Math Lab 1 MS 2
Substituting in Formulas

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

NOTE PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There are two Preparatory Math Skills Labs in a separate book entitled *PT Resource Manual* that contain concepts your students should have mastered before they begin this Math Skills Lab. These two preparatory labs are coded "PMS3" and "PMS4." They are titled "Learning How to Multiply and Divide Numbers and Units" and "Learning How to Write Numbers in Scientific Notation." Encourage students who need these skills to study the material in PMS3 and PMS4.

RESOURCE MATERIALS

Student Text: Math Skills Lab

Resource Manual: Preparatory Math Skills

CLASS GOALS

1. Given two of three quantities in an equation, solve the equation for the third quantity.
2. Substitute numerical values in equations for density and fluid pressure, and solve the equation for the unknown quantity.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete the activities. Students already should have read the discussion material and looked at the examples for each activity before coming to class. You should summarize the main points in each activity, work an example or two, and have students complete the Practice Exercises for each activity on their own.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 1F1, "Measuring Specific Gravity," as homework.

Math Skills Laboratory

Lab 1 MS 2

MATH ACTIVITY

Substituting in Formulas

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. ***Given two of three quantities in an equation, solve the equation for the third quantity.***
2. ***Substitute numerical values in equations for density and fluid pressure, and solve the equation for the unknown quantity.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Examples A and B.***
3. ***Work the problems.***

ACTIVITY

Substituting in Formulas

MATERIALS

For this activity, you'll need a hand calculator.

1. The mass density of any substance is equal to the mass of the substance divided by the volume of the substance. In equation form, density is given as follows:

(In words) $\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$

(In symbols) $D = \dfrac{M}{V}$

where: D = density
M = mass
V = volume

2. If the density of a substance is known, you can always find how much mass of that substance is contained in a given volume by rearranging the formula for density as follows:
(In words) Mass = Density × Volume
(In symbols) $M = D \times V$
where: M = mass
D = density
V = volume

3. The pressure of any fluid is equal to the force acting on a given area divided by the area being acted on. In equation form, pressure is given as follows:

(In words) $\text{Pressure} = \dfrac{\text{Force}}{\text{Area}}$

(In symbols) $p = \dfrac{F}{A}$

where: p = pressure
F = force
A = area

4. Pressure of fluid at some depth in a tank is equal to the weight density of the fluid times the height of the fluid above the point where the pressure is measured.
(In words)
Pressure = Weight Density × Height
(In symbols) $p = \rho_w h$
where: p = pressure
ρ_w = weight density
h = height (or depth)

5. Total pressure equals the sum of atmospheric pressure and gage pressure.
(In words)
Total Pressure = Atmospheric Pressure + Gage Pressure
(In symbols) $p_{tot} = p_{atm} + p_g$

SOLVED EXAMPLES

Study the examples that follow. Each example shows how to calculate a certain unknown quantity by substitution in a known formula.

Example A: ***Calculating Density***

Given: A volume of three cubic feet of sea water weighs 192 pounds.

Find: The weight density of sea water.

Solution:

Step 1: Write down the formula for *weight density* in symbol form.

$$\rho_w = \frac{w}{V}$$

Step 2: Identify what is "given" and what needs to be "found."

Given: Weight (w) = 192 lb
Volume (V) = 3 cubic feet = 3 ft^3

Find: Weight density (ρ_w)

Step 3: Substitute "given" values for w and V in the formula, including both the numerical value and units.

$$\rho_w = \frac{w}{V} = \frac{192\ \text{lb}}{3\ \text{ft}^3}$$

Step 4: Use a calculator to perform the indicated division. Write out the answer, including both the numerical result and the units.

$$\rho_w = 64\ \frac{\text{lb}}{\text{ft}^3}$$

Conclusion: The weight density of sea water has been calculated to be 64 pounds per cubic foot. Note that the correct answer includes both a number (64) and units (lb/ft^3).

ANSWERS TO PRACTICE EXERCISES

NOTE: In Problem 7, we have given a weight density of $0.005616/ft^3$. Explain to students that this is an **average** weight density. That's because the density of air varies with altitude. So we've made the problem simpler by providing single value--an average value.

Problem 1:

$$D = \frac{M}{V}$$

$$D = \frac{68\ gm}{5\ cm^3}$$

$$D = 13.6\ gm/cm^3.$$

Problem 2:

$$\rho_w = \frac{W}{V}$$

$$\rho_w = \frac{400\ lb}{8\ ft^3}$$

$$\rho_w = 50\ lb/ft^3$$

Problem 3:

$$M = D \times V$$

$$M = 19.3 \frac{gm}{\cancel{cm^3}} \times 10\ \cancel{cm^3}$$

$$M = 193\ gm.$$

Problem 4:

$$M = D \times V$$

$$M = 0.9 \frac{gm}{\cancel{cm^3}} \times 1000\ \cancel{cm^3}$$

$$M = 900\ gm.$$

Example B: *Calculating Mass*

Given: Antifreeze (ethylene glycol) has a density of 1.125 gm/cm^3 at the freezing temperature of water.

Find: The mass of one liter of antifreeze. (One liter = 1000 cm^3.)

Solution:

Step 1: Write down the formula for mass in symbol form. $M = D \times V$

Step 2: Identify what is **given** and what is to be **found**.

Given: Density (D) = 1.125 gm/cm^3

Volume (V) = 1000 cm^3

Find: Mass (M)

Step 3: Substitute given values for D and V in the formula, including numbers and units.

$$M = D \times V$$

$$M = 1.125 \frac{gm}{cm^3} \times 1000\ cm^3$$

Step 4: Use your calculator to complete the indicated multiplication. Write out numbers and units.

$$M = 1125 \frac{gm \cdot \cancel{cm}^3}{\cancel{cm}^3}$$ (Cancel identical units.)

$$M = 1125\ gm$$

Conclusion: The mass of one liter of ethylene glycol at 32°F is 1125 gm or 1.125 kg.

Note: The units $\frac{gm \cdot cm^3}{cm^3}$ are correct, but can be simplified by canceling the cm^3 in the numerator with cm^3 in the denominator, leaving the unit *gm*. Since *gm* is a mass unit, and the calculation was to determine a mass, the units come out correctly.

PRACTICE EXERCISES

Problem 1: Mercury is used as a liquid in thermometers. A volume of five cubic centimeters (5 cm^3) of mercury has a mass of 68 grams (68 gm). What is the density of mercury in gm/cm^3?

(Use the formula, $D = \frac{M}{V}$.)

Problem 2: Oak wood floats in water. A 400-lb chunk of oak wood occupies a volume of about 8 cubic feet (8 ft^3). What is the weight density of oak wood in lb/ft^3?

(Use the formula, $\rho_w = \frac{w}{V}$.)

Problem 3: Gold is a heavy metal. It has a density of 19.3 gm/cm^3. What is the mass of 10 cubic centimeters (10 cm^3) of gold?

(Use the formula, $M = D \times V$.)

Problem 4: Oil floats on water. Oil has a density of 0.9 gm/cm^3. What is the mass of one liter (1000 cm^3) of oil?

(Use the formula, $M = D \times V$.)

ANSWERS TO PRACTICE EXERCISES, Continued

Problem 5:

$$p = \frac{F}{A}$$

$$p = \frac{5000\ \text{lb}}{500\ \text{in}^2}$$

$$p = 10\ \frac{\text{lb}}{\text{in}^2}.$$

Problem 6:

$$F = p \times A$$

$$F = 480\ \frac{\text{lb}}{\cancel{\text{in}^2}} \times 6\ \cancel{\text{in}^2}$$

$$F = 2880\ \text{lb}.$$

Problem 7:

$$p = \rho_w h$$

$$p = 0.056\ \frac{\text{lb}}{\cancel{\text{ft}^3}} \times 30{,}000\ \cancel{\text{ft}}$$

$$p = 1680\ \frac{\text{lb}}{\text{ft}^2}.$$

Problem 8:

$$P_{tot} = P_{atm} + Pg$$

$$P_{tot} = 14.7\ \frac{\text{lb}}{\text{in}^2} + 40\ \frac{\text{lb}}{\text{in}^2}$$

$$P_{tot} = 54.7\ \frac{\text{lb}}{\text{in}^2}$$

Problem 5: Air is stored under pressure in a tank. The air exerts a total force of 5000 lb against the tank wall. The total surface area of the tank wall is 500 in^2. What is the tank pressure?

(Use the formula, $p = \frac{F}{A}$.)

Problem 6: What is the force exerted by a pressure of 480 lb/in^2 on a hydraulic piston that has a surface area of 6 in^2?

(Use the formula, $F = p \times A$.)

Problem 7: Find the pressure of air at the bottom of a column of air 30,000 ft high with a weight density of 0.056 lb/ft^3.

(Use the formula, $p = \rho_w \times h$.)

Problem 8: At sea level, atmospheric pressure is 14.7 lb/in^2. What is the total pressure of air in an inner tube that has a gage pressure of 40 lb/in^2?

(Use the formula, $p_{tot} = p_{atm} + p_g$.)