PREPARATORY MATH SKILLS LAB

Lab PMS 5

MATH ACTIVITY

Learning How to Measure Angles

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. ***Explain what's meant by measuring angles in radians.***
2. ***Use a circle to show an angle of one radian measured at the center of a circle.***
3. ***Compare an angle in degrees to an angle in radians.***
4. ***Convert an angle in degrees to an equal angle in radians.***
5. ***Convert an angle in radians to an equal angle in degrees.***

MATERIALS

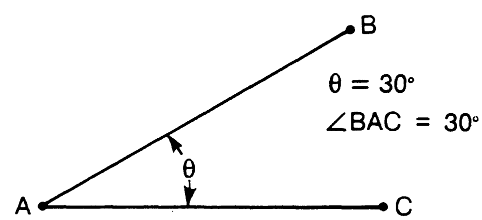
For this activity, you'll need a sharp pencil, a compass, a protractor and a straightedge.

HOW DO YOU MEASURE ANGLES IN DEGREES?

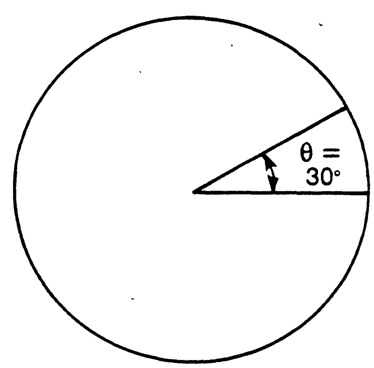
Angles often are measured in degrees. Figure 1a shows an angle θ—also written $\angle BAC$—equal to 30°. Figure 1b shows the same angle as part of a circle.

The entire circle contains 360°. The unit angle is one degree, or $\frac{1}{360}$ of a circle. The unit angle, a 90° angle, and a 180° angle are shown in Figure 2. Note that a 90° angle corresponds to one-fourth of the circle. A 180° angle corresponds to one-half of the circle. A one-degree angle corresponds to $\frac{1}{360}$ of the circle.

In most cases, as you learned in Activity 2, "Learning How to Draw and Measure Angles," we measure angles between two lines (or line segments)—like sides BA and CA in Figure 1a. A protractor, marked off in degrees, is used to read the angle. In Figure 2, the sides of the 90° angle are AO and BO. The sides of the 180° angle are AO and CO. The sides of the small, 1° angle are DO and EO.



a. Angle of 30°



b. Angle drawn as part of a circle

Fig. 1 Angle measure in degrees.

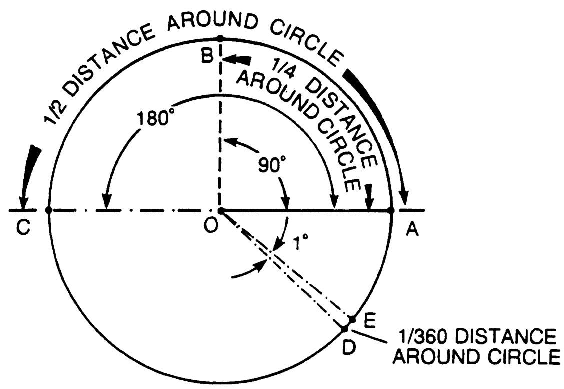


Fig. 2 Typical angles in degrees.

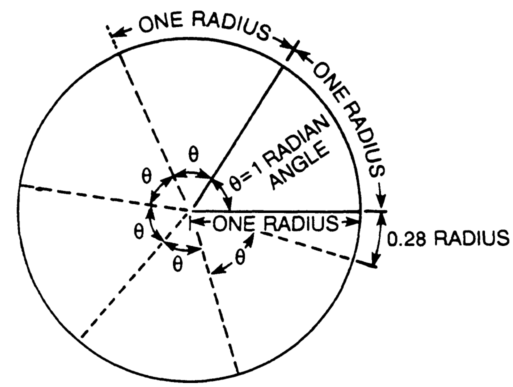


Fig. 3 Angle measure in radians.

HOW DO YOU MEASURE ANGLES IN RADIANS?

There's another angle measure that's used in science and technology. It's an important measure of an angle called a "radian." An angle of one radian is much larger than an angle of one degree. In fact, one radian equals 57.3 degrees. The idea behind a radian, based on the circle, is shown in Figure 3.

The *circumference* of a circle is the distance around the circle. The circumference is equal to $2\pi r$. In this case, r is the radius of the circle. Pi (π) is a constant equal to about 3.1416. (The symbol, "π," is pronounced "pie." The Greek word for this symbol is pi. Let's use $\pi = 3.14$ for short.) The circumference ($2\pi r$) is then the same as 6.28r.

Suppose a distance equal to one radius is marked off on the circle. (This is shown in Figure 3.) Two radius lines are drawn from the center of the circle to the edges of the one-radius length on the circle. The angle formed by the two radii is ***one radian.*** If you measure this angle—called "θ" in Figure 3—with a protractor, you'll find it's close to 57.3°.

Suppose we continue to mark off lengths on the circle—each equal to one radius. (This is shown in Figure 3.) We can fit 6 complete lengths and still have one piece left over. The piece left over is equal to 0.28 r. That's because the circumference is equal to 6.28 r, or 6r + 0.28r. For each of the six equal sections shown in Figure 3, the angle (θ) is equal to one radian. For the small piece left over, the angle is equal to 0.28 radian. So the complete angle in a circle is equal to 6.28 radians (2π radians). In terms of degrees, it's equal to 360°. We then have two useful equations:

2π (or 6.28 rad) = 360°
and
One rad = 57.3°
(Let "rad" stand for radian.)

The two equations in the box say the same thing. We can use the top one to get other useful information.

1. Start with $2\pi = 360°$, or rather, $360° = 2\pi$ rad.
2. Divide each side by 2 to get: $\frac{360°}{2} = \frac{2\pi}{2}$ rad, or $180° = \pi$ rad
3. Divide each side by 4 to get: $\frac{360°}{4} = \frac{2\pi}{4}$ rad, or $90° = \frac{\pi}{2}$ rad
4. Divide each side by 6 to get: $\frac{360°}{6} = \frac{2\pi}{6}$ rad, or $60° = \frac{\pi}{3}$ rad
5. Divide each side by 8 to get: $\frac{360°}{8} = \frac{2\pi}{8}$ rad, or $45° = \frac{\pi}{4}$ rad
6. Divide each side by 6.28 to get: $\frac{360°}{6.28} = \frac{2\pi}{6.28}$ rad, or $57.3° = 1$ rad

HOW DO YOU CONVERT FROM DEGREES TO RADIANS?

Suppose an angle is given in degrees and you want to get the same angle in radians. You can use the following equation:

$$\boxed{\text{Angle in rad} = \text{Angle in degrees} \times \left(\frac{1\text{ rad}}{57.3^\circ}\right)}$$

Example: *Change 72° to radians.*

$$\text{Angle in rad} = 72^\circ \times \left(\frac{1\text{ rad}}{57.3^\circ}\right) = \frac{72\cancel{^\circ}}{57.3\cancel{^\circ}} = \mathit{1.26\ rad} \qquad \text{(Cancel degrees.)}$$

Example: *Change 150° to radians.*

$$\text{Angle in rad} = 150^\circ \times \left(\frac{1\text{ rad}}{57.3^\circ}\right) = \frac{150\cancel{^\circ}}{57.3\cancel{^\circ}} = \mathit{2.62\ rad} \qquad \text{(Cancel degrees.)}$$

HOW DO YOU CONVERT FROM RADIANS TO DEGREES?

Suppose an angle is given in radians and you want to get the same angle in degrees. You can use the following equation:

$$\boxed{\text{Angle in degrees} = \text{Angle in rad} \times \left(\frac{57.3^\circ}{1\text{ rad}}\right)}$$

Example: *Change 1.5 radians to degrees.*

$$\text{Angle in degrees} = 1.5\,\cancel{\text{rad}} \times \left(\frac{57.3^\circ}{1\,\cancel{\text{rad}}}\right) \qquad \text{(Cancel rad.)}$$

$$\text{Angle in degrees} = 1.5 \times 57.3^\circ = \mathit{85.9^\circ}$$

Example: *Change 0.785 radian to degrees.*

$$\text{Angle in degrees} = 0.785\,\cancel{\text{rad}} \times \left(\frac{57.3^\circ}{1\,\cancel{\text{rad}}}\right) \qquad \text{(Cancel rad.)}$$

$$\text{Angle in degrees} = 0.785 \times 57.3^\circ = \mathit{45^\circ}$$

PRACTICE EXERCISES

1. With a compass, draw a circle of radius 2 inches. On the circumference, mark off a curved length approximately equal to the radius, 2 inches. From each edge of this curved length, draw a line to the center of the circle. The angle between the lines is equal to about one radian. Measure the angle to the nearest ½ degree with a protractor. Is your result near 57°?
2. With a compass, draw a circle of radius one inch. On this circle, draw an angle of 90° and an angle of 2 radians. Label each angle.

3. Convert the following angles in degrees to angles in radians.

   a. $130° =$ _______ rad

   b. $45° =$ _______ rad

   c. $180° =$ _______ rad

   d. $320° =$ _______ rad

4. Convert the following angles in radians to angles in degrees.

   a. 0.5 rad = _______ degrees

   b. 1.8 rad = _______ degrees

   c. $\pi/4$ rad = _______ degrees (Remember, $\pi/4 = 3.14/4 = 0.785$.)

   d. π rad = _______ degrees (π rad = 3.14 rad)

   e. 2π rad = _______ degrees (2π rad $= 2 \times 3.14 = 6.28$ rad)

   f. $\pi/2$ rad = _______ degrees