Math Lab 10 MS 2

Solving Energy-Conversion Problems for Fluid Energy Convertors

For best results, print this document front-to-back and place it in a three-ring binder. Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Review the procedures for converting units in one system to the units of another system.
2. Teach students how to calculate the efficiency of a fluid energy convertor.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to this class. (How much you will accomplish will depend on the math skills your students already have.) Summarize the explanatory material for Activity 1: "Solving Energy-Conversion Problems for Fluid Energy Convertors." Then have students complete the Practice Exercises given at the end of Activity 1.
3. Supervise student progress. Have students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 10*3, "Converting Light to Electricity with Solar Panels."

Math Skills Laboratory

Lab 10 MS 2

MATH ACTIVITY

Activity: ***Solving Energy-Conversion Problems for Fluid Energy Convertors***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve energy-conversion problems for fluid energy convertors.***
2. ***Substitute correct numerical values and units in energy-conversion equations. Solve the equations for an unknown numerical value with the proper units.***
3. ***Find the efficiency of fluid energy convertors.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

ACTIVITY

Solving Energy-Conversion Problems for Fluid Energy Convertors

MATERIALS

For this activity, you'll need a calculator.

In this Math Skills Lab, you'll review the important formulas for work and energy. You'll review the units used to measure work, energy and power. You'll solve problems that involve fluid energy convertors.

NOTE: Tables 1, 2 and 3 are the same as those in Math Skills Lab 10MS1. Students will have to review the contents of the tables in order to answer the 14 short questions that follow the tables. The purpose of the review is to focus your students' attention on basic formulas, symbols and units. This renewed attention--from time to time--will help your students get used to formulas and symbols. It will also help them get used to all of the strange units.

LET'S REVIEW FORMULAS AND UNITS!

Examine Tables 1, 2 and 3. (They are the same as those in Math Skills Lab 10MS1.)

Table 1 lists the important formulas for work and energy that you've already studied. Table 2 sums up common units that you'll use in energy-conversion problems. Table 3 lists some conversions between units.

You'll find these conversions helpful when you solve the problems in this activity.

TABLE 1. REVIEW OF BASIC FORMULAS FOR WORK AND ENERGY

Energy System	Formulas	Definition of Symbols
MECHANICAL		
Linear Work	$W = F \times D$	F = applied force D = distance moved
Rotational Work	$W = T \times \theta$	T = torque applied θ = angular distance rotated
Gravitational Potential Energy	$E_p = w \times h$	w = weight h = height raised above reference level
Elastic Potential Energy	$E_p = \frac{1}{2} kd^2$	k = force or spring constant d = distance spring is stretched or compressed
Linear Kinetic Energy	$E_k = \frac{1}{2} mv^2$	m = mass v = speed
Rotational Kinetic Energy	$E_k = \frac{1}{2} I\omega^2$	I = moment of inertia ω = angular speed
FLUID		
Fluid Work	$W = p \times (\Delta V)$; $W = (\Delta p) \times V$	p; Δp = pressure or pressure difference ΔV; V = volume of fluid moved
Fluid Kinetic Energy	$E_k = \frac{1}{2} (\rho V)v^2$	ρ = mass density V = volume v = speed
ELECTRICAL		
Electrical Work	$W = q \times \Delta V$	q = electrical charge moved ΔV = voltage difference
Electrical Energy	$E_{elec} = P_{elec} \times t$	P_{elec} = electrical power used t = time power is used
THERMAL		
Thermal Energy	$H = mc\Delta T$	m = mass c = specific heat ΔT = temperature difference

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TABLE 2. SUMMARY OF COMMON UNITS USED IN ENERGY CONVERSION CALCULATIONS

Quantity	Symbol	English Unit	SI Unit
Mass	m	slug or (lb·sec^2/ft)	kg
Weight	w	lb	N
Force	F	lb	N
Work	W	ft·lb	N·m or J
Kinetic Energy	E_k	ft·lb	N·m or J
Potential Energy	E_p	ft·lb	N·m or J
Power	P	(ft·lb)/sec; hp	(N·m)/sec or watt
Spring Constant	k	lb/ft; oz/in.	N/m; kgf/m *
Angular Displacement	θ	rad	rad
Angular Speed	ω	rad/sec	rad/sec
Linear Speed	v	ft/sec	m/sec
Mass Density	ρ	slug/ft^3; lbm/ft^3 **	kg/m^3
Moment of Inertia	I	slug·ft^2	kg/m^2
Pressure or Pressure Difference	p; Δp	lb/ft^2; oz/in^2	N/m^2
Electrical Charge	q	—	coulomb (C)
Torque	T	lb·ft;lb·in.	N·m
Fluid Volume	V; ΔV	ft^3	m^3
Electrical Current	I	—	ampere (A)
Electrical Voltage	V; ΔV	—	volt (V)
Electrical Resistance	R	—	ohm (Ω)
Heat Energy	H	Btu	cal
Temperature	T	°F	°C
Temperature Difference	ΔT	F°	C°
Specific Heat	c	$\frac{\text{Btu}}{\text{lb·F°}}$	$\frac{\text{kcal}}{\text{kg·C°}}$

* kgf ⇒ kilogram force
** lbm ⇒ pound mass

TABLE 3. USEFUL CONVERSIONS FOR UNITS

1 slug	=	32 pound mass (32 lbm)
1 kilogram force (kgf)	=	9.8 newtons
1 newton·meter	=	1 joule
1 volt·coulomb	=	1 joule
1 ampere·second	=	1 coulomb
1 radian	=	57.3°
1 horsepower	=	550 ft·lb/sec
1 horsepower	=	746 watts
1 watt	=	1 newton·meter per second
1 watt	=	1 joule per second
1 watt	=	1 volt·ampere
1 watt·sec	=	1 joule = 0.7376 ft·lb
1 kilowatt·hour (kWh)	=	3.6×10^6 joules
1 calorie	=	3.086 foot·pounds
1 calorie	=	4.184 joules
1 Btu	=	778 foot·pounds
1 Btu	=	1055 joules

SOLUTIONS TO EQUATIONS/UNITS QUESTIONS

1. $W = p \times (\Delta V)$ or $W = (\Delta p) \times V$
2. $E_k = 1/2\ (\rho V)v^2$
3. $E_k = 1/2\ I\omega^2$
4. $W = q \times (\Delta V)$
5. $E_{elec} = P_{elec} \times t$
6. Volume
7. fluid convertor energy output, or work done
8. 1 cal = 4.184 joules (from Table 3)

9. $$\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\% = \frac{83.7\ \cancel{J}}{100\ \cancel{J}} \times 100\%$$

   $\text{Eff (\%)} = 0.837 \times 100\%$
   $\text{Eff (\%)} = 83.7\%.$

10. 778 ft•lb = 1 Btu = 1054 J. (See Table 3.)

11. (from Question 10)
    $E_{in} = E_{out} = 1054$ J; therefore, Eff (%) = 100%.

12. Units for "ρ" are kg/m^3.
    Units for "V" are m^3.
    Units for "ρV" are $\left(\frac{kg}{\cancel{m^3}}\cancel{m^3}\right)$ = kg.

13. $1\ \frac{N\cdot m}{sec} = 1$ watt

14. $$\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\% = \frac{890 \text{ watts}}{1000\ \frac{N\cdot m}{sec}} \times 100\% = \frac{890\ \cancel{\text{watts}}}{1000\ \cancel{\text{watts}}} \times 100\%$$

    (since $1\ \frac{N\cdot m}{sec} = 1$ watt)

    $\text{Eff (\%)} = 0.890 \times 100\%$
    $\text{Eff (\%)} = 89\%.$

Use the contents of Tables 1, 2 and 3 to help you answer the following questions.

1. The formula for fluid work is ____.
2. The formula for fluid kinetic energy is ____.
3. The formula for rotational mechanical kinetic energy is ____.
4. The formula for electrical work is ____.
5. The formula for electrical energy used by a device, given the electrical power and time of operation, is ____.
6. The kinetic energy of a moving fluid may be found if you know the mass and speed of the fluid. It may also be found if you know the mass density of the fluid, the ____ of the fluid, and the speed of the fluid.
7. In a fluid energy convertor that is 100% efficient, the fluid kinetic energy of the input equals the ____.
8. A fluid energy convertor changes 100 joules of input fluid energy into 83.7 joules of output rotational mechanical energy. At the same time, 3.896 calories of energy are wasted as heat. One calorie of energy equals ____ joules of energy.
9. The fluid energy convertor in Question 8 has an efficiency of ____ %.
10. A fluid energy convertor changes 778 ft·lb of input fluid energy into 1054 joules of output mechanical energy. A fluid energy of 778 ft·lb equals ____ joules of energy.
11. The fluid energy convertor in Question 10 has an efficiency of ____ %.
12. The quantity ρV can be used for mass in fluid kinetic-energy problems. The SI units of ρ are ____. The SI units of V are ____. Therefore, SI units of ρV are ____.
13. A fluid energy convertor changes 1000 N·m of fluid energy each second into 890 watts of electrical power. One N·m per second equals ____ watt(s).
14. The fluid energy convertor in Question 13 has an efficiency of ____%.

ENERGY-CONVERSION PROBLEMS THAT INVOLVE FLUID ENERGY CONVERTORS

Study the two examples of energy-conversion problems given here. Then solve the ***Practice Exercises*** that follow.

Example A: Efficiency of a Windmill Rotor (Fluid-to-Mechanical Energy Convertor)

Given: A windmill rotor consists of aluminum blades, gears and an output shaft. The windmill extracts 60,000 N·m of energy from the air each second while the wind blows past the blades at a speed of 12.5 m/sec. The windmill output shaft delivers 48,000 J of energy to the input shaft of a water pump.

Find: The efficiency of the windmill rotor as an energy convertor.

Solution: The efficiency of the windmill rotor is given as:

$$\text{Eff}(\%) = \frac{E_{OUT}}{E_{IN}} \times 100\% \qquad \text{where: } E_{OUT} = 48{,}000\text{ J},\ E_{IN} = 60{,}000\text{ N·m or } 60{,}000\text{ J}$$

$$\text{Eff}(\%) = \frac{48{,}000\ \cancel{\text{J}}}{60{,}000\ \cancel{\text{J}}} \times 100\% \qquad \text{(Cancel units.)}$$

$$\text{Eff}(\%) = 0.8 \times 100\% = 80\%$$

Thus, this windmill is 80% efficient.

NOTE: Your students may not be familiar with pressure in units of N/cm^2. The problem solution converts from the less familiar N/cm^2 to the more familiar N/m^2. With access to the *Student Resource Book*, your students can find that 1 psi = $6.895 \times 10^3\ N/m^2$. Then

$$1000 \text{ psi} = 6.895 \times 10^6\ N/m^2 = 6.895 \times 10^6 \frac{N}{\cancel{m^2}} \times \frac{1\ \cancel{m^2}}{10^4\ cm^2} = 689.5\ N/cm^2.$$

This value is close to the 690 N/cm^2 given in the problem as equivalent to 1000 psi.

Example B: Efficiency of a Hydraulic Motor (Fluid-to-Mechanical Energy Convertor)

Given: One-sixth of a gallon (631 cm^3) of hydraulic fluid moves through a hydraulic motor each second at a pressure of 1000 psi (690 N/cm^2). The hydraulic motor converts 3610 J of fluid energy each second to mechanical energy of the motor output shaft.

Find:
a. The energy contained in the fluid that enters and drives the hydraulic motor.
b. The efficiency of the hydraulic motor as an energy convertor. (Neglect any internal friction in the system.)

Solution:
a. Energy in the fluid that enters and drives the motor equals the work done on the fluid. Therefore, use the following formula.

$$E_{IN} = W_{IN} = p \times \Delta V \qquad \text{where:} \quad p = 690\ N/cm^2, \quad \Delta V = 631\ cm^3$$

So,

$$E_{IN} = 690\ N/cm^2 \times 631\ cm^3$$

$$E_{IN} = (690 \times 631)\left[\frac{N}{\cancel{cm^2}} \times cm^{\cancel{3}}\right] \qquad \text{(Cancel units.)}$$

$$E_{IN} = 435{,}390\ N{\cdot}cm$$

Change to N·m by using 1 cm = 10^{-2} m.

$$E_{IN} = 435{,}390\ N{\cdot}\cancel{cm} \times \left[\frac{10^{-2}\ m}{1\ \cancel{cm}}\right] \qquad \text{(Cancel units.)}$$

$$E_{IN} = 4353.9\ N{\cdot}m$$

$$E_{IN} = 4354\ N{\cdot}m \text{ or } 4354\ J \text{ (rounded)} \qquad \text{(Remember 1 N·m = 1 J.)}$$

b. Efficiency of the hydraulic motor equals the mechanical output energy E_{OUT} divided by the total input energy of the fluid E_{IN}.

$$Eff(\%) = \frac{E_{OUT}}{E_{IN}} \times 100\%$$

where: E_{OUT} = 3610 J, the fluid energy converted to mechanical energy

E_{IN} = 4354 J, the total energy of the fluid that enters the hydraulic motor

$$Eff(\%) = \frac{3610\ \cancel{J}}{4354\ \cancel{J}} \times 100\% \qquad \text{(Cancel units.)}$$

$$Eff(\%) = 0.829 \times 100\% = 82.9\%$$

Thus, this hydraulic motor is 82.9% efficient.

Examples A and B deal with two energy convertors—each of which converts fluid energy to rotational mechanical energy. The windmill's mechanical energy output is an intermediate stage in energy conversion. That's because the rotational mechanical energy becomes input energy for conversion into electrical energy in a generator—or fluid energy in a pump.

The mechanical energy output from the hydraulic motor described in Example B most often is used to do rotational mechanical work. A hydraulic-powered gearbox or winch is a good example of uses for hydraulic motor output energy.

SOLUTIONS TO PRACTICE EXERCISES

Problem 1:

$$\text{Water Turbine Eff (\%)} = \frac{\text{Rotational Mech Energy of Output Shaft}}{\text{Fluid Kinetic Energy of Input Water}} \times 100\%$$

$$\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\% \quad \text{where: } E_{out} = 2800 \text{ Btu}$$

$$E_{in} = 2{,}341{,}248 \text{ ft}\cdot\text{lb} \approx 2.34 \times 10^6 \text{ ft}\cdot\text{lb}$$

Since energy values aren't in the same units, efficiency can't be calculated. Therefore, change "E_{out} = 2800 Btu" to "ft•lb." Use Table 3.

$$E_{out} = 2800 \text{ Btu} \times 778 \frac{\text{ft}\cdot\text{lb}}{1 \text{ Btu}} = (2800 \times 778) \frac{\cancel{\text{Btu}}\cdot\text{ft}\cdot\text{lb}}{\cancel{\text{Btu}}}$$

$$E_{out} = 2{,}178{,}400 \text{ ft}\cdot\text{lb} \approx 2.18 \times 10^6 \text{ ft}\cdot\text{lb}.$$

Then,

$$\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\%$$

$$\text{Eff (\%)} \approx \frac{2.18 \times 10^6 \cancel{\text{ft}\cdot\text{lb}}}{2.34 \times 10^6 \cancel{\text{ft}\cdot\text{lb}}} \times 100\% \approx \frac{2.18 \times 10^6}{2.34 \times 10^6} \times 100\%$$

$$\text{Eff (\%)} \approx 0.93 \times 100\%$$

$$\text{Eff (\%)} \approx 93\%.$$

Problem 2:

$$\text{Electrical Generator Efficiency (\%)} = \frac{\text{Electrical Energy Output}}{\text{Generator Shaft Input}} \times 100\%$$

$$\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\% \quad \text{where: } E_{out} = 2.8 \times 10^6 \text{ J}$$

$$E_{in} = 2800 \text{ Btu (from Problem 1)}$$

Again, energy values aren't given in the proper units. "Eff" can't be calculated. Therefore, change "E_{in} = 2800 Btu" to "joules." Use Table 3.

(**Note:** You could change "$E_{out} = 2.8 \times 10^6$ J" to "Btu." The answer would be the same. Units cancel in efficiency problems. That's because the "E_{out}/E_{in}" portion is always a ratio.)

$$E_{in} = 2800 \text{ Btu} \times \frac{1054 \text{ J}}{\text{Btu}} = (2800 \times 1054) \frac{\cancel{\text{Btu}}}{\cancel{\text{Btu}}} \text{ J}$$

$$\approx 2{,}951{,}200 \text{ J} \simeq 2.95 \times 10^6 \text{ J}$$

$$\text{Eff (\%)} \approx \frac{2.8 \times 10^6 \cancel{\text{J}}}{2.95 \times 10^6 \cancel{\text{J}}} \times 100\% = \left(\frac{2.8 \times 10^6}{2.95 \times 10^6}\right) 100\%$$

$$\text{Eff (\%)} \approx 0.95 \times 100\%$$

$$\text{Eff (\%)} \approx 95\%.$$

Problem 3: $\text{Eff (\%)} = \dfrac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = P_{out} \times t$

$t = 1$ sec

Eff (%) = 78%

Solve for "E_{in}." Rearrange equation to isolate "E_{in}." Multiply each side of the efficiency equation by [E_{in}/Eff (%)]. Cancel appropriately. Rearrange the equation to get:

$$E_{in} = \frac{(P_{out} \times t) \times 100\%}{\text{Eff\%}}$$

$$E_{in} = \frac{(50 \frac{\text{ft}\cdot\text{lb}}{\text{sec}} \times 1 \text{ sec})}{78\%} \times 100\% = \frac{50 \times 100\cancel{\%}}{78\cancel{\%}} \frac{\text{ft}\cdot\text{lb}\cdot\cancel{\text{sec}}}{\cancel{\text{sec}}}$$

$E_{in} = 64$ ft·lb.

Problem 4: $\text{Eff (\%)} = \dfrac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = 2.8 \times 10^4$ J

E_{in} = kinetic energy contained in fluid

a. $E_{in} = W_{in} = P \times \Delta V$

where: $p = 2 \times 10^3 \text{ N/cm}^2$

$\Delta V = 1575 \text{ cm}^3$

$$E_{in} = 2 \times 10^3 \frac{\text{N}}{\text{cm}^2} \times 1575 \text{ cm}^3 = 2 \times 1575 \times 10^3 \frac{\text{N}\cdot\text{cm}^{\cancel{3}}}{\cancel{\text{cm}^2}}$$

$E_{in} = 3.15 \times 10^6$ N·cm

Change 3.15×10^6 N·cm to N·m. There are 10^{-2} m per cm.

$$E_{in} = 3.15 \times 10^6 \text{ N}\cdot\text{cm} \times \frac{10^{-2} \text{ m}}{1 \text{ cm}} = 3.15 \times 10^{6-2} \frac{\text{N}\cdot\cancel{\text{cm}}\cdot\text{m}}{\cancel{\text{cm}}}$$

$E_{in} = 3.15 \times 10^4$ N·m $= 3.15 \times 10^4$ J. (1 N·m = 1 J)

b. $$\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\% = \frac{2.8 \times 10^4 \cancel{\text{J}}}{3.15 \times 10^4 \cancel{\text{J}}} \times 100\%$$

$$\text{Eff (\%)} = \left(\frac{2.8 \times 10^{4-4}}{3.15}\right) \times 100\%$$

Eff (%) = 0.889 x 100%

Eff (%) = 88.9%

SOLUTIONS TO PRACTICE EXERCISES, Continued

Problem 5: $\text{Eff}\ (\%) = \dfrac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = 30 \times 10^4$ ft•lb

$E_{in} = 400$ Btu

Change 400 Btu to ft•lb to get "E_{out}" and "E_{in}" in same units. Use Table 3.

$$E_{in} = 400 \text{ Btu} \times \frac{778 \text{ ft•lb}}{1 \text{ Btu}} = \left(\frac{400 \times 778}{1}\right) \frac{\cancel{\text{Btu}}\text{•ft•lb}}{\cancel{\text{Btu}}} = 311{,}200 \text{ ft•lb}$$

$E_{in} = 31.12 \times 10^4$ ft•lb.

$$\text{Eff}\ (\%) = \frac{E_{out}}{E_{in}} \times 100\% = \frac{30 \times 10^4 \cancel{\text{ft•lb}}}{31.12 \times 10^4 \cancel{\text{ft•lb}}} \times 100\%$$

$$\text{Eff}\ (\%) = \frac{30 \times 10^{4-4}}{31.12} \times 100\% = \frac{30}{31.12} \times 100\%$$

$\text{Eff}\ (\%) = 0.964 \times 100\%$

$\text{Eff}\ (\%) = 96.4\%$

PRACTICE EXERCISES

Problem 1: Given: Over a certain period of time, a dam releases 2,341,248 ft·lb of fluid kinetic energy onto the wheel of a water turbine. The turbine output shaft is coupled to an electric power generator input shaft. During the same time period, the water turbine changes the fluid kinetic energy into 2800 Btu of output mechanical energy to the input shaft of the generator.

(***Note:*** The output mechanical energy normally **wouldn't be rated in Btu.** We're using that unit in this problem to give you practice with conversion of energy units.)

Find: The water turbine efficiency.

Solution: (***Hint:*** Change all energy values into the same units.)

Problem 2: Given: The electric generator in Problem 1 produces 2.80×10^6 J of electrical energy from the input mechanical energy it receives.

Find: The electric generator efficiency.

Solution:

Problem 3: Given: A vane-type air motor is chosen to operate a paint stirrer. The motor is rated at a shaft power of 50 ft·lb/sec. The efficiency of the pneumatic-to-rotational mechanical energy conversion is 78%.

Find: The fluid energy converted into rotational mechanical energy.

Solution: (***Hint:*** Power = Energy/Time; so Energy = Power × Time, or E = P × t.)

Problem 4: Given: The fluid volume displacement (movement) through a hydraulic gear motor is 1575 cm^3 each second at a constant pressure of 2×10^3 N/cm^2. The gear motor converts 2.8×10^4 J of fluid energy each second to output shaft mechanical energy.

Find:

a. The kinetic energy contained in the fluid that enters the gear motor.

b. The efficiency of the gear motor.

Solution: (***Hint:*** Model solution after Example B.)

Problem 5: Given: Aircraft carriers use steam catapults to launch planes. One of the catapults extracts 400 Btu of energy from superheated steam each second during a launch. The catapult delivers 30×10^4 ft·lb of thrust energy to the plane during each second of the launch.

Find: The efficiency of the steam catapult.

Solution:

SOLUTION TO STUDENT CHALLENGE PROBLEM

Problem 6: $\text{Eff}\ (\%) = \frac{E_{out}}{E_{in}} \times 100\%$ where: E_{in} = energy contained in the steam

$E_{out} = 46.363 \times 10^6$ J

E_{in} = heat to raise steam from 212°F to 970°F

$E_{in} = mc\Delta T$ where: $m = 62.4$ lbm

$c = 1$ Btu/lbm•F°

$\Delta T = 970°F - 212°F = 758$ F°

$$E_{in} = 62.4 \text{ lbm} \times 1 \frac{\text{Btu}}{\text{lbm}\cdot\text{F°}} \times 758 \text{ F°}$$

$$E_{in} = (62.4 \times 1 \times 758)\left(\frac{\cancel{\text{lbm}}\cdot\text{Btu}\cdot\cancel{\text{F°}}}{\cancel{\text{lbm}}\cdot\cancel{\text{F°}}}\right)$$

E_{in} = 47,299 Btu. (This answer has been rounded-off.)

In units of joules, "E_{in}" is equal to:

$$E_{in} = 47{,}299 \text{ Btu} \times 1054 \frac{\text{J}}{\text{Btu}} = (47{,}299 \times 1054)\left(\cancel{\text{Btu}} \times \frac{\text{J}}{\cancel{\text{Btu}}}\right)$$

$E_{in} = 49.85 \times 10^6$ J.

Then,

$\text{Eff}\ (\%) = \frac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = 46.363 \times 10^6$ J

$E_{in} = 49.85 \times 10^6$ J

$$\text{Eff}\ (\%) = \left(\frac{46.363 \times 10^6 \cancel{\text{J}}}{49.85 \times 10^6 \cancel{\text{J}}}\right) \times 100\%$$

Eff (%) = 93%.

Student Challenge

Problem 6: Given: A mass of steam equal to 62.4 lbm at a temperature of 212°F is superheated to steam at 970°F and 4500 psi. All of the energy gained by the steam is extracted by a steam turbine. The steam returns to the boiler condenser line, cooled back down to 212°F. The steam turbine develops 46.363×10^6 J of output mechanical energy in the turbine shaft.

(***Note:*** The specific heat of steam is nearly 1 Btu/lbm·F° at pressures as high as 4500 psi.)

Find: The efficiency of the turbine.

Solution: