Math Skills Laboratory

Lab 5 MS 2

MATH ACTIVITIES

Activity 1: Reviewing Examples of Kinetic Energy Problems

Activity 2: Solving Practical Problems Involving Kinetic Energy and Work in Mechanical and Fluid Energy Systems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Rearrange the equation to solve for mass (m), or linear speed (v), when given the equation for linear kinetic energy, $E_k = \frac{1}{2} mv^2$.***
2. ***Rearrange the equation to solve for moment of inertia (I), or rotational speed (ω), when given the equation for rotational kinetic energy, $E_k = \frac{1}{2} I\omega^2$.***
3. ***Substitute correct numerical values and units in energy equations. Solve the equations for a numerical value with the proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Examples A, B and C.***
3. ***Work the problems.***

ACTIVITY 1

Reviewing Examples of Kinetic Energy Problems

MATERIALS

For this activity, you'll need a calculator.

During classroom discussions, you learned some general methods of "storing" kinetic energy. You also learned that stored potential energy can be converted to kinetic energy to do work. Activity 1 of this Math Skills Lab explains methods that you can use to solve technical problems. In Activity 2, you'll solve problems similar to those a technician might have to solve.

In this lab, you'll solve problems that involve linear and rotational kinetic energy. You'll also use equations for the moment of inertia for the particular shape of object involved. Moment of inertia equations for various shapes are given in Figure 5-13 of the text.

Let's look at the relationship of various physical quantities in the linear kinetic energy equation and the rotational kinetic energy equation.

Linear Kinetic Energy = ½ Mass × $(\text{Speed})^2$

This relationship helps us determine the value of one physical quantity if we know the value and units of the other two physical quantities. The relationship is often written with symbols rather than words, as follows:

$$E_k = \tfrac{1}{2} mv^2 \qquad \textbf{\textit{Equation 1}}$$

where: E_k = linear kinetic energy (in ft·lb or N·m)
m = mass (in kg or $^W\!/_g$ in $\text{lb}\cdot\text{sec}^2/\text{ft}$)
v = linear speed (ft^2/sec^2 or m^2/sec^2)

Equation 1 describes the linear kinetic energy of an object in terms of the object's mass and speed. Equation 2 describes the rotational kinetic energy of an object in terms of the moment of inertia of a spinning mass, and its rotational speed. This equation can be written as:

Rotational Kinetic Energy = ½ Moment of Inertia × $(\text{Angular Speed})^2$

This relationship helps us find the value of one physical quantity if we know the value and units of the other physical quantities. The relationship often is written with symbols rather than words, as follows:

$$E_k = \tfrac{1}{2} I\omega^2 \qquad \textbf{\textit{Equation 2}}$$

where: E_k = rotational kinetic energy (ft·lb, in·lb or N·m)
I = moment of inertia ($\text{ft}\cdot\text{lb}\cdot\text{sec}^2$ or $\text{kg}\cdot\text{m}^2$)
ω = rotational speed (rad/sec)

Table 1 sums up the units used with each of the physical quantities given in Equations 1 and 2. Study Table 1. Compare the units for each system.

TABLE 1. KINETIC ENERGY UNITS

	System of Units: English	System of Units: SI
Equation 1: $E_k = \frac{1}{2} mv^2$		
E_k	ft·lb	N·m, J, or $\frac{kg \cdot m^2}{sec^2}$
m	$\frac{w}{g} = \frac{lb}{ft/sec^2} = \frac{lb \cdot sec^2}{ft}$	kg
v	ft/sec	m/sec
v^2	ft^2/sec^2	m^2/sec^2
Equation 2: $E_k = \frac{1}{2} I\omega^2$		
E_k	ft·lb	N·m, or $\frac{kg \cdot m^2}{sec^2}$
I	$ft \cdot lb \cdot sec^2$	$kg \cdot m^2$
ω	rad/sec	rad/sec
ω^2	rad^2/sec^2	rad^2/sec^2

LET'S REVIEW UNITS!

Use Table 1 to answer the following questions. Fill in the blank with the correct word or words.

a. In the English system, linear kinetic energy is measured in units of ______ (ft·lb, N·m).

b. In the English system, rotational kinetic energy is measured in units of ______ (ft·lb, N·m).

c. In the rotational kinetic energy equation, the moment of inertia has units of ______ (N·m, $kg \cdot m^2$) in SI units.

d. In the English system of units, the quantity (I) is measured in ______ ($ft \cdot lb \cdot sec^2$, $kg \cdot m^2$).

PRACTICE EXERCISES FOR ACTIVITY 1

Example A: *Kinetic Energy of a Falling Object*

Note: The kinetic energy of a moving object is related to the mass and linear speed of the moving object.

Given: A construction elevator raises an 800-lb load of bricks from the ground to the third floor of a building. The third floor is 24 ft above the ground. The load of bricks on the elevator becomes unbalanced. The load falls to the ground. When the bricks hit the ground, they're falling with a speed of 39.19 ft/sec.

Find: Total kinetic energy of bricks at the instant they hit the ground.

Solution: In equation form, the linear kinetic energy is:

$$E_k = \frac{1}{2} mv^2$$

where: E_k = linear kinetic energy

$$m = \frac{w}{g} = \frac{800 \text{ lb}}{32 \text{ ft/sec}^2} = 25 \frac{\text{lb} \cdot \text{sec}^2}{\text{ft}}$$

$$v = 39.19 \text{ ft/sec}$$

Substitute values into the equation.

$$E_k = \tfrac{1}{2}\left[25\frac{\text{lb}\cdot\text{sec}^2}{\text{ft}}\right]\left[39.19\ \frac{\text{ft}}{\text{sec}}\right]^2$$

$$E_k = (\tfrac{1}{2})\,(25)\,(1536)\left[\frac{\text{lb}\cdot\cancel{\text{sec}^2}}{\cancel{\text{ft}}}\right]\left[\frac{\cancel{\text{ft}^2}}{\cancel{\text{sec}^2}}\right] \qquad \text{(Cancel units.)}$$

$$E_k = 19{,}200 \text{ ft}\cdot\text{lb (kinetic energy at moment of impact)}$$

Note: When the bricks hit the ground, their total kinetic energy (19,200 ft·lb) is the same as the stored potential energy (19,200 ft·lb) the bricks had at the 24-ft height. This makes sense. The falling bricks simply returned the energy that was expended (work done) in lifting them.

Example B: *Kinetic Energy of a Flywheel*

Note: The kinetic energy of an object rotating about its axis is related to its moment of inertia and its angular speed.

Given: The flywheel on an automobile weighs 35 pounds and has a diameter of 14 inches. The engine turns at 2200 rpm.

Find: Total kinetic energy of the rotating flywheel.

Solution: First, you must find the moment of inertia (I) of the flywheel. (See Figure 5-13.) An automotive flywheel has the shape of a solid cylinder. Thus, according to Figure 5-13;

$$I = \tfrac{1}{2}\,mr^2$$

$$\text{where:}\quad m = \frac{w}{g} = \frac{35 \text{ lb}}{32 \text{ ft/sec}^2} = 1.094\,\frac{\text{lb}}{\text{ft/sec}^2}$$

$$r = \text{radius} = \frac{d}{2} = \frac{14 \text{ in.}}{2} = 7 \text{ in. or } 0.583 \text{ ft}$$

Substitute values and solve for I.

$$I = \tfrac{1}{2}\left[1.094\ \frac{\text{lb}}{\text{ft/sec}^2}\right](0.583 \text{ ft})^2$$

$$I = \tfrac{1}{2}\,(1.094)\ (0{,}583)^2\ \frac{\text{lb}}{\text{ft/sec}^2} \times \text{ft}^2$$

$$I = 0.186 \text{ lb}\cdot\frac{\text{sec}^2}{\cancel{\text{ft}}}\cdot\text{ft}^{\cancel{2}} \qquad \text{(Cancel like units.)}$$

$$I = 0.186 \text{ lb}\cdot\text{ft}\cdot\text{sec}^2$$

Next, you must convert the engine speed from rpm to rad/sec.

$$\omega = 2200\ \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\left[\frac{1\ \cancel{\text{min}}}{60 \text{ sec}}\right]\left[\frac{6.28 \text{ rad}}{\cancel{\text{rev}}}\right] \qquad \text{(Cancel like units.)}$$

$$\omega = 230 \text{ rad/sec}$$

By substituting the values of I and ω, you can now solve the equation for kinetic energy.

$$E_k = \tfrac{1}{2}\,I\omega^2$$

$$E_k = \tfrac{1}{2}\,(0.186 \text{ ft}\cdot\text{lb}\cdot\text{sec}^2)\,(230 \text{ rad/sec})^2$$

$$E_k = \tfrac{1}{2}\ (0.186)\cdot(230)^2\ \text{ft}\cdot\text{lb}\cdot\cancel{\text{sec}^2} \times \frac{\text{rad}^2}{\cancel{\text{sec}^2}} \qquad \text{(Cancel like units. Drop rad}^2\text{.)}$$

$$E_k = 4920 \text{ ft}\cdot\text{lb}$$

The 35-lb, 14-in.-diameter flywheel, turning at 2200 rpm, develops 4920 ft·lb of rotational kinetic energy.

Example C: *Withdrawal of Kinetic Energy from a Flywheel*

Note: The kinetic energy stored in a rotating flywheel may be withdrawn to do work.

Given: A commercial building contractor has a portable generator equipped with a flywheel. The flywheel has a total mass of 50 kg. It's a hollow cylinder, with inner radius r_1 = 20 cm and outer radius r_2 = 30 cm. The normal operating speed of the generator is 1500 rpm. Many machines with electric motors are connected to the generator. The rotational speed of the generator slows down to 1250 rpm when all of the machines are operating at the same time.

Find: The amount of energy removed from the flywheel to operate the motors.

Solution: First, find the moment of inertia of the flywheel. (See Figure 5-13.) This flywheel has the shape of an annular (ring) cylinder. Therefore:

$$I = \tfrac{1}{2} m (r_1^2 + r_2^2)$$

where:

$$m = 50 \text{ kg}$$
$$r_1 = 20 \text{ cm} = 0.2 \text{ m}$$
$$r_2 = 30 \text{ cm} = 0.3 \text{ m}$$
$$I = \tfrac{1}{2} (50 \text{ kg}) [(0.2 \text{ m})^2 + (0.3 \text{ m})^2] = 25\,(0.13) \text{ kg·m}^2$$
$$I = 3.25 \text{ kg·m}^2$$

Next, convert the rotational speed in rpm to rad/sec.

Initial angular speed:

$$\omega_i = 1500 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \left(\frac{1 \cancel{\text{min}}}{60 \text{ sec}}\right) \left(\frac{6.28 \text{ rad}}{\cancel{\text{rev}}}\right) \qquad \text{(Cancel like units.)}$$
$$\omega_i = 157 \text{ rad/sec}$$

Final angular speed:

$$\omega_f = 1250 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \left(\frac{1 \cancel{\text{min}}}{60 \text{ sec}}\right) \left(\frac{6.28 \text{ rad}}{\cancel{\text{rev}}}\right) \qquad \text{(Cancel like units.)}$$
$$\omega_f = 130.8 \text{ rad/sec}$$

Now, use the moment of inertia, the initial angular speed and the final angular speed to determine the amount of kinetic energy removed from the flywheel.

$$E_k \text{ (initial)} = \tfrac{1}{2} I(\omega_i)^2$$
$$= \tfrac{1}{2} (3.25 \text{ kg·m}^2) \left(157 \frac{\text{rad}}{\text{sec}}\right)^2 \qquad \left(\textbf{\textit{Note:}}\ 1 \frac{\text{kg·m}^2}{\text{sec}^2} = 1 \text{ J.}\right)$$
$$= 40{,}054.6 \text{ J}$$

$$E_k \text{ (final)} = \tfrac{1}{2} I(\omega_f)^2$$
$$= \tfrac{1}{2} (3.25 \text{ kg·m}^2) \left(130.8 \frac{\text{rad}}{\text{sec}}\right)^2$$
$$= 27{,}801.5 \text{ J}$$

Energy removed from flywheel equals E_k (initial) minus E_k (final).

E_k (initial) − E_k (final) = 40,054.6 J − 27,801.5 J

Energy removed = 12,253.1 J.

ACTIVITY 2

Solving Problems That Involve Energy and Work in Mechanical and Fluid Energy Systems

MATERIALS

For this lab, you'll need a calculator.

Problem 1: Cash Construction Company is installing some natural gas lines. One gas line must run under a heavily traveled street. To do this, a horizontal boring machine will bore a hole under the roadway. To help stabilize the boring auger while it's rotating, the auger will be attached to a solid sphere, with the shaft of the auger going through the center of the sphere.

Given: The sphere has a mass of 8 kg, is 0.26 m in diameter and rotates at 400 rpm with the auger.

Find:
a. Moment of inertia of the sphere. (***Note:*** From Figure 5-13 of the text, $I = \frac{2}{5} mr^2$ for a sphere.)
b. Kinetic energy stored in the rotating sphere.

Solution:

Check the units of your solution against the units in Table 1. Are the units correct? For each problem of this Math Skills Lab, you should compare the units of the solution to the units in Table 1. Do this. It will help you make sure the solution units are correct.

Problem 2: Electricity is generated at a dam by storing water as potential energy at some height above the generator. The water is turned into kinetic energy when released, and strikes a turbine blade that makes the generator turn and generate electricity.

Given: A dam releases 4000 liters of water stored 50 meters above the turbine. Assume that all of the stored potential energy will be converted to kinetic energy. (Water has a mass of 1 kilogram/liter.)

Find: Kinetic energy available to turn the turbine.

Solution:

(Don't forget to compare the units in your solution to Table 1!)

Student Challenge

The following problems review some concepts learned earlier. You may want to refer to the equations given in Table 5-5 at the end of the Summary of Unit 5.

Problem 3: Given: Modern aircraft have brakes built into the wheels. The manufacturer of a small plane wants to test a new tire design on a plane that weighs 2400 lb. A pilot lands the plane at 90 mph and applies the brakes. The "anti-brake-lock system" operates correctly. The wheels do not lock—they continue to turn. In a table of coefficients of friction, a technician finds that the value of μ is 0.75 between rubber and dry concrete.

Find:
a. Frictional force between the rubber tires and the concrete runway.
b. When the technician looked up the correct coefficient of friction, did the technician look under "static" or "kinetic" coefficients of friction? Why?

Solution:

Problem 4: Given: The same conditions as Problem 3, except that the wheels *do lock* (the wheels stop turning) and the value of μ from the table is found to be 0.70 between rubber and dry concrete.

Find:
a. The frictional force.
b. Is the value of μ a static friction coefficient or a sliding friction coefficient?

Solution:

Problem 5: Given: Making sheets of either steel or plastic involves a similar industrial process. Sheet thickness is made uniform by running the material through a set of rollers—something like using a rolling pin to roll out pie dough. Both steel and plastic are very soft when they go through the rollers. As the sheets come out of the rollers, they go onto a conveyor belt. To avoid stretching the sheets, technicians adjust the linear belt speed of the conveyor to 4 feet/second. The conveyor belt is driven by a 12-inch-diameter drum that's powered by a direct-drive motor.

Find: Motor shaft angular speed in revolutions per minute (rpm) to achieve the 4 feet/second linear speed of the conveyor belt. (***Note:*** The relationship between angular speed [ω] and linear speed [v] is given by the equation, $\omega = {}^{v}\!/_{r}$, where r is the drive-drum radius.)

Solution: