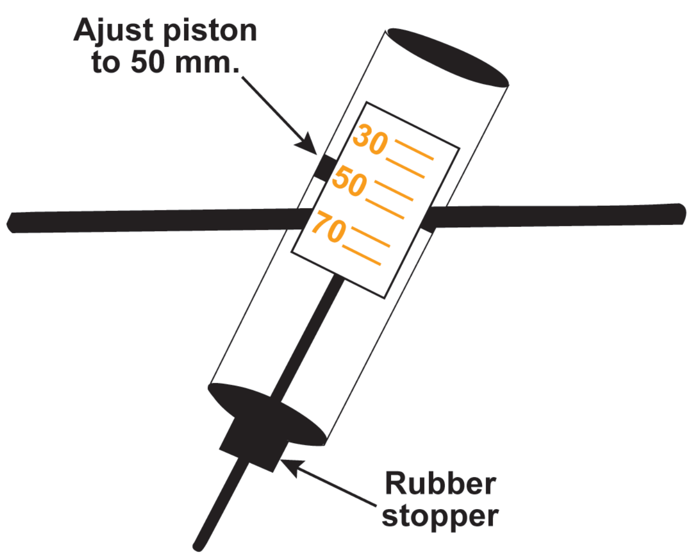


Figure 13
Adjusting the rubber stopper

- ❑ 7. Measure mass of the ball in grams using a triple beam balance. Record the mass in Data Table 3.
- ❑ 8. Measure the height of the table. Record the mass in Data Table 3.
- ❑ 9. Note the position of the piston. Record this value as the initial piston position ℓ_i in Data Table 3. (The value should be near 50.)
- ❑ 10. Hold the projectile apparatus with one hand and pull the piston all the way into the cylinder. Record the value as the final piston position (ℓ_f) in Data Table 3.
- ❑ 11. Orientate the projectile apparatus so the ball will land on the floor.
- ❑ 12. Assign each student to a task.

  A. One student to insert the ball into the projectile apparatus.

  B. One student to measure the time the ball is in flight and to estimate the maximum height the ball is above the table.

  C. One student to observe the spot where the ball lands and to mark that spot on the floor. The horizontal displacement is measured as shown in Figure 14.

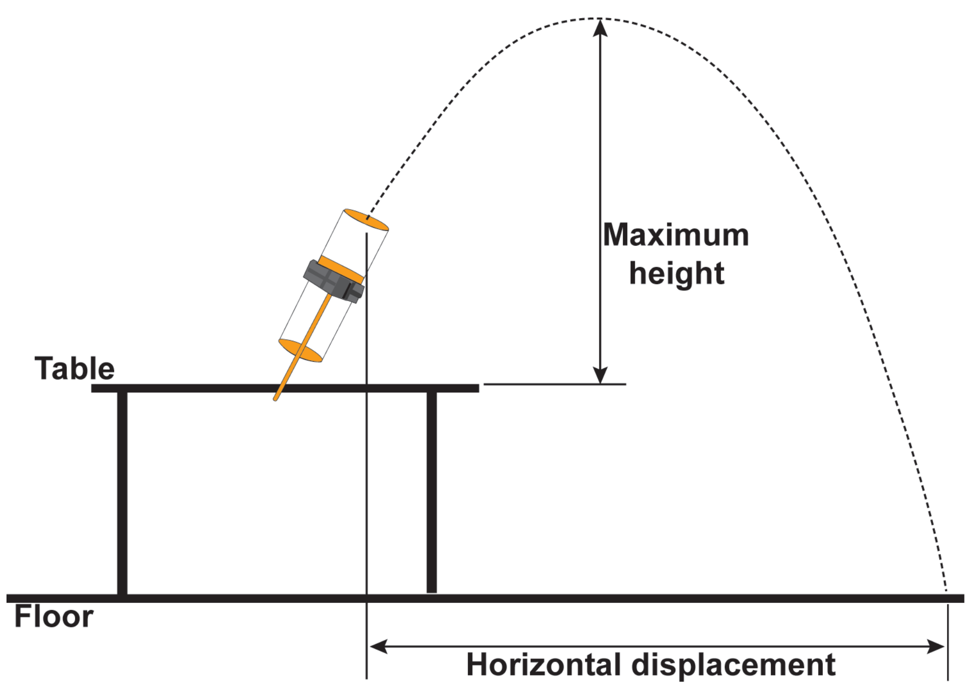


Figure 14
Measuring the horizontal displacement and estimating the maximum height

❑ 13. Place the ball in the projectile apparatus. Hold the projectile apparatus and pull the spring all the way back. With the other students ready for action, release the spring.

❑ 14. Enter the time of flight, the horizontal displacement, and the estimated height above the tablein Data Table 4 .

❑ 15. Repeat steps 12 through 13 two more times.

Observations and Calculations

Enter your answers in the Student Journal.

Finding the Ejection Velocity

❑ 1. From Data Table 4, find the average flight time, the average horizontal displacement, and the average estimated height above the table. Enter your answers in Data Table 4.

❑ 2. In the projectile motion experiment the potential energy of the stretched spring is converted into the kinetic energy given to the ball. This conversion can be stated as:

$$\mathbf{KE_{ball} = PE_{spring}}$$

❑ 3. The equation for the Potential Energy of the spring is:

$$\mathbf{PE_{spring} = {}^{1}/_{2}\, k(\Delta\ell)^2}$$

Where

k = spring constant

$\Delta\ell$ = Initial velocity in meters/second

❑ 4. The kinetic energy developed by the spring is imparted not only to the ball but also to the piston and shaft.

$$\mathbf{m_{piston\ \&\ shaft} = 56.9\ gm}$$

Enter the total mass in Data Table 5.

$$\mathbf{m_{total} = m_{ball} + 56.9\ gm}$$

- ❑ 5. Before we can solve the previous equation we need to convert the total mass of the moving mechanism from grams to kilograms. Enter your answer in Data Table 5.

$$\mathbf{1\ gram = {}^{1}/_{1000}\ kilogram}$$

- ❑ 6. Likewise we need to find the distance the piston moves. Enter your answer in Data Table 5.

$$\Delta\ell = \ell_{\mathbf{f}} - \ell_{\mathbf{i}}$$

- ❑ 7. Convert $\Delta\ell$ from millimeters to meters. Enter your answer in Data Table 5.

- ❑ 8. The value of the spring constant is:

$$\mathbf{k = 170\ N/m}$$

- ❑ 9. If you have a scientific calculator you can now solve this equation for v. To remind us that the mass is the total mass, the equation has been changed to reflect the total mass.

$$\mathbf{{}^{1}/_{2}\ m_{total}v^{2} = {}^{1}/_{2}\ k(\Delta\ell)^{2}}$$

If you do not have a scientific calculator you can use this form of the above equation:

$$v = \sqrt{\frac{k\left(\Delta\ell\right)^{2}}{m_{total}}}$$

Enter the ejection velocity in Data Table 5.

Determining the Final Velocity

❑ 1. To work with the projectile path we need to divide the path into sections that identify the important points. See Figure 15.

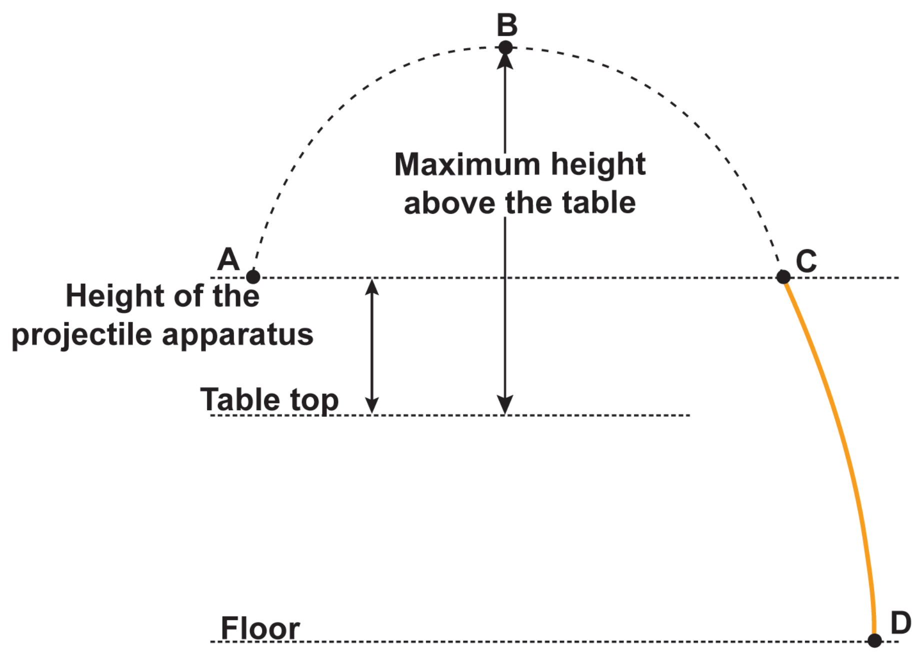


Figure 15
The important points of the projectile path

❑ 2. To find the final velocity of ball prior to hitting the floor at point D, we will work with the projectile path from points C to D. The formula that will gives us the final velocity from points C to D is:

$$v_f^2 = v_i^2 + 2ad$$

Before we can use this equation we need to rework the equation and prepare the data as outlined in the following steps.

❑ 3. At point C the projectile has gained all the velocity it had at point A if we neglect friction. Also the initial projectile velocity is the ejection velocity from the spring.

$$v_{i\ (\text{projectile velocity})} = v_{(\text{ejection velocity})}$$

Enter the initial projectile velocity in Data Table 6.

❑ 4. At point C the angle of the velocity will be the same as the angle at the projectile apparatus. Because of the angle of the projectile velocity at point C, the vertical component is the one to use in the equation. See Figure 16.

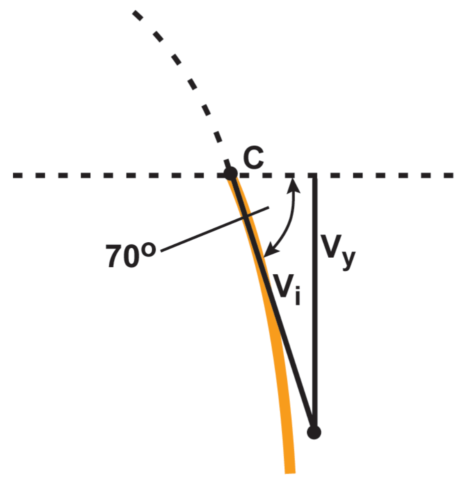


Figure 16
The angle of the velocity determines the vertical component at Point C

❑ 5. Applying trigonometry to point C, the vertical component of vi can be found as follows:

$$\frac{v_y}{v_i} = \sin(70°)$$

$$v_y = v_i \sin(70°)$$

Enter the value of v_Y in Data Table 6 in your Student Journal.

❑ 6. Convert the table height to meters. Enter the table height in Data Table 6.

❑ 7. Convert the height of the projectile apparatus above the table to meters. Enter in Data Table 6.

❑ 8. Find the height between points C and D. Enter it in DataTable 6.

❑ 9. To find the vertical component of the final velocity at point D, we can rewrite the equation in terms of the vertical components:

$$(v_{yfinal})^2 = v_y^2 + 2ay$$

Where

v_{yfinal} = vertical velocity at point D

y = the distance between Points C and D

v_i = v_y (the vertical component of v_i at point C)

a = 9.81 m/s2 (the acceleration due to gravity)

Enter the value of v_{yfinal} in Data Table 6.

❑ 10. To find the final projectile velocity at point D, we need to do vector addition of the vertical and horizontal components. Without friction the horizontal component remains the same through all the horizontal positions.

$$\frac{v_x}{v_i} = \cos(70°)$$

$$v_x = v_i \cos(70°)$$

Enter the value of v_x in Data Table 6.

❑ 11. Use the Pythagorean theorem to find V_f (right before the ball hits the ground). Record this value in Data Table 6.

$$v_f^2 = v_x^2 + v_{yfinal}^2$$

$$v_f = \sqrt{v_x^2 + v_{yfinal}^2}$$

Determine the Maximum Height the Ball Reaches

❑ 1. So far we only know the total flight time. However, we can calculate the time it took for the ball to fall to the ground from point C to point D. See Figure 17. Use the vertical components of the velocities. Enter the value of t(C to D) in Data Table 7 of your Student Journal.

$$v_y + at_{(C\text{ to }D)} = v_{yfinal}^2$$

$$at_{(C\text{ to }D)} = v_{yfinal}^2 - v_y$$

$$t_{(C\text{ to }D)} = \frac{v_{yfinal} - v_y}{a}$$

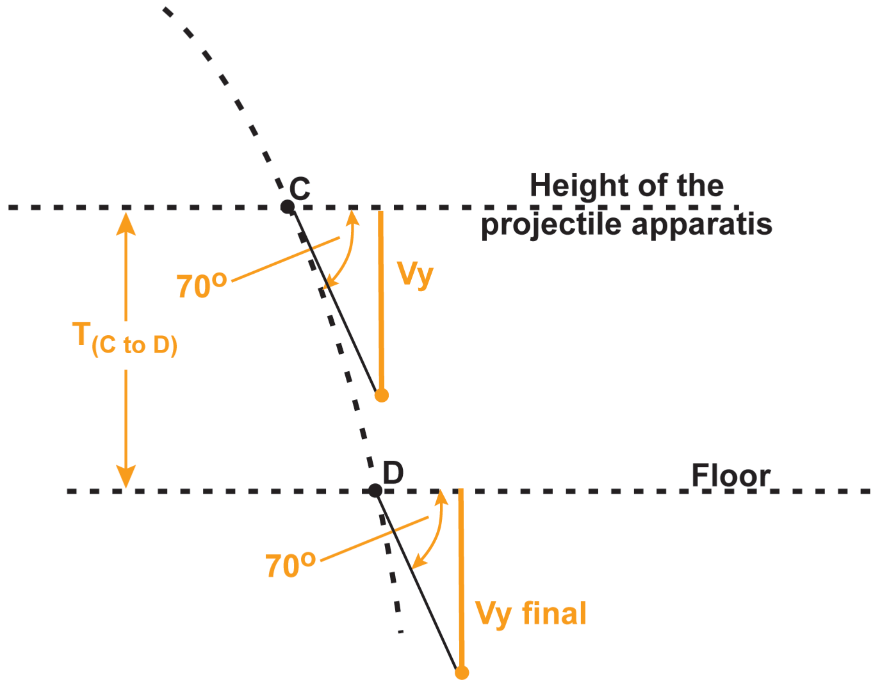


Figure 17
Finding $t_{(C\ to\ D)}$

- ❑ 2. Subtract $t_{(C\ to\ D)}$ from the stopwatch time. The result is $t_{(A\ to\ C)}$, the time for the ball to travel from point A to point C. Enter $t_{(A\ to\ C)}$ in Data Table 7.

$$t_{(A\ to\ C)} = t_{stopwatch} - t_{(C\ to\ D)}$$

- ❑ 3. Figure 18 shows the data needed to find the maximum height that the ball reaches. The time $t_{(A\ to\ B)}$ is one half of the time from point A to point C. Enter $t_{(A\ to\ B)}$ in Data Table 7.

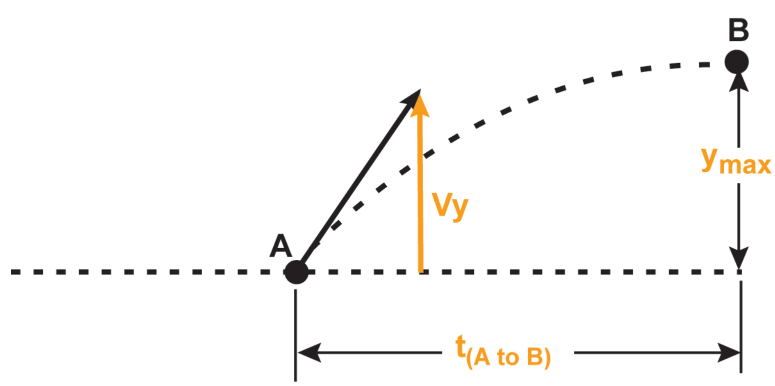


Figure 18
Finding the maximum height

- ❑ 4. Previously we found the vertical component of the ejection velocity. Enter the vertical component v_y in Data Table 7.

- ❑ 5. This is an application for the following formula.

$$\mathbf{d = v_i\, t + 0.5\, at^2}$$

We can rewrite the equation as:

$$\mathbf{y_{max} = v_y\, t + 0.5\, at_{(A\ to\ B)}{}^2}$$

Enter the height y_{max} in Data Table 7.

- ❑ 6. Determine the total height above the table. Enter the value in Data Table 7.

- ❑ 7. Enter the estimated total height above the table in Data Table 7.

- ❑ 8. In your Student Journal list some of the factors that would account for a large difference between the calculated height and the estimated height.

Calculating the Horizontal Displacement

- ❑ 1. Enter the horizontal velocity in Data Table 8 of the Student Journal.

- ❑ 2. Enter the average flight time in Data Table 8.

- ❑ 3. Calculate the displacement based on the measured flight time. Enter your answer in Data Table 8.

$$\textbf{Horizontal displacement} = \mathbf{v_x \times \textbf{flight time}}$$

- ❑ 4. Enter the measured horizontal displacement in meters in Data Table 8.

- ❑ 5. Compare the measured horizontal displacement to the calculated horizontal displacement. Enter your answer in the Student Journal.

Questions and Interpretations

1. A basketball player throws a ball from the same distance to the basket with the same projection velocity. For angles other than 45º there are two possible angles. Will the low angle shot take more or less time than a high angle shot? ____
____
____
____

2. You are playing deep center field and you need to make a throw to home plate. However, you have very little strength. At want angle do you throw the ball to get the most distance? Neglect air resistance? ________________________
____
____
____
____

3. Neglecting air resistance, if somebody dropped a sheet of paper and a bowling ball from a very tall building which would hit the ground first? _______
____
____
____
____

Notes