Math Lab 8 MS 2

Solving Angular Momentum Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach students how to substitute values and units in angular momentum equations.
2. Teach students how to use angular impulse and angular momentum equations to solve problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to class. (How much is accomplished depends on the math skills that your students already have.)
   a. Summarize the explanatory material for "Activity: "Solving Angular Momentum Problems."
   b. Then have students complete Practice Exercises given at the end of the Math Activity.
3. Supervise student progress. Have students obtain the correct answers.
4. Before the class ends, tell your students to read the "double" experiment, Lab 8F1/8F2, "Impulse and Momentum in Fluid Systems."

Math Skills Laboratory

Lab 8 MS 2

MATH ACTIVITY
Solving Angular Momentum Problems

MATH SKILLS LAB OBJECTIVES
When you complete this activity, you should be able to do the following:

1. ***Substitute appropriate numerical values and units in angular momentum equations. Solve the equations for unknown numerical values with the proper units.***
2. ***Use the following equations to solve angular momentum problems.***
   a. $L_{mom} = I \times \omega$
   b. $T \times \Delta t = \Delta(I\omega)$
   c. $Ang\ Imp = T\Delta t$

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

ACTIVITY
Solving Angular Momentum Problems

MATERIALS
For this activity, you'll need a calculator.

In this lab, you'll review basic units. You'll also solve problems that involve angular momentum, angular impulse and changes in angular momentum. The important equations, with typical units, are summarized on the following page.

NOTE: Have students refer to Equations 1 through 4 and the associated units to check their problem-solving procedures and final answers when working the assigned exercises.

NOTE: Your students may still have difficulty understanding that (lb·ft)·sec is equivalent to slug·ft^2/sec, and that (N·m)·sec equals kg·m^2/sec. To help them understand these relationships, you may want to review the following unit analysis.

For English Units:

$$\text{Ang Imp} = T \times \Delta t$$
$$\text{Ang Imp} = (F \times \ell) \times \Delta t$$
$$\text{Ang Imp} = \text{lb} \times \text{ft} \times \text{sec}$$
$$\text{Ang Imp} = (\text{lb}\cdot\text{ft})\cdot\text{sec}$$

where: $T = F \times \ell$
F is in lb
ℓ is in ft
Δt is in sec

$$\Delta(\text{Ang Mom}) = I\Delta\omega$$

where: $I \approx mr^2$

$$\Delta(\text{Ang Mom}) = mr^2 \times \Delta\omega$$

$m = \dfrac{w}{g}$
w is in lb

$$\Delta(\text{Ang Mom}) = \frac{w}{g} \times r^2 \times \Delta\omega$$

g is in $\dfrac{\text{ft}}{\text{sec}^2}$
r is in ft

$$\Delta(\text{Ang Mom}) = \left(\frac{\text{lb}}{\frac{\text{ft}}{\text{sec}^2}}\right) \times \text{ft}^2 \times \frac{\text{rad}}{\text{sec}}$$

$\Delta\omega$ is in $\dfrac{\text{rad}}{\text{sec}}$

$$\Delta(\text{Ang Mom}) = \frac{\text{lb}\cdot\text{sec}^{\cancel{2}}}{\cancel{\text{ft}}} \times \text{ft}^{\cancel{2}} \times \frac{\text{rad}}{\cancel{\text{sec}}}$$

$$\Delta(\text{Ang Mom}) = (\text{lb}\cdot\text{ft})\cdot\text{sec}$$ (The unit "rad" has been dropped.)

For SI units:

$$\text{Ang Imp} = T \times \Delta t$$
$$\text{Ang Imp} = (F \times \ell) \times t$$
$$\text{Ang Imp} = (\text{N}\cdot\text{m})\cdot\text{sec}$$

where: $T = F \times \ell$
F is in N
ℓ is in m
Δt is in sec

$$\text{Ang Imp} = \left(\frac{\text{kg}\cdot\text{m}}{\text{sec}^{\cancel{2}}}\right)\cdot \text{m}\cdot\cancel{\text{sec}}$$

$1\ \text{N} = 1\ \dfrac{\text{kg}\cdot\text{m}}{\text{sec}^2}$

$$\text{Ang Imp} = \frac{\text{kg}\cdot\text{m}^2}{\text{sec}}$$

where: $I \approx mr^2$

$$\Delta(\text{Ang Mom}) = \Delta(I\omega) = I\Delta\omega$$
$$\Delta(\text{Ang Mom}) = mr^2 \times \Delta\omega$$

m is in kg
r is in m

$$\Delta(\text{Ang Mom}) = \text{kg}\cdot\text{m}^2 \times \frac{\text{rad}}{\text{sec}}$$

$\Delta\omega$ is in $\dfrac{\text{rad}}{\text{sec}}$

$$\Delta(\text{Ang Mom}) = \frac{\text{kg}\cdot\text{m}^2}{\text{sec}}$$ (The unit "rad" has been dropped.)

a. **Angular Momentum: $L_{mom} = I\omega$** ***Equation 1***

$$\left\{\begin{matrix}\text{Angular}\\ \text{Momentum}\end{matrix}\right\} = \left\{\begin{matrix}\text{Moment of Inertia}\\ \text{of Object}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Angular Velocity}\\ \text{of Object}\end{matrix}\right\}$$

English Units	*SI Units*
I in *slugs·ft^2	I in kg·m^2
ω in rad/sec	ω in rad/sec
L_{mom} in $\frac{\text{slug·ft}^2}{\text{sec}}$	L_{mom} in $\frac{\text{kg·m}^2}{\text{sec}}$

b. **Angular Impulse: Ang Imp = T × Δt** ***Equation 2***

$$\text{Angular Impulse} = \left\{\begin{matrix}\text{Torque Acting on}\\ \text{Object (or Fluid)}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Time That}\\ \text{Torque Acts}\end{matrix}\right\}$$

English Units	*SI Units*
T in lb·ft	T in N·m
Δ t in sec	Δ t in sec
Ang Imp in (lb·ft) sec	Ang Imp in (N·m) sec

c. **Angular Impulse and Angular Momentum Change: T × Δt = I × $\Delta\omega$** ***Equation 3***

$$\left\{\begin{matrix}\text{Torque Acting}\\ \text{on Object}\\ \text{(or Fluid)}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Time}\\ \text{During Which}\\ \text{Torque Acts}\end{matrix}\right\} = \left\{\begin{matrix}\text{Moment of Inertia}\\ \text{of Object}\\ \text{(or Fluid)}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Change in Angular}\\ \text{Velocity Caused}\\ \text{by the Torque}\end{matrix}\right\}$$

English Units	*SI Units*
T in lb·ft	T in N·m
Δ t in sec	Δt in sec
I in slug·ft^2	I in kg·m^2
$\Delta\omega$ in rad/sec	$\Delta\omega$ in rad/sec

Note: Since $T\Delta t = I\Delta\omega$, it's important to remember that the following relationships exist between units.

$$\text{(lb·ft)·sec} = \frac{\text{slug·ft}^2}{\text{sec}} \quad \text{and} \quad \text{(N·m)·sec} = \frac{\text{kg·m}^2}{\text{sec}}$$

d. **Conservation of Angular Momentum: $L_{mom\ before} = L_{mom\ after}$** ***Equation 4***

$$\left\{\begin{matrix}\text{Angular Momentum of Isolated}\\ \text{System BEFORE an Interaction}\end{matrix}\right\} = \left\{\begin{matrix}\text{Angular Momentum of Same}\\ \text{System AFTER the Interaction}\end{matrix}\right\}$$

English Units	*SI Units*
L_{mom} in $\frac{\text{slug·ft}^2}{\text{sec}}$	L_{mom} in $\frac{\text{kg·m}^2}{\text{sec}}$

* The mass of an object in slugs is equal to the weight of the object in pounds divided by 32 ft/sec^2.
In units, 1 slug = 1 lb·sec^2/ft.

NOTE: In the *Summary of Units* at the top of the page, the *change in angular momentum* is given as I x $\Delta\omega$. We have been using $\Delta(\text{Ang Mom}) = \Delta(I\omega)$. We showed earlier that $\Delta P_{mom} = \Delta(mv) = m\Delta v$ (when the mass is constant and only v changes). For $\Delta(L_{mom})$ we can write, from calculus, with complete validity:

$$\Delta(L_{mom}) = \Delta(I\omega) = I\Delta\omega + \omega\Delta I$$

But if the moment of inertia I does not change during the motion,

$$\Delta(I\omega) = I\Delta\omega,$$

since $\Delta I = 0$ causes the second term $\omega\Delta I$ to vanish. For most of the "industrial" problems involved in this unit, the moment of inertia I will be constant so $\Delta(I\omega) = I\Delta\omega$. (For the figure skater we discussed earlier, it was the moment of inertia I that changed as the skater moved arms in or out! And for Lab 8M1, the weights were moved to different positions along the spinning arm, so there too, the moment of inertia of the spinning assembly changed!)

SOLUTIONS TO REVIEW OF UNITS

a. $\dfrac{\text{slug}\cdot\text{ft}^2}{\text{sec}}$

b. $\dfrac{\text{kg}\cdot\text{m}^2}{\text{sec}}$

c. mass

d. (lb·ft)·sec

e. (N·m)·sec

f. $\dfrac{\text{slug}\cdot\text{ft}^2}{\text{sec}}$

g. $\dfrac{\text{kg}\cdot\text{m}^2}{\text{sec}}$

e. Summary of Units

Quantity	Symbol	English Units	SI Units
Moment of inertia	I	$slug \cdot ft^2$	$kg \cdot m^2$
Angular velocity (speed)	ω	rad/sec	rad/sec
Change in angular velocity	$\Delta\omega$	rad/sec	rad/sec
Torque	T	lb·ft	N·m
Time interval	Δt	sec	sec
Angular momentum	$I \times \omega$	$\frac{slug \cdot ft^2}{sec}$	$\frac{kg \cdot m^2}{sec}$
Change in angular momentum	$I \times \Delta\omega$	$\frac{slug \cdot ft^2}{sec}$	$\frac{kg \cdot m^2}{sec}$
Angular impulse	$T \times \Delta t$	(lb·ft)·sec	(N·m)·sec

LET'S REVIEW UNITS!

Before studying the ***Practice Exercises*** and solving the ***Problems,*** answer the following questions to check your understanding of the units given above. Fill in the blanks with the correct word or words.

a. The units for angular momentum in the English system are ____.
b. The units for angular momentum in SI are ____.
c. The slug is an English unit for ____ (mass; weight).
d. The units for angular impulse in the English system are ____.
e. The units for angular impulse in SI are ____.
f. The equation $T\Delta t = I\Delta\omega$ tells us that the units for angular impulse and change in angular momentum are equivalent. That means that 1 (lb·ft)·sec is equivalent to ____.
g. Based on Question f above, we can also say that 1 (N·m)·sec is equivalent to ____.

PRACTICE EXERCISES

Example 1: *Angular Momentum*

Given: An energy-storing flywheel (disk shape) on a punch press has a mass of 300 kg. It has a radius of 0.8 m. At full speed, the flywheel turns at 250 rpm.

Find: The angular momentum of the flywheel at full speed.

Solution: Use Equation 1 for the angular momentum of an object rotating about its own axis.

$$L_{mom} = I \times \omega$$

where: $I = \frac{1}{2} mr^2$ for a disk

$$m = 300 \text{ kg}$$
$$r = 0.8 \text{ m}$$
$$\omega = 250 \text{ rpm} = 250 \text{ rev/min}$$

Substitute for I and ω in the equation and solve.

$$L_{mom} = \left(\frac{1}{2} mr^2\right) \times \omega$$

$$L_{mom} = \left[0.5 \times 300 \text{ kg} \times (0.8 \text{ m})^2\right] \times \left[250 \frac{\cancel{rev}}{\cancel{min}} \times \frac{6.28 \text{ rad}}{1 \cancel{rev}} \times \frac{1 \cancel{min}}{60 \text{ sec}}\right]$$

$$L_{mom} = 96 \text{ kg}\cdot\text{m}^2 \times 26.17 \text{ rad/sec}$$

$$L_{mom} = 2512 \text{ kg}\cdot\text{m}^2/\text{sec}$$

At full speed, the flywheel has an angular momentum of 2512 $kg \cdot m^2/sec$.

SOLUTIONS TO ACTIVITY 1

Problem 1: Use equation 1.

$L_{mom} = I\omega$ where: $I = 1.15\ g\cdot cm^2$, $\omega = 70$ rad/sec

$$L_{mom} = 1.15\ g\cdot cm^2 \times 70\ \frac{rad}{sec}$$

$$L_{mom} = 1.15 \times 70\ g\cdot cm^2\ \frac{rad}{sec}$$

$$L_{mom} = 80.5\ \frac{g\cdot cm^2}{sec}$$

Problem 2: a. Use Equation 1. Solve for "I."

$L_{mom} = I\omega$ where: $L_{mom} = 0.455\ \frac{slug\cdot ft^2}{sec}$, $\omega = 15.7$ rad/sec

$$I = \frac{L_{mom}}{\omega}$$

$$I = \frac{0.455\left(\frac{slug\cdot ft^2}{sec}\right)}{15.7\ rad/sec} =$$

$$I = \frac{0.455}{15.7}\ \frac{slug\cdot ft^2}{\cancel{sec}}\cdot\frac{\cancel{sec}}{rad} \quad \text{(drop “rad”)}$$

$$I = 0.029\cdot slug\cdot ft^2$$

b. Use equation for a solid cylinder.

$$I = \frac{1}{2}mr^2 \quad \text{where: } m = \frac{w}{g} = \frac{120\ lb}{32\ \frac{ft}{sec^2}} = 3.75\ \frac{lb\cdot sec^2}{ft} = 3.75\ slug$$

$$r = \frac{D}{2} = \frac{3\ in.}{2} = 1.5\ in.$$

$$r = 1.5\ \cancel{in.} \times \frac{1\ ft}{12\ \cancel{in.}} = 0.125\ ft$$

$$I = (\frac{1}{2})(3.75\ slug)(0.125\ ft)^2$$

$$I = 0.5 \times 3.75 \times 0.0156\ slug\cdot ft^2$$

$I = 0.029\ slug\cdot ft^2$. Yes, the values of "I" are equal.

Example 2: *Angular Impulse and Change in Angular Momentum*

Given: The punch press in Example 1. When the operator engages the punch to shape or punch out a part, the energy-storing flywheel slows from 250 rpm (26.17 rad/sec) to 150 rpm (15.7 rad/sec) during the 6 seconds the punch is engaged.

Find:
a. The change in angular momentum.
b. The angular impulse.

Solution:
a. The change in angular momentum $\Delta(I\omega)$ comes from the change in angular velocity $\Delta\omega$. Therefore the change in angular momentum = $I\Delta\omega$, or

$$\Delta(I\omega) = I\Delta\omega$$

where: $I = \frac{1}{2}mr^2$ for a disk = 96 kg·m^2 (from Example 1)

$\Delta\omega$ = change in angular velocity, $\omega_f - \omega_i$

ω_f = 15.7 rad/sec, the final angular velocity

ω_i = 26.17 rad/sec, the initial angular velocity

First determine the change in angular velocity, $\Delta\omega$.

$$\Delta\omega = \omega_f - \omega_i$$
$$\Delta\omega = 15.7 \text{ rad/sec} - 26.17 \text{ rad/sec}$$
$$\Delta\omega = -10.47 \text{ rad/sec}$$

Ignore the minus sign. It simply tells us that the flywheel is *slowing down.* Substitute the values for I and $\Delta\omega$ into the equation.

$$\Delta(I\omega) = I\Delta\omega$$
$$\Delta(I\omega) = 96 \text{ kg·m}^2 \times (10.47 \text{ rad/sec})$$
$$\Delta(I\omega) = (96 \times 10.47)(\text{kg·m}^2 \times \text{rad/sec})$$
$$\Delta(I\omega) = 1005 \text{ kg·m}^2\text{/sec}$$

b. From the equation, $T \times \Delta t = \Delta(I\omega)$, we know that the angular impulse $T \times \Delta t$ is equal to the change in angular momentum $\Delta(I\omega)$. Since we calculated $\Delta(I\omega)$ in Part a above, we have the answer to Part b.

Angular Impulse $T\Delta t = \Delta(I\omega)$

Angular Impulse $T\Delta t$ = 1005 kg·m^2/sec, or 1005 (N·m)·sec

Problem 1: Given: A bullet fired from a rifle is given a rotational motion of 70 rad/sec by the spiral grooves in the rifle bore. The moment of inertia of the bullet is 1.15 g·cm^2.

Find: The angular momentum of the bullet.

Solution:

Problem 2: Given: The 120-lb drive shaft of a gravel conveyor has a diameter of 3 inches. It has an angular momentum of 0.455 slug·ft^2/sec when rotating at 150 rpm (15.7 rad/sec).

Find:
a. The moment of inertia I of the drive shaft by solving for I in the equation, $L_{mom} = I \times \omega$.
b. The moment of inertia I of the drive shaft, calculated using the equation for a cylinder revolving about its center axis, $I = \frac{1}{2}mr^2$. Remember: $m = \frac{w}{g}$ and 1 slug = 1 lb·sec^2/ft.

Solution:

SOLUTIONS TO ACTIVITY 1, Continued

Problem 3: a. $I = \frac{1}{2} mr^2.$ where: $m = 10\ g$, $r = 0.48\ cm$

$I = (\frac{1}{2})\ (10\ g)\ (0.48\ cm)^2$

$I = 0.5 \times 10 \times 0.23\ g{\cdot}cm^2$

$I = 1.15\ g{\cdot}cm^2$

b. $L_{mom} = I\omega$ where: $L_{mom} = 57.6\ \frac{g{\cdot}cm^2}{sec}$, $\omega = 50\ rad/sec$

$I = \frac{L_{mom}}{\omega}$

$$I = \frac{57.6\ \frac{g\cdot cm^2}{sec}}{50\ \frac{rad}{sec}}$$

$I = 1.15\ \frac{g{\cdot}cm^2 \cdot \cancel{sec}}{\cancel{sec}}$ (drop the "rad" unit)

$I = 1.15\ g{\cdot}cm^2$

Problem 4: Use Equation 3.

$T\Delta t = \Delta(I\omega)$ where: $T = F \times L$ (Solve for "F.")

$(F \times L)\Delta t$ $T\Delta t = 50\ (lb{\cdot}ft){\cdot}sec$

$L = 3\ in = 0.25\ ft$

$\Delta t = 2\ sec$

$F = \frac{\Delta I\omega}{L \times \Delta t}$

$F = \frac{50\ (lb{\cdot}ft){\cdot}sec}{0.25\ ft \times 2\ sec}$

$F = \frac{50}{0.25 \times 2}\ \frac{lb{\cdot}\cancel{ft}{\cdot}\cancel{sec}}{\cancel{ft}{\cdot}\cancel{sec}}$

$F = 100\ lb$

The solutions to Problems 5 and 6 are on Page T-57c.

SOLUTIONS TO ACTIVITY 1, Continued

Problem 5: Use Equation 2.

$$\text{Ang Imp} = T\Delta t$$

where:
$T = F \times L$
$F = 40 \text{ lb}$
$L = 11\text{-in. radius}$
$\Delta t = 20 \text{ sec}$

$$\text{Ang Imp} = F \times L \times \Delta t$$
$$\text{Ang Imp} = 40 \text{ lb} \times 11 \text{ in.} \times 20 \text{ sec}$$
$$\text{Ang Imp} = (40 \times 11 \times 20) \text{ lb}\cdot\text{in.}\cdot\text{sec}$$
$$\text{Ang Imp} = 8800 \text{ lb}\cdot\text{in.}\cdot\text{sec}$$

Change units to lb·ft·sec.

$$\text{Ang Imp} = 8800 \text{ lb}\cdot\text{in.}\cdot\text{sec} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{8800 \times 1}{12}\left(\frac{\text{lb}\cdot\cancel{\text{in.}}\cdot\text{sec}\cdot\text{ft}}{\cancel{\text{in.}}}\right)$$

$$\text{Ang Imp} = 733.3 \text{ lb}\cdot\text{ft}\cdot\text{sec.}$$

Problem 6: Use Equation 2 to find angular impulse of the air.

$$\text{Ang Imp} = T\Delta t$$

where:
$T = F \times L$
$F = 30 \text{ N}$
$L = 7.5 \text{ cm}$ = radius of intake turbine
$= 0.075 \text{ m}$
$\Delta t = 1 \text{ sec}$

$$\text{Ang Imp} = F \times L \times \Delta t$$
$$\text{Ang Imp} = 30 \text{ N} \times 0.075 \text{ m} \times 1 \text{ sec}$$
$$\text{Ang Imp} = 30 \times 0.075 \times 1 \quad \text{N}\cdot\text{m}\cdot\text{sec}$$
$$\text{Ang Imp} = 2.25 \text{ N}\cdot\text{m}\cdot\text{sec.}$$

Problem 3: Given: A 10-g bullet fired from a rifle has a rotational motion of 50 rad/sec. It has an angular momentum of 57.6 $g{\cdot}cm^2/sec$. The 155-grain (10-g) bullet is a 38-caliber (0.38 inch in diameter) bullet. The radius of the bullet is 0.48 cm.

Find:

a. The moment of inertia I of the bullet using $I = \frac{1}{2} mr^2$ —the formula for a solid cylinder. (Assume that the bullet shape approximates that of a solid cylinder.)

b. The moment of inertia I of the bullet by solving for I in the equation, $L_{mom} = I\,\omega$.

Solution:

Problem 4: Given: An air cylinder operates a ball-type water valve on a pollution-control scrubber. The valve handle is a 3-inch lever arm. In 2 seconds, it must turn 90° (1.57 rad) from fully open to fully closed. The angular momentum of the valve goes from zero to 50 $slug{\cdot}ft^2/sec$ when the force is applied.

Find: The force F applied by the air cylinder to close the valve.

Solution: [***Hint:*** Use the equation, $T\Delta t = I\Delta\omega$. Substitute $F \times \ell$ for T. Rearrange the equation to isolate F. Also remember that 50 $slug{\cdot}ft^2/sec$ = 50 $(lb{\cdot}ft){\cdot}sec$.]

Problem 5: Given: To balance an automobile wheel without removing the wheel from the car, Ron jacks up the car wheel and uses a portable wheel-spinning machine. This machine applies a 40-lb force to the tread part of the 11-in.-radius wheel and tire for 20 seconds.

Find: The angular impulse given to the wheel.

Solution: (***Hint:*** Use the equations, Ang Imp = $T\Delta t$ and $T = F\ell$.)

Problem 6: Given: Turbochargers are used to increase the horsepower in large diesel engines. The engine exhaust gas is used to spin a turbine with blades of radius 7.5 cm in the intake air passage of the engine. The tips of the intake turbine blades apply a 30-N force to the free air entering the engine. This gives the air an angular momentum around the inside perimeter of the turbocharger intake housing.

Find: The angular impulse delivered by the blades to the air in one second. (Use the equations, Ang Imp = $T\Delta t$ and $T = F\ell$.)

Solution: