

# Math Skills Laboratory

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Lab 10<sup>M</sup>4<sup>S</sup>

## **MATH ACTIVITY**

**Activity:** *Solving Energy-Conversion Problems  
for Thermal Energy Convertors*

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## **MATH SKILLS LAB OBJECTIVES**

*When you complete these activities, you should be able to do the following:*

- 1. Solve energy-conversion problems for thermal energy convertors.*
  - 2. Substitute correct numerical values and units in appropriate equations. Solve equations for an unknown numerical value with the proper units.*
  - 3. Find the efficiency of thermal energy convertors.*
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## **LEARNING PATH**

- 1. Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
  - 2. Study the examples.*
  - 3. Work the problems.*
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## **ACTIVITY**

### **Solving Energy-Conversion Problems for Thermal Energy Convertors**

#### **MATERIALS**

For this activity, you'll need a calculator.

In this Math Skills Lab, you'll review the important formulas for work and energy. You'll review the units used to measure work, energy and power. You'll also solve problems that involve thermal energy convertors.

### LET'S REVIEW FORMULAS AND UNITS!

Examine Tables 1, 2 and 3. These are the same tables included in Math Skills Labs 10MS1, 10MS2 and 10MS3.

Table 1 lists the important formulas for work and energy that you've already studied. Table 2 sums up common units that you'll use in energy-conversion problems. Table 3 lists some conversions between units.

You'll find these conversions helpful when you solve the problems in this activity.

TABLE 1. REVIEW OF BASIC FORMULAS FOR WORK AND ENERGY

Energy System	Formulas	Definition of Symbols
<b>MECHANICAL</b>		
Linear Work	$W = F \times D$	F = applied force D = distance moved
Rotational Work	$W = T \times \theta$	T = torque applied $\theta$ = angular distance rotated
Gravitational Potential Energy	$E_p = w \times h$	w = weight h = height raised above reference level
Elastic Potential Energy	$E_p = \frac{1}{2} k d^2$	k = force or spring constant d = distance spring is stretched or compressed
Linear Kinetic Energy	$E_k = \frac{1}{2} m v^2$	m = mass v = speed
Rotational Kinetic Energy	$E_k = \frac{1}{2} I \omega^2$	I = moment of inertia $\omega$ = angular speed
<b>FLUID</b>		
Fluid Work	$W = p \times (\Delta V);$ $W = (\Delta p) \times V$	p; $\Delta p$ = pressure or pressure difference $\Delta V$ ; V = volume of fluid moved
Fluid Kinetic Energy	$E_k = \frac{1}{2} (\rho V) v^2$	$\rho$ = mass density V = volume v = speed
<b>ELECTRICAL</b>		
Electrical Work	$W = q \times \Delta V$	q = electrical charge moved $\Delta V$ = voltage difference
Electrical Energy	$E_{elec} = P_{elec} \times t$	$P_{elec}$ = electrical power used t = time power is used
<b>THERMAL</b>		
Thermal Energy	$H = mc\Delta T$	m = mass c = specific heat $\Delta T$ = temperature difference

**TABLE 2. SUMMARY OF COMMON UNITS USED IN  
ENERGY CONVERSION CALCULATIONS**

<b>Quantity</b>	<b>Symbol</b>	<b>English Unit</b>	<b>SI Unit</b>
Mass	m	slug or (lb·sec <sup>2</sup> /ft)	kg
Weight	w	lb	N
Force	F	lb	N
Work	W	ft·lb	N·m or J
Kinetic Energy	E <sub>k</sub>	ft·lb	N·m or J
Potential Energy	E <sub>p</sub>	ft·lb	N·m or J
Power	P	(ft·lb)/sec; hp	(N·m)/sec or watt
Spring Constant	k	lb/ft; oz/in.	N/m; kgf/m *
Angular			
Displacement	$\theta$	rad	rad
Angular Speed	$\omega$	rad/sec	rad/sec
Linear Speed	v	ft/sec	m/sec
Mass Density	$\rho$	slug/ft <sup>3</sup> ; lbm/ft <sup>3</sup> **	kg/m <sup>3</sup>
Moment of Inertia	I	slug·ft <sup>2</sup>	kg/m <sup>2</sup>
Pressure or			
Pressure Difference	p; $\Delta p$	lb/ft <sup>2</sup> ; oz/in <sup>2</sup>	N/m <sup>2</sup>
Electrical Charge	q	—	coulomb (C)
Torque	T	lb·ft; lb·in.	N·m
Fluid Volume	V; $\Delta V$	ft <sup>3</sup>	m <sup>3</sup>
Electrical Current	I	—	ampere (A)
Electrical Voltage	V; $\Delta V$	—	volt (V)
Electrical Resistance	R	—	ohm ( $\Omega$ )
Heat Energy	H	Btu	cal
Temperature	T	°F	°C
Temperature			
Difference	$\Delta T$	F°	C°
Specific Heat	c	$\frac{\text{Btu}}{\text{lb} \cdot \text{F}^\circ}$	$\frac{\text{kcal}}{\text{kg} \cdot \text{C}^\circ}$

\* kgf  $\Rightarrow$  kilogram force

\*\* lbm  $\Rightarrow$  pound mass

**TABLE 3. USEFUL CONVERSIONS FOR UNITS**

1 slug	=	32 pound mass (32 lbm)
1 kilogram force (kgf)	=	9.8 newtons
1 newton-meter	=	1 joule
1 volt-coulomb	=	1 joule
1 ampere-second	=	1 coulomb
1 radian	=	57.3°
1 horsepower	=	550 ft·lb/sec
1 horsepower	=	746 watts
1 watt	=	1 newton-meter per second
1 watt	=	1 joule per second
1 watt	=	1 volt-ampere
1 watt·sec	=	1 joule = 0.7376 ft·lb
1 kilowatt-hour (kWh)	=	$3.6 \times 10^6$ joules
1 calorie	=	3.086 foot-pounds
1 calorie	=	4.184 joules
1 Btu	=	778 foot-pounds
1 Btu	=	1055 joules

Review the contents of Tables 1, 2 and 3. Use these tables to help you answer the following questions.

1. Consider a body of mass  $m$  and specific heat  $c$  that undergoes an increase in temperature of  $\Delta T$ . The formula for the amount of heat energy absorbed by this body is \_\_\_\_.
2. The formula for electrical work is \_\_\_\_.
3. One volt-coulomb of work equals one \_\_\_\_ of work.
4. One watt-sec of electrical energy is equal to one \_\_\_\_.
5. The formula for electrical energy used, given the electrical power and time of operation, is \_\_\_\_.
6. A thermal energy convertor changes 2 Btu of thermal energy into 1400 ft·lb of rotational mechanical energy. Two Btu of energy equals \_\_\_\_ ft·lb of energy.
7. The percent efficiency of the thermal energy convertor in Question 6 is \_\_\_\_ %.
8. A thermal energy convertor changes  $0.9 \times 10^6$  calories of thermal energy into 1 kWh of electrical energy.
  - a. One calorie equals \_\_\_\_ joules.
  - b. One kWh equals \_\_\_\_ joules.
  - c.  $0.9 \times 10^6$  calories equals \_\_\_\_ joules.
9. The percent efficiency of the thermal energy convertor in Question 8 is \_\_\_\_ %.
10. A thermal energy convertor changes 1 Btu of thermal energy into 1000 N·m of fluid energy. One Btu equals \_\_\_\_ N·m.
11. The percent efficiency of the thermal energy in Question 10 is \_\_\_\_ %.

#### EXAMPLES OF ENERGY-CONVERSION PROBLEMS.

Study these two examples of energy-conversion problems. Then solve the exercises that follow.

##### Example A: Efficiency of a Fuel-Oil-Fired Boiler

**Given:** An oil-fired boiler rated at 250 hp output uses 354 Btu of heat energy each second at the input while burning the oil. The output energy is changed to fluid energy in steam that will eventually be used to run a turbine.

**Find:** The boiler efficiency.

**Solution:** The formula for efficiency is

$$\text{Eff}(\%) = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% \quad \text{where: } P_{\text{OUT}} = \text{Boiler output power} = 250 \text{ hp}$$

$$P_{\text{IN}} = \text{Boiler input power} = 354 \text{ Btu/sec}$$

Since  $P_{\text{OUT}}$  and  $P_{\text{IN}}$  are in different units, we must convert them to the *same form* before finding the efficiency. Let's convert  $P_{\text{IN}}$  from Btu/sec to hp. From Table 3 we get the conversions used below.

$$P_{\text{IN}} = \frac{354 \text{ Btu}}{\text{sec}} \times \frac{1055 \text{ J}}{1 \text{ Btu}} \times \frac{1 \text{ watt}}{1 \text{ J/sec}} \times \frac{1 \text{ hp}}{746 \text{ watts}}$$

$$P_{\text{IN}} = \left[ \frac{354 \times 1055 \times 1 \times 1}{1 \times 1 \times 746} \right] \left[ \frac{\cancel{\text{Btu}} \times \cancel{\text{J}} \times \cancel{\text{watt}} \times \text{sec} \times \text{hp}}{\cancel{\text{sec}} \times \cancel{\text{Btu}} \times \cancel{\text{J}} \times \cancel{\text{watt}}} \right] \quad (\text{Cancel units.})$$

$$P_{\text{IN}} = 501 \text{ hp (rounded)}$$

So,  $P_{\text{IN}} = 354 \text{ Btu/sec} = 501 \text{ hp}$ . Now find efficiency of the boiler.

$$\text{Eff}(\%) = \frac{250 \cancel{\text{hp}}}{501 \cancel{\text{hp}}} \times 100\% \quad (\text{Cancel units.})$$

$$\text{Eff}(\%) = 0.499 \times 100\%$$

$$\text{Eff}(\%) = 50\% \text{ (rounded)}$$

### Example B: Thermo-Photo-Voltaic (TPV) Energy Converter

**Given:** A TPV energy converter uses heat energy to produce electrical energy. In one particular application, the infrared radiation from an incandescent filament is focused onto the 1.8-cm<sup>2</sup> sensitive area of a TPV semiconducting material. The resulting thermal energy is changed by the semiconductor material to 51.9 millijoules of electrical energy each second. The efficiency of the TPV energy converter is known to be 35%.

**Find:** The input thermal energy in calories produced each second as a result of the radiation from the incandescent source striking the semiconductor material.

**Solution:** The efficiency of this energy converter is given by the equation:

$$\text{Eff}(\%) = \frac{E_{\text{OUT}}}{E_{\text{IN}}} \times 100\% \quad \text{where: } \begin{aligned} \text{Eff}(\%) &= 35\% \\ E_{\text{OUT}} &= 51.9 \times 10^{-3} \text{ J each second} \\ E_{\text{IN}} &= \text{unknown input thermal energy} \end{aligned}$$

Solve for  $E_{\text{IN}}$ .

$$E_{\text{IN}} = \frac{E_{\text{OUT}}}{\text{Eff}(\%)} \times 100\%$$

Substitute the known values to find  $E_{\text{IN}}$  in joules.

$$E_{\text{IN}} = \frac{51.9 \times 10^{-3} \text{ J}}{35\%} \times 100\% \quad (\text{Cancel \% symbols.})$$

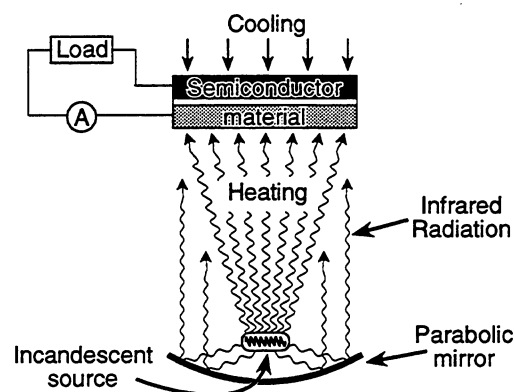
$$E_{\text{IN}} = 0.148 \text{ J (rounded)}$$

Then convert  $E_{\text{IN}}$  to calories.

$$E_{\text{IN}} = 0.148 \cancel{\text{ J}} \times \frac{1 \text{ cal}}{4.184 \cancel{\text{ J}}} \quad (\text{Cancel units.})$$

$$E_{\text{IN}} = 0.0354 \text{ cal (rounded)}$$

The thermal energy that falls on the sensitive area of the semiconductor material each second equals 0.0354 calorie.



### PRACTICE EXERCISES

**Problem 1:** **Given:** The diesel engine in an 18-wheel tractor-trailer produces 482 hp (that is, 360 kW of power). The dashboard tachometer shows that the engine is operating at 2100 rpm. The output power is stored as rotational mechanical energy in the engine flywheel. It's used to do work to run the tractor-trailer. The fuel burned each second within the engine provides  $1.1 \times 10^6 \text{ J}$  of thermal input energy. It's converted to mechanical energy by the engine.

- Find:**
- The diesel-engine efficiency in changing thermal input energy to mechanical output energy.
  - The amount of thermal energy lost as waste heat through the cooling system and exhaust.

**Solution:**

**Problem 2:** Given: A power-plant, low-speed, diesel engine operates at 110 rpm. It delivers 12,500 hp (that is, 9.33 megawatts) each second to the shaft of the power-plant generator. The efficiency of this type of diesel engine is in a range of 22% to 30%.

Find: The amount of heat energy in Btu converted to mechanical energy at a 25% efficiency.

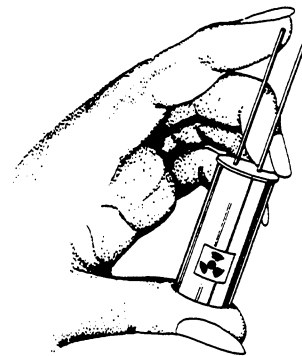
Solution:

**Problem 3:** Given: A thermoelectric generator is used to provide power for a weather satellite. It uses enriched uranium to provide heat energy and a silicon-germanium thermopile to convert the heat energy into electrical energy. The uranium provides heat energy equal to 3 calories each second. The thermopile generates 10 watts of power from the heat energy provided.

Find: The efficiency of the thermoelectric generator.

Solution:

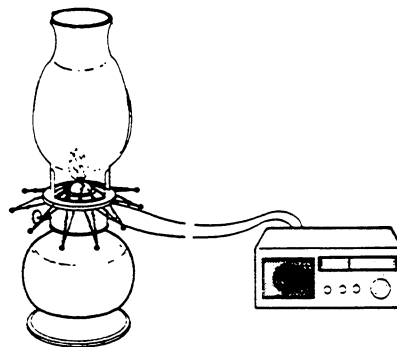
**Problem 4:** Given: An implantable, heart pacemaker uses a nuclear isotope battery as a heat source for the warm junction of a thermopile. The other junction uses the body temperature of 98°F as the cold junction. Five such batteries provide  $60 \times 10^{-3}$  Btu of thermal energy each second to the thermopile. The thermopile provides 40 watt-sec of energy to operate the pacemaker.



Find: The efficiency of the pacemaker power-supply unit.

Solution:

**Problem 5:** Given: In the 1930s and 1940s, rural Russians used an ordinary oil lamp to provide light. At the same time, an array of thermocouples (a thermopile) provided power to operate a radio, as shown in the drawing. The radio needed 40 watts of power to operate. The thermoelectric generator was 40% efficient.



Find: The heat energy in Btu the oil lamp had to provide each second to the hot junction of the thermopile to operate the radio.

Solution: