Math Lab 11 MS 4

Solving Problems That Involve Thermal Transducers

For best results, print this document front-to-back and place it in a three-ring binder. Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach your students how to solve problems that involve thermal system transducers.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to this class. (How much you accomplish will depend on the math skills your students already have.) Summarize the explanatory material for the activity: "Solving Problems That Involve Thermal Transducers." Then have students complete the Practice Exercises given at the end of the activity.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 11T1/11T2, "Building a Temperature-Control System". This is a design laboratory that will extend over two lab periods. Stimulate their interest! Be sure to impress upon them the importance of reading about the design lab before coming to class.

Math Skills Laboratory

Lab 11 MS 4

MATH ACTIVITY

Activity: Solving Problems That Involve Thermal Transducers

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve problems that involve the selection and use of thermal transducers.***
2. ***Rearrange equations and substitute correct units to solve for an unknown quantity.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Work the problems.***

ACTIVITY

Solving Problems That Involve Thermal Transducers

MATERIALS

For this activity, you'll need a pencil, paper and a calculator.

DISCUSSION

In this lab, you'll practice solving problems that involve thermal transducers. The problems are similar to those a technician might meet while troubleshooting a thermal system. They're also the type of problems technicians face when they select thermal transducers for a new system.

Thermal transducers are used in many ways, and there are many different types available. Knowing about transducer characteristics such as ***sensitivity, linearity*** and ***temperature range*** (usually spelled out on the specification sheet) is important. Understanding these characteristics will help you choose the right transducer for the right job.

ANSWERS TO PRACTICE EXERCISES

Problem 1: a. $I = \dfrac{V}{R}$ where: $V = 1406 \text{ mV at } 25°C = 1406 \times 10^{-3} \text{ V}$
$R = 2250\ \Omega \text{ at } 25°C$

$$I = \frac{1406 \times 10^{-3} \text{ V}}{2250\ \Omega} = 0.6248 \times 10^{-3} \text{ A} = 0.625 \times 10^{-3} \text{ A}$$

$I = 625\ \mu A$ or 0.625 milliamps.

b. ΔV = change in temperature x sensitivity

$$\Delta V = (37°C - 25°C) \times 30 \frac{\text{mV}}{°C}$$

$$\Delta V = 12\ C° \times 30 \frac{\text{mV}}{°C} = 360 \text{ mv} = 360 \times 10^{-3} \text{ V}$$ (a voltage decrease)

Therefore, the change in resistance ΔR is:

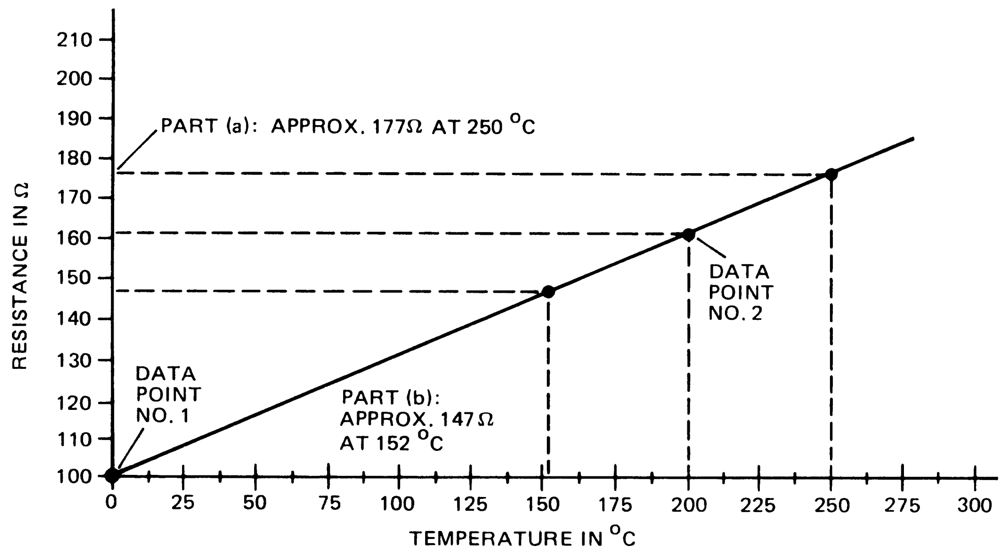
$$\Delta R = \frac{\Delta V}{I} = \frac{360 \times 10^{-3} \text{ V}}{625 \times 10^{-6} \text{ A}} = 0.576 \times 10^{3} \frac{V}{A} = 576\ \Omega; \ (1 \frac{V}{A} = 1\ \Omega)$$

$\Delta R = 576\ \Omega$ (a resistance decrease)

The resistance at 37°C is the resistance at 25°C **MINUS** the change in resistance due to the 12C° temperature **INCREASE.** Therefore: $R = 2250\ \Omega - 576\ \Omega = 1674\ \Omega$ at 37°C.

Answer to Problem 2 follows on Page T-112c.

Problem 2: a. Given data points 100 Ω at 0°C and 161.5 Ω at 200°C. Plot and draw a straight line through the points. Then read approximate resistance from graph for a temperature of 250°. The answer is close to 177 Ω, as shown below.



b. Calculate the sensitivity (ΔR/ΔT).

where: $\Delta R = 61.5\ \Omega$

$\Delta T = 200\ C°$

$$\frac{\Delta R}{\Delta T} = \frac{61.5\ \Omega}{200\ C°} = 0.3075\ \Omega/C°$$

To find "R" at 152°, use Data Point 1 (100 Ω at 0°C) and sensitivity data as follows:

$$R_{152°} = R_{0°C} + \frac{\Delta R}{\Delta T}(152°C - 0°C) = 100\ \Omega + 0.3075\ \frac{\Omega}{\cancel{C°}}\ (152\ \cancel{C°})$$

$$R_{152°} = 100\ \Omega + 46.74\ \Omega$$

$R_{152°} = 146.7\ \Omega$. Check on graph for part (b) shows calculation to be correct. See graph above for resistance at 152°.

PRACTICE EXERCISES

Solve the following problems.

Problem 1: The electrical resistance of thermistors (devices constructed of semiconductor materials) ***decreases*** with an ***increase*** in temperature. They're often used in the temperature probes that are part of a solid-state thermometer.

Given: A thermistor probe used with an oral thermometer has the following data on its specifications sheet.

Sensitivity: 30 mV/C°
Linear Accuracy: ±0.1% from 0°C to 100°C
Resistance: 2250 Ω at 25°C
Voltage: 1406 mV at 25°C

Find:

a. The current flowing in the thermistor at 25°C, with voltage and resistance as given on the specification sheet.
b. The resistance of the thermistor when it shows normal body temperature (37°C). (To solve this part, assume that the solid-state thermometer works as a ***constant-current*** device. That means that the voltage and resistance change in step, as the temperature increases or decreases. The current remains at the constant value calculated for Part "a" of this problem.)

Solution:

a. ***Hint:*** Use Ohm's law ($I = V/R$) to find I.
b. ***Hint:*** 1. First, use the data from the specification sheet to find the change in voltage ($\triangle V$) as the temperature increases from 25°C to 37°C. (Think carefully about whether the voltage will increase or decrease.)
2. Then use the change in voltage ($\triangle V$), the constant current (I) from Part "a," and Ohm's law ($\triangle R = \triangle V/I$) to find the change in resistance ($\triangle R$). (Does the resistance increase or decrease?)

Problem 2: The resistance-temperature detector (RTD) used by industry is a 99.9%-pure platinum wire that's wound around a ceramic or glass core. It's sealed within a glass or ceramic capsule. It works on the principle of change in electrical wire resistance as a function of temperature. For this device as temperature ***rises*** the resistance ***increases.*** (Note that this is opposite from the way a thermistor behaves.)

Given: The plastic extruder—shown in the video making plastic key-ring tabs—uses RTD probes in the 0°C to 300°C range. The RTD probes control the temperature of the zone heaters that melt the plastic for extrusion. At a temperature of 0°C, the RTD resistance is 100 Ω. At 200°C, the RTD resistance is 161.5 Ω.

Find:

a. The RTD resistance that the zone-heater control responds to at a temperature of 482°F (250°C). (Assume that the response of the RTD is ***linear*** over the entire temperature range from 0°C to 300°C. This will allow you to plot temperature versus resistance on a graph and read the resistance at 250°C from the curve directly. Use the two data points given above to plot the straight line.)

Problem 3:

a. Select the 0.032 inch E-type or the 0.005 inch K-type. The E-type has the highest sensitivity.

b. The K-type at 0.032 inches will reach temperatures up to 1800°F. If temperatures are near the 2500°F end of the range, the R-type will have to be used, with a sacrifice in mV output. This means, of course, less sensitivity.

c. At 0°F, the mV output is 0. At 1000°F, the mV output is about 38 mV. The sensitivity is equal to $\Delta V/\Delta T$:

where: $\Delta V = 38 \text{ mV} - 0 \text{ mV} = 38 \text{ mV}$

$\Delta T = 1000°F - 0°F = 1000 \text{ F°}$

Therefore,

$$\frac{\Delta V}{\Delta T} = \frac{38 \text{ mV}}{1000 \text{ F°}}$$

$$\frac{\Delta V}{\Delta T} = 0.038 \text{ mv/F°}.$$

d. A type-K thermocouple is a good, all-around choice. The slope of the curve is linear with 0.032 in wire, and max temperatures of 1800°F can be measured. Furthermore, the sensitivity (about 0.22 mV/F°) is not too bad! If more sensitivity is desired, a type-E thermocouple can be used. However, upper temperature measurements should not exceed 1100°F (for the 0.032-inch wire size).

b. The RTD resistance at 152°C. Remember that the resistance increased from 100 Ω at 0°C to 161.5 Ω at 200°C. That's a difference of 61.5 Ω for a 200-C° temperature range. Remember also that the RTD operates as a ***linear*** device. (Use the graph only to check the ***"Calculations."***)

Solution:

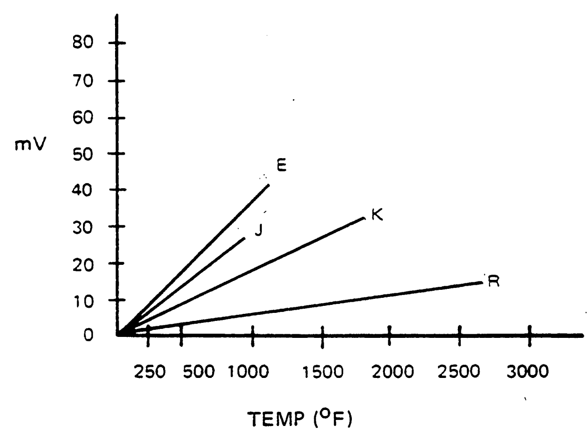
Problem 3: Thermocouples made of unsheathed, beaded-junction, fine wire have many uses in biophysics, metal cutting, scientific instruments, internal combustion engines, calorimetry, thermoelectric cooling and other areas. The voltage and temperature range is determined by the wire material that's used to make the thermocouples. The graph shown gives output voltage (mV) versus junction temperature difference for different thermocouples. The American National Standards Institute (ANSI) symbols for each thermocouple are given below the graph. The table shows maximum service temperatures for various wire sizes of different thermocouple types.

Given: Data in graph and table.

Find:

a. The ***type*** and ***size(s)*** thermocouple to use for a voltage response to temperature change in the 500°F to 1000°F range.
b. The thermocouple to choose for temperatures from 0°F to 2500°F.
c. The sensitivity of a type-E thermocouple (from the graph) in mV/F°.
d. Which type of thermocouple is a good all-around choice and why.

Solution:



MAXIMUM SERVICE TEMPERATURE

Thermocouple	Wire Diameters (in inches)			
Type	0.005	0.015	0.020	0.032
J	600 °F	700 °F	700 °F	900 °F
K	1100	1600	1600	1800
T	400	400	400	500
E	700	800	800	1100
R,S	— —	— —	2642	2642

Problem 4: $V = k\Delta T$ $\qquad k = 50\ \mu V/C°$ (sensitivity)

$\Delta T = (100°C - 20°C) = 80\ C°$

$$V = \frac{50 \times 10^{-6}\ V}{\cancel{C°}} \times 80\ \cancel{C°}$$

$$V = 4000 \times 10^{-6}\ V = 4 \times 10^{-3}\ V = 4\ mV$$

Problem 5: Use the equation, $\ell_F = \ell_O\ (1 + \tau \times \Delta T)$.

where: $\ell_O = 20$ cm

$$\tau = \frac{25 \times 10^{-6}}{C°}$$

$$\Delta T = (100°C - 20°C) = 80\ C°$$

$$\ell_F = 20\ cm\ (1 + \frac{25 \times 10^{-6}}{\cancel{C°}} \times 80\ \cancel{C°})$$

$$\ell_F = 20\ cm\ (1 + 2000 \times 10^{-6})\ ; \quad (2000 \times 10^{-6} = 0.002)$$

$$\ell_F = 20\ cm\ (1.002)$$

$$\ell_F = 20.04\ cm.$$

The 20-cm aluminum rod increases in length by 0.04 cm for the 80 C° temperature increase.

Note: Some students may be confused by the addition of $(1 + 2000 \times 10^{-6})$. They may think the answer is 2001×10^{-6}. Be sure to point out that the number 1 is added to a small number (2000×10^{-6}). Therefore, $1 + 2000 \times 10^{-6} = 1 + 0.002 = 1.002$.

Problem 6: Use the equation of ratios:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

where: $p_1 = 120$ psi

$p_2 =$ unknown

$T_1 = 20°C + 273.15 = 293.15\ K$

$T_2 = 100°C + 273.15 = 373.15\ K$

Rearrange the equation, $\frac{p_1}{T_1} = \frac{p_2}{T_2}$, to isolate the pressure (p_2).

Then solve for "p_2":

$$p_2 = \frac{p_1 T_2}{T_1}$$

$$p_2 = \frac{120\ psi \times 373.15\ \cancel{K}}{293.15\ \cancel{K}} = 120\ psi \times 1.2728$$

$p_2 = 152.7$ psi (about 153 psi).

Problem 4: Given: A certain type thermocouple has a sensitivity (voltage out/temperature difference) of $k = 50\ \mu V/C°$. The voltage generated equals the thermocouple sensitivity times the temperature difference between junctions. (Voltage Out = $k \times \Delta T$)

Find: The voltage generated when the thermocouple junctions are at temperatures of 20°C and 100°C.

Solution:

Problem 5: Given: A solid object expands in size in proportion to its temperature. The new length can be predicted from the following equation:

$$\ell_{final} = \ell_{original}\,(1 + \tau \times \Delta T)$$

where: ℓ = length

τ = the thermal coefficient of linear expansion for a particular material

T = temperature

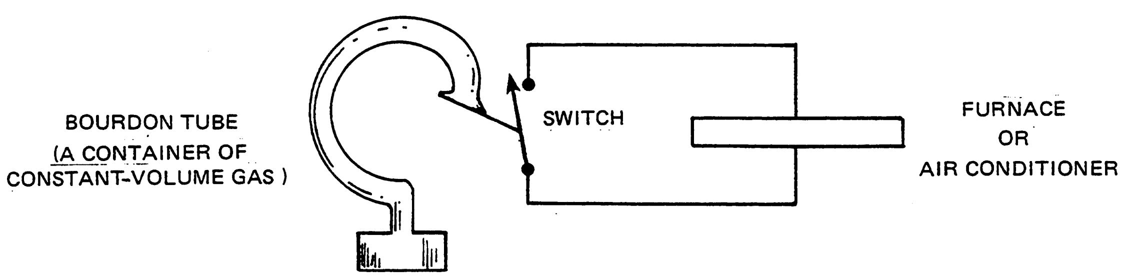
$\Delta T = T_{new} - T_{original}$

Find: The new length of an aluminum rod 20 cm long after it's heated from 20°C to 100°C. The thermal coefficient of linear expansion for aluminum is $25 \times 10^{-6}/C°$, and the original length of the rod is 20 cm at 20°C.

Solution:

Problem 6: Discussion—A "gas thermostat" is a thermal transducer that opens and closes a set of electrical contacts when the pressure of the confined gas reacts to temperature changes.

If a gas is kept in a container at constant volume, and only the pressure and temperature are allowed to vary, the ratio of gas pressure and temperature is a constant (C) expressed by $C = P_1/T_1 = P_2/T_2$. Here, the subscript "1" indicates absolute pressure and temperature in °K in state 1. Subscript "2" indicates the same values in state 2 of the gas. Temperatures in degrees Kelvin (°K) can be found by simply adding 273.15 to the temperature in degrees Celsius. That is, °K = °C + 273.15.



Given: The gas in a constant-volume gas thermostat has a pressure of 120 psi at a temperature of 20°C (state 1).

Find: The pressure of the same gas at 100°C (state 2).

Solution: First, convert temperature in °C to °K. Then use the ratio, $P_1/T_1 = P_2/T_2$, to solve for "P_2."