# Math Lab 14 MS 1

## Plotting and Analyzing an Exponential Decay Curve

## Comparing the General Exponential Equation, $Q = Q_0e^{-t/\tau}$, with Other Specific Decay Processes

## Using a Calculator to Evaluate Exponential Equations, Such as $V = V_0e^{-t/\tau}$

For best results, print this document front-to-back and place it in a three-ring binder. Corresponding teacher and student pages will appear on each opening.

## TEACHING PATH - MATH SKILLS LAB - CLASS M

### RESOURCE MATERIALS

Student Text: Math Skills Lab

### CLASS GOALS

1. Teach students how to plot and analyze an exponential decay curve.
2. Teach students how to compare the general exponential equation $Q = Q_0 e^{-t/\tau}$ with specific exponential equations for decay processes.
3. Teach students how to evaluate an exponential equation such as $V = V_0 e^{-t/\tau}$ with the help of a calculator.

### CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to this class. (How much you accomplish will depend on the math skills your students already have.) Summarize the explanatory material for the three math activities. Then have students complete the Practice Exercises given at the end of the Math Skills Lab.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 14*1, "Time Constants in Damped Vibrations." ALSO, tell them that Lab 14*1, is a demonstration-type laboratory, with only one setup of equipment. You will show them the equipment and demonstrate its operation. You will then provide them with data that they are to analyze. It should be a "fun lab."

# Math Skills Laboratory

Lab 14 MS 1

## *MATH ACTIVITIES*

***Activity 1:*** ***Plotting and Analyzing an Exponential Decay Curve***

***Activity 2:*** ***Comparing the General Exponential Equation, $Q = Q_0 e^{-t/\tau}$, with Other Specific Decay Processes***

***Activity 3:*** ***Using a Calculator to Evaluate Exponential Equations, Such as $V = V_0 e^{-t/\tau}$***

## *MATH SKILLS LAB OBJECTIVES*

***When you complete these activities, you should be able to do the following:***

1. ***Recognize an exponential decay curve and use it to find:***
   - ***half-life time constant ($T_{1/2}$)***
   - ***1/e time constant ($\tau$)***
   - ***time to complete just over 99% of the change (5 $\tau$)***
2. ***Plot an exponential decay curve from data given in a table.***
3. ***Recognize the equations of several processes as exponential decay functions.***

## *LEARNING PATH*

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Read the discussion in the Activities and study the examples.***
3. ***Work the problems.***

**NOTE:** Repetition develops familiarity! So be sure to cover the material in "Let's Review." And especially, point out the decay curve, the equation $Q = Q_0 e^{-t/\tau}$, the time constants $T_{1/2}$, $\tau$ and the five time constant interval $5\tau$. Figure 1 is a good recap of much of the material covered in Subunit 1.

**Note:** This is not a simple math lab. The work outlined in Activities 1, 2, and 3 require concentration and careful attention. But when your students work through the examples given in the three activities--and then complete the Practice Exercises at the end of the Math Skills Lab, they will have a much better understanding of exponential decay curves and time constants. Challenge them--encourage them--and help them!

### *WHAT DO THE 1/e AND $T_{1/2}$ TIME CONSTANTS MEAN? AND WHAT'S THE EXPONENTIAL DECAY EQUATION?*

Let's review the meanings of the half-life ($T_{1/2}$) and 1/e ($\tau$) time constants. Let's also review the general form of the exponential decay equation and the meaning of the terms in the equation.

The ***half-life*** time constant is the time required for a process to go halfway (50%) to completion. It's represented by the symbol, $T_{1/2}$.

The ***1/e*** time constant is the time required for an exponential process to go 63% of the way to completion. It's represented by the symbol, $\tau$. The ***exponential decay equation*** is represented by the formula: $Q = Q_0 e^{-t/\tau}$.

where: $Q$ = a quantity that varies with time
$Q_0$ = the initial value of the quantity (value of $Q$ at $t = 0$)
$t$ = time in the same units as $\tau$
$\tau$ = the time constant for the process
$e$ = 2.71828...(a number, like $\pi$, that often occurs in natural processes)

The time interval required for an exponential process to be 99% complete is equal to five 1/e time constants (5 $\tau$). Figure 1 shows a general exponential decay curve with the values of $T_{1/2}$, $\tau$, and 5 $\tau$ labeled.

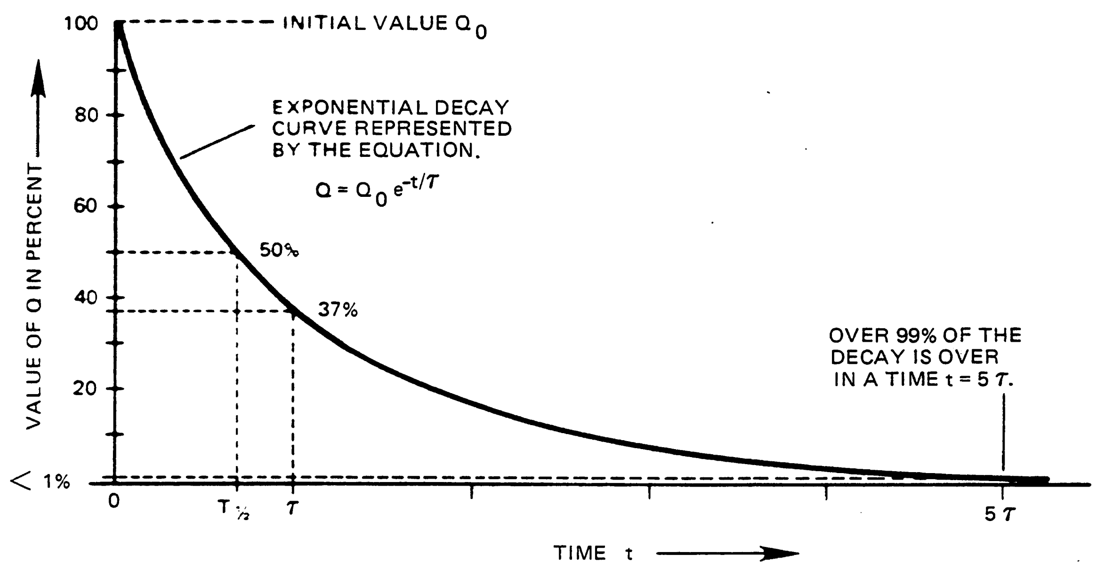



***Fig. 1*** Time constants of an exponential decay curve.

### *ACTIVITY 1*

## Plotting and Analyzing an Exponential Decay Curve

You've learned that many processes have a constant rate. These are called ***linear processes*** because a plot of the quantity that's changing versus time gives a ***straight*** line. The rate is given by the quantity that's changing (length, volume, and charge, for example) divided by the elapsed time.

**NOTE:** The contents of Tables 1A and 1B should be clear. However, the columns headed by $Q/Q_0$ may confuse some of your students. So point out that the percentage values below the heading represent the fraction $Q/Q_0$. Thus, for the value 6.25% at 4 seconds in Table 1A, the meaning is that Q is 6.25% of the original value $Q_0$ after a time lapse of 4 seconds, and so on for all the other values.

Common rates are *miles per hour, gallons per second* and *coulombs per second.* The rate also is indicated by the ***slope*** of the line in a graph that shows the quantity plotted against time. The steeper the slope, the higher the rate. All straight-line graphs have a constant slope.

Many processes are ***nonlinear.*** Some of these give an ***exponential decay*** curve. An exponential decay curve is described by the following equation:

$$Q = Q_0\, e^{-t/\tau}$$

There are some characteristics of these curves that help us recognize them as ***exponential decay*** curves.

Consider the two sets of data presented in Tables 1A and 1B. The letter, Q, stands for any quantity that's changing—like temperature, voltage, flow rate and so forth. The values for Q are given in percent. Note that Q has a value equal to $Q_0$ ($Q = 100\%\ Q_0$) at the time t = 0. As time changes, the value of Q takes on a decreasing fraction—or percentage of $Q_0$. After a long time, Q approaches zero.

| TABLE 1A | | TABLE 1B | |
|---|---|---|---|
| $Q/Q_o$ | t | $Q/Q_o$ | t |
| 100% | 0 sec | 100% | 0 sec |
| 50% | 1 sec | 50% | 1 sec |
| 25% | 2 sec | 36% | 2 sec |
| 12.5% | 3 sec | 25% | 3 sec |
| 6.25% | 4 sec | 20% | 4 sec |
| 3.12% | 5 sec | 17% | 5 sec |
| 1.56% | 6 sec | 14% | 6 sec |
| 0.78% | 7 sec | 12.5% | 7 sec |
| 0.39% | 8 sec | 11.5% | 8 sec |
| 0.20% | 9 sec | 10.5% | 9 sec |
| 0.10% | 10 sec | 9.5% | 10 sec |

The data in Table 1A is plotted in Figure 2. The data in Table 1B is plotted in Figure 3. The data in Table 1A describes an exponential decay process. The data in Table 1B doesn't, even though it's a nonuniform change. How can you tell when a nonuniform process is an exponential change?

First, look at the graph in Figure 2. You can see from the graph that the time for the quantity (Q) to decay to 50% is one second, to decay to 25% is two seconds, to decay to 12.5% is three seconds, and to decay to 6.25% is four seconds. Each decay is one half—or 50%—of the previous value.

The time between each "halving" of the quantity is ***constant*** and is equal to one second, the half-life time constant of the process ($T_{1/2}$ = 1 sec.) *Conclusion: Whenever the successive half-lives for a nonuniform process are equal, the process is exponential.* The process is described by the equation, $Q = Q_0\, e^{-t/\tau}$.

Now look at the curve in Figure 3. The time required for the quantity to decay to 50% is one second, to 25% is three seconds, and to 12.5% is seven seconds. In this case, the time between each halving isn't constant. The time interval to go from 100% to 50% is one second, from 50% to 25% is another two seconds, and from 25% to 12.5% is an additional four seconds. Here the "half-life" doubles with each halving. *Conclusion: This process can't be described by the general equation for exponential decay.*

**NOTE:** Use Figures 2 and 3 to reinforce the idea that a **non-linear change is exponential** if the time intervals for successive 50% decreases are all **equal**. Thus in Figure 2, the time to go from 100 to 50 (50% decrease) is <u>one second</u>. The time to go from 50 to 25 (another 50% decrease) is also <u>one second</u>, and so on. By contrast, the changes shown in Figure 3 do not follow such a pattern. That is, the time intervals for successive *50% decreases do not remain equal to one another*.

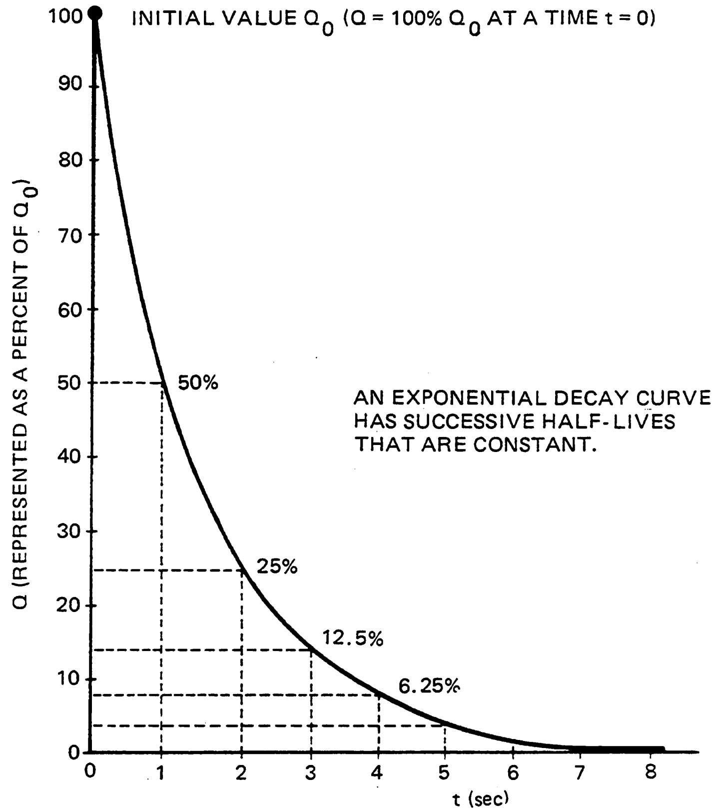



*Fig. 2* A nonuniform process that is an exponential change.

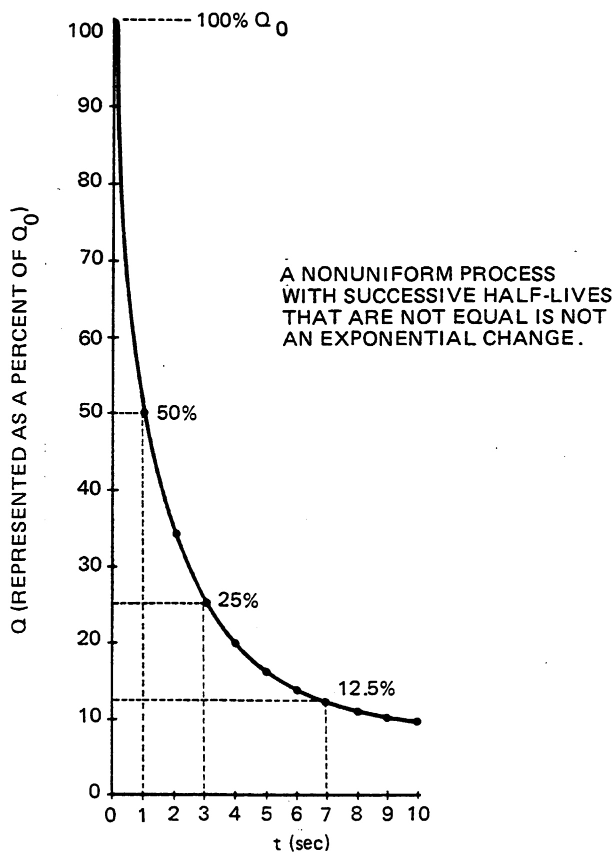



*Fig. 3* A nonuniform process that isn't an exponential change.

**NOTE:** In part b of the solution for Example 1, the numerical value of $\tau$ = 2.9 seconds was obtained from "curve b" as follows:

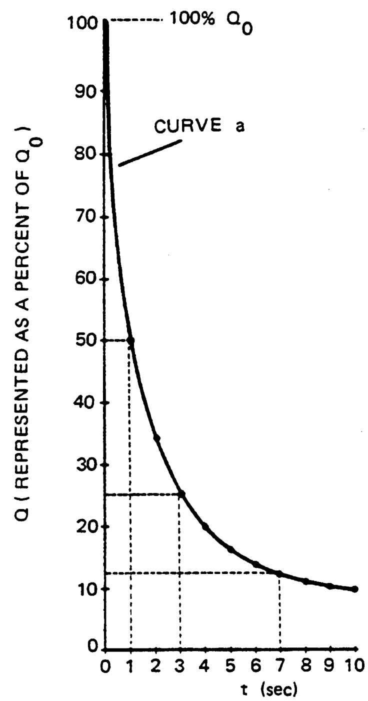
1. Locate the 37% point on the Q-axis (vertical axis).
2. Draw a horizontal line from this point over to the decay curve. Mark the point where the horizontal line intersects the curve.
3. From the intersection point, draw a vertical line down to the time axis. Mark the point where the vertical line crosses the time axis.
4. Based on the time scale, this point should be just less than three--or 2.9 sec.

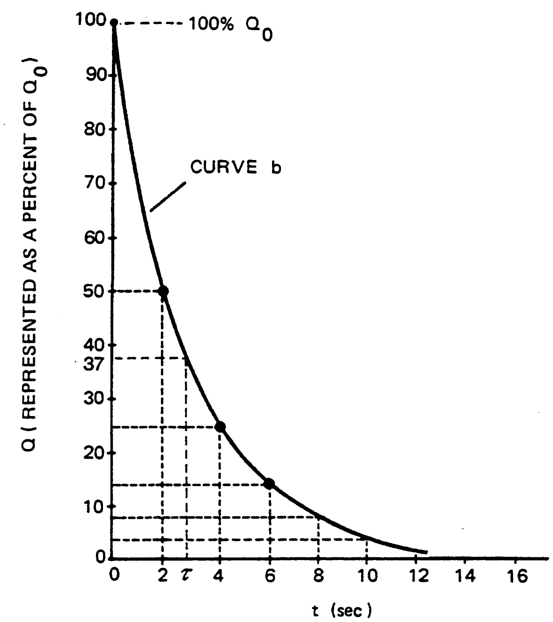
**This is shown with the dotted lines connecting 37% on the "y-axis" and $\tau$ on the "x-axis."**

Now study the two graphs given in Example 1. See if you can tell which is an exponential change and which isn't. For the one that's exponential, you can determine a constant half-life ($T_{1/2}$) and the 1/e time constant ($\tau$).

**Example 1: *Recognizing and Analyzing an Exponential Decay Curve***

Given: The two graphs here.




Find:

a. Which curve can be described by the general equation for an exponential decay — $Q = Q_0\, e^{-t/\tau}$?

b. The value of $T_{1/2}$, $\tau$, and 5 $\tau$ for the curve that's exponential.

Solution: Compare successive half-life intervals for each curve.

a. Curve **a**: Q goes from 100% to 50% in one second.
Q goes from 50% to 25% in another two seconds.
Q goes from 25% to 12.5% in another four seconds.

The successive half-life time intervals are not constant. Therefore, curve **a** isn't described by the equation, $Q = Q_0\, e^{-t/\tau}$. It ***is not*** an exponential curve.

Curve **b**: Q goes from 100% to 50% in two seconds.
Q goes from 50% to 25% in two seconds.
Q goes from 25% to 12.5% in two seconds.

The successive half-life time intervals are constant. The half-life time constant ($T_{1/2}$) equals two seconds. Therefore, curve **b** described by the equation, $Q = Q_0\, e^{-t/\tau}$, ***is*** an exponential curve.

For curve **b**, the following values are obtained from the graph.

b. $T_{1/2}$ = 2 sec (time at which Q = 50%)

$\tau \approx$ 2.9 sec (time at which Q = 37%)

$5\tau$ = 5(2.9 sec) = 14.5 sec. This is the time it takes to complete just over 99% of the total change from $Q_0$ to zero—or some final value.

**NOTE:** In Table 2, be sure to point out that the equation $Q_V = Q_{V_0} e^{-t/\tau}$ is only an approximation for the tank-emptying process. The true equation is a *quadratic*. Its shape more closely resembles an exponential decay as the flow more closely resembles laminar flow.

*ACTIVITY 2*

# Comparing the General Exponential Equation, $Q = Q0\ e^{-t/\tau}$, with Other Specific Decay Processes

Many processes can be described by the general equation for exponential decay. (Recall that the general equation is $Q = Q_0\ e^{-t/\tau}$.)

These processes include the damping of a mechanical vibration, the charging of a compressed gas cylinder, the draining of a liquid from an open tank, the discharging of a capacitor, the cooling of a warm object to room temperature, and the decay of a radioactive substance.

The specific equations for these processes are given in Table 2.

TABLE 2. EQUATIONS FOR EXPONENTIAL DECAY PROCESSES

| *Process* | *Equation* |
|---|---|
| Damping of vibration | $A = A_0\ e^{-t/\tau}$ |
| Charging rate of compressed gas cylinder | $Q_m = Q_{m0}\ e^{-t/\tau}$ |
| Draining of a liquid from an open tank | $Q_v = Q_{v0}\ e^{-t/\tau}$ |
| Discharge of a capacitor | $V = V_0\ e^{-t/\tau}$ |
| Cooling warm object to ambient temperature | $(\Delta T) = (\Delta T)_0\ e^{-t/\tau}$ |
| Radioactive decay | $a = a_0\ e^{-t/\tau}$ |
| *General equation* | $Q = Q_0\ e^{-t/\tau}$ |

where:
- $A$ = amplitude of a vibration
- $Q_m$ = mass-flow rate
- $Q_v$ = volume-flow rate
- $V$ = voltage across the capacitor being charged or discharged
- $\Delta T$ = temperature difference
- $a$ = decay rate of a radioactive sample

A term-by-term comparison of each equation with the general equation shows a high degree of similarity. Some of the similarities are listed in Table 3.

TABLE 3. SIMILARITIES IN EXPONENTIAL DECAY EQUATIONS

| *Changing Quantity Similar to "Q"* | *Initial Quantity Similar to "$Q_0$"* | *1/e Time Constant Describing Change* |
|---|---|---|
| $A$ | $A_0$ | $\tau$ |
| $Q_m$ | $Q_{m_0}$ | $\tau$ |
| $Q_v$ | $Q_{v_0}$ | $\tau$ |
| $V$ | $V_0$ | $\tau$ |
| $\Delta T$ | $(\Delta T)_0$ | $\tau$ |
| $a$ | $a_0$ | $\tau$ |

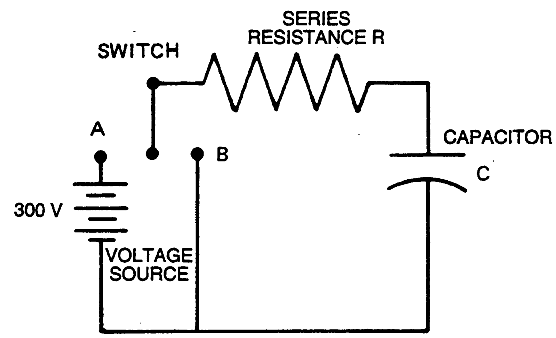
Example 2 shows how to get specific information about an exponential change from the formula for the change.

**NOTE:** If your students are with you in Unit 14, it's our guess that they will have had enough PT under their belts to see that in Example 2, switch position A leads to the charging process, while switch position B leads to the discharging process.

**Example 2: *Getting Information from an Exponential Change Equation***

Given: A capacitor, in the simple series RC circuit shown, is discharged when the switch is moved to position B. You're told that the equation for the voltage across the discharging capacitor is given by:

$$V = (300\text{ volts})\ e^{-t/0.003\text{ sec}}$$




Find:

a. The initial voltage across the capacitor at time t = 0.

b. The numerical value of the 1/e time constant ($\tau$).

c. The time for the capacitor to be just over 99% discharged.

Solution: Compare the given equation, $V = 300\ e^{-t/0.003\text{ sec}}$, with either the general equation, $Q = Q_0\ e^{-t/\tau}$, or the voltage equation, $V = V_0\ e^{-t/\tau}$. See Table 2. A term-by-term comparison shows that:

a. Initial voltage ($V_0$) is equal to 300 volts.

b. The 1/e time constant ($\tau$) is equal to 0.003 second (3 milliseconds).

c. Therefore, the time to complete 99% of the discharge (5 $\tau$) must be equal to $5 \times 0.003$ second, or 0.015 second.

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***ACTIVITY 3***

## Using a Calculator to Evaluate Exponential Equations, Such as $V = V_0\ e^{-t/\tau}$

As you saw in Table 2 and Example 2, the equation for the changing voltage across a discharging capacitor is given by:

$$V = V_0\ e^{-t/\tau}$$

where:
- $V$ = value of the changing voltage across the capacitor
- $V_0$ = initial voltage across the capacitor before discharge begins
- $t$ = time in seconds
- $\tau$ = time constant in seconds

For a series resistance-capacitance (RC) circuit, if you know the value of the series resistance (R) in ohms and the capacitance (C) in farads, you can calculate the time constant ($\tau$). This is done with the equation, $\tau = R \times C$. The answer you get is equal to the 1/e time constant ($\tau$) in seconds. (Recall that the term *ohm·farad* [Ω·F] is equal to 1 second.)

Example 3 shows how to calculate the value of the decreasing voltage across a capacitor at any time during the discharge process.

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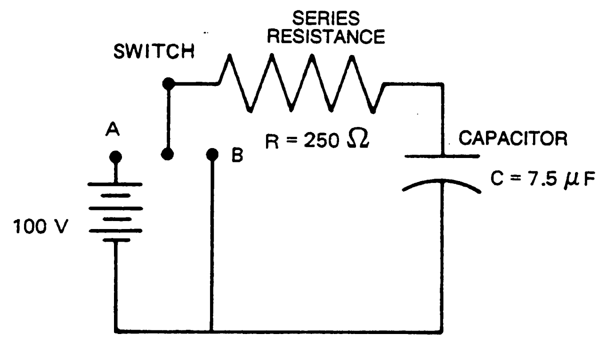
**Example 3: *Voltage Across a Capacitor During a Discharge Process***

Given: A series RC circuit consists of a 100-V dc source, a 250-ohm resistor, a 7.5-$\mu$F capacitor, and a switch. The capacitor initially has a voltage of 100 V.




Find:

a. The 1/e time constant ($\tau$) of the circuit.

b. The voltage across the capacitor at one-millisecond intervals after the switch is moved from the charge position A to the discharge position B. Do this for a total time of 10 milliseconds.

c. The half-life of the discharge process.

Solution:

a. Determine the time constant.

$\tau = R \times C$

$\tau = (250\ \Omega)(7.5 \times 10^{-6}\ \mathrm{F})$

$\tau = (250 \times 7.5 \times 10^{-6})(\Omega \cdot \mathrm{F})$ (note that $1\ \Omega \cdot \mathrm{F} = 1$ sec)

$\tau = 1.875 \times 10^{-3}$ sec

$\tau = 1.875$ msec

b. Use the equation to find V as time increases. Start with the equation, $V = V_0\, e^{-t/\tau}$. Substitute $V_0 = 100$ V and $\tau = 1.875$ msec.

Now write the equation:

$V = 100\ e^{-t/1.875\ \mathrm{msec}}$

Let's solve this equation with the help of a scientific calculator.

A calculator that has an $e^x$ key can be used to evaluate the exponential term ($e^{-t/\tau}$) in the general decay equation, $Q = Q_0\, e^{-t/\tau}$. Therefore, the values of V in the equation, $V = 100\ e^{-t/1.875\ \mathrm{msec}}$ can be obtained with the help of the $e^x$ key. First, solve for the voltage V across the capacitor at time t = 0. That is, solve the equation:

$V\ (\text{at } t = 0) = e^{-0/1.875\ \mathrm{msec}}$.

Step 1. First calculate the exponent, $\dfrac{t}{1.875\ \mathrm{msec}}$ for t = 0. Divide 0 on the calculator by 1.875. The answer is 0 (obviously).

Step 2. Press "0" on the calculator so that it shows on the display. Then press the "change sign" key. The display now changes "0" to "–0." You have changed "x" to "– x.")

Step 3. Press the $e^x$ key for x = (–0). The calculator display gives "1." The answer for $e^{-0/1.875\ \mathrm{msec}}$ is 1.0.

Step 4. Multiply the display "1.0" by 100 to get "V". The answer is 100 V. You have just evaluated the equation, $V = 100\ e^{-0/1.875\ \mathrm{msec}}$. The answer is 100 V—as you know it should be. That's because V = 100 volts at time (t) = 0.

Next solve the voltage equation for t = 1 msec. That means, solve the equation, $V = 100\ e^{-1\ \mathrm{msec}/1.875\ \mathrm{msec}}$, for the value (V).

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Step 1. Evaluate the exponent $\frac{1}{1.875 \text{ msec}}$. The units cancel—as they always *must!*

The fraction $\frac{1}{1.875}$ evaluated on the calculator, gives 0.533 (to three places). Enter this into the calculator. It's now on the display.

Step 2. Change the display from 0.533 to –0.533 by pressing the "change sign" or "+/–" key.

Step 3. With the display showing –0.533, press the $e^x$ key on the calculator. The display gives 0.587 (to three places). You now have evaluated the exponential term, $e^{-1/1.875}$.

Step 4. To get the final voltage, multiply the display (0.587) by 100 V. The answer is 58.7 volts. (You have completed the evaluation of the equation, $V = 100\ e^{-1 \text{ msec}/1.875 \text{ msec}}$).

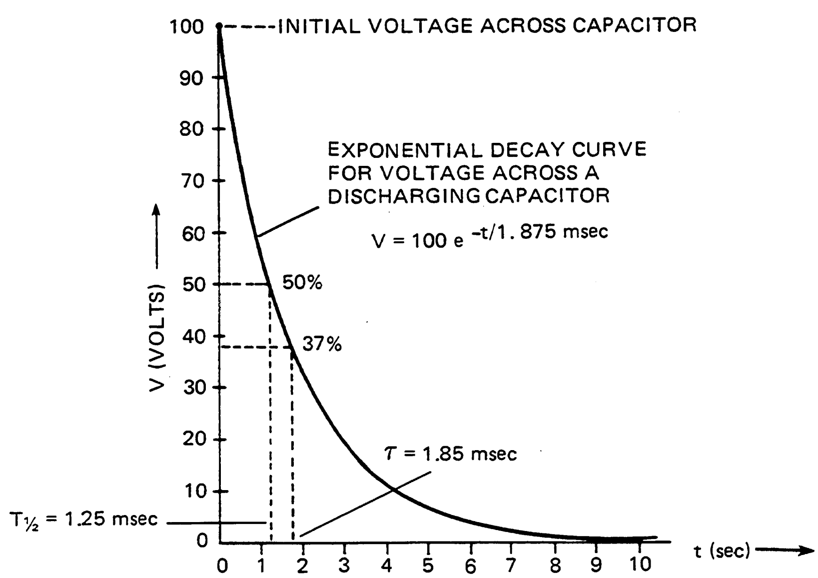
Continuing in the same way for t = 2 msec, t = 3 msec and so on, you should get the values listed in the following table. If you have trouble using your calculator to do this, see you teacher.

TABLE 4. VOLTAGE ACROSS CAPACITOR DURING DISCHARGE PROCESS

| *t (msec)* | *V (Volts)* |
|---|---|
| 0.00 | 100.00 |
| 1.00 | 58.7 |
| 2.00 | 34.4 |
| 3.00 | 20.1 |
| 4.00 | 11.8 |
| 5.00 | 6.9 |
| 6.00 | 4.1 |
| 7.00 | 2.4 |
| 8.00 | 1.4 |
| 9.00 | 0.8 |
| 10.00 | 0.5 |

c. Finding the half-life.

Plot the values of V and t in Table 4 on a graph. The resulting curve is shown here. From this curve, read the value of t for $V = \frac{1}{2} V_0$ (50 volts). The approximate result is $T_{1/2}$ = 1.25 msec. With this curve you can also read the value of t for $V = 0.37\ V_0$ (37 volts). The approximate result is t = 1.85 msec. It should, of course, be $\tau$ = 1.875 msec. But one can't read the graph that accurately.



## SOLUTIONS TO PRACTICE EXERCISES

1. Advise your supervisor to proceed as follows. On the time axis of the given decay curve, mark off the time values that correspond to successive half lives $T_{1/2}$. Then compare the values of each successive half life. For an exponential curve, the successive half lives will all be <u>equal</u>. For a curve that is not exponential, the successive half lives will not be equal.

2. Compare the given equation $V = 5\ e^{-t/0.15\ sec}$ with the general equation $V = V_0 e^{-t/\tau}$, term by term. Then it is apparent that:

   $V_0 = 5$ volts

   $\tau = 0.15$ sec $= 150$ milliseconds

3. a. $\tau = R \times C$

   $\tau = (1 \times 10^6\ \Omega)(4 \times 10^{-6}\ F)$

   $\tau = (1 \times 4 \times 10^{6-6})(\Omega \cdot F)$ (Note: $1\ \Omega \cdot F = 1$ sec)

   $\tau = 4 \times 10^0$ sec (Note: $10^0 = 1$)

   $\tau = 4$ sec

   b. $T_{1/2} = 0.693 \times \tau$

   $T_{1/2} = 0.693 \times 4\ \text{sec} = 2.77\ \text{sec}$

   c. Discharge is 99% completed in 5 $\tau$.

   $5\ \tau = 5 \times 4\ \text{sec} = 20\ \text{sec}$

4. a. $5\ \tau = 20$ min (given since 99% change occurs in a time equal to 5 $\tau$)

   $$\tau = \frac{20\ \text{min}}{5} = 4\ \text{min}$$

   b. After one time constant, the initial pressure has dropped by 63%, to a value equal to 37% of 1500 psi. Therefore,

   $P(t = \tau) = 1500\ \text{psi} \times 0.37$

   $P(t = \tau) = 555\ \text{psi}$

***PRACTICE EXERCISES***

**Problem 1:** Given: The following situation. Your supervisor has two nonuniform decay curves. One is an exponential curve; the other isn't. She comes to you and asks if you can tell her how to define which is an exponential curve and which isn't.

Find: Explain to your supervisor how to tell an exponential decay from a nonexponential decay.

Solution: (***Hint:*** Review Example 1.)

**Problem 2:** Given: Tom works with an engineer who is designing a special electronic circuit. The engineer points out an RC series circuit that's part of the overall circuit. The engineer tells Tom that the discharge voltage across the capacitor in the circuit is described by the equation, $V = 5\ e^{-t/0.15\ \text{sec}}$.

Find: The initial voltage ($V_0$) across the capacitor and the 1/e time constant ($\tau$), in milliseconds.

Solution: (***Hint:*** Review Example 2.)

**Problem 3:** Given: Susan learns that the resistance in a simple RC series circuit is equal to $1 \times 10^6\ \Omega$ (1 megohm) and the capacitance is 4.0 $\mu$F. She charges the capacitor to an initial voltage of 500 V.

Find:

a. The 1/e time constant ($\tau$) for the RC circuit.

b. The half-life ($T_{1/2}$), knowing that $T_{1/2} = 0.693\ \tau$.

c. When the capacitor discharging process will be just over 99% complete.

Solution:

**Problem 4:** Given: John works for a diving manufacturing company that builds scuba tanks. John is told that a certain air tank has an initial pressure of 1500 psi (gage). The tank discharges through a valve (partially opened) and is 99% empty in 20 minutes. The pressure decay equation is given by $P = 1500\ e^{-t/\tau}$.

Find:

a. The 1/e time constant $\tau$ (in minutes).

b. The pressure after one time constant.

Solution:

5. (Student Challenge)
Begin with the equation $P = 1500\ e^{-t/\tau}$ and solve for P.

$P = 1500\ e^{-t/\tau}$ where: $\tau = 4$ min (from Problem 4)
$t = 6$ min (given)

$P = 1500\ e^{-6/4} = 1500\ e^{-1.5}$

$P = 1500\ (0.2231)$ ; (since $e^{-1.5}$ on calculator = 0.2231)

$P = 334.6$

Thus, after 6 minutes, the air pressure (gage) in the tank is about 335 psi.

6. (Student Challenge)

a. Since half-life is 8 days, we know that:
after 8 days, 1/2 is left
after 16 days, 1/2 x 1/2 or 1/4 is left
after 24 days, 1/2 x 1/2 x 1/2 or 1/8 is left
after 32 days, 1/2 x 1/2 x 1/2 x 1/2 or 1/16 is left
Thus, after 32 days, 1/16th of the original radioactive Iodine-131 is still present.

b. Given that $T_{1/2} = 0.693\ \tau$, isolate $\tau$ and solve the equation.

$T_{1/2} = 0.693\ \tau$

Divide each side by 0.693 and rearrange the equation.

$$\frac{T_{1/2}}{0.693} = \frac{0.693\ \tau}{0.693}$$

$$\tau = \frac{T_{1/2}}{0.693} \quad \text{where } T_{1/2} = 8 \text{ days}$$

$$\tau = \frac{8 \text{ days}}{0.693} = 11.54 \text{ days}$$

c. 99% disappears in a time interval equal to 5 $\tau$.

$5\ \tau = 5 \times (11.54 \text{ days}) = 57.7$ days

So after about 2 months, less than 1% is still around.

**Problem 5:** Given: The same condition described in Problem 4.

Find: The pressure in the air tank at the time (t) = 6 minutes. (***Hint:*** Solve the equation, $P = 1500\ e^{-t/\tau}$, using a calculator. Review the procedure outlined in Example 3. Use the value of $\tau$ [in minutes] calculated in Problem 4.)

Solution:

**Problem 6:** Given: During the Chernobyl nuclear power plant accident, radioactive iodine went into the atmosphere. Much of it was carried across Russia and Europe, in clouds, causing great concern. The ***half-life*** of the radioactive iodine isotope I-131 ($_{53}I^{131}$) is about 8.0 days.

Find:

a. What fraction of the initial I-131 released on the day of the accident was still around after 32 days?

b. The 1/e time constant ($\tau$) if you're given the relationship that $T_{1/2} = 0.693\ \tau$.

c. How many days must pass before 99% of the radioactive iodine has disappeared?

Solution: