Math Skills Laboratory

Lab 8 MS 1

MATH ACTIVITY

Solving Linear Momentum Problems

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. ***Substitute correct numerical values and units in momentum equations. Solve the equations for an unknown numerical value with the proper units.***
2. ***Use the following equations to solve linear momentum problems.***
   a. $P_{mom} = m \times v$ b. $Imp = F\Delta t$ c. $F\Delta t = \Delta(mv) = m\Delta v$
   d. P_{mom} ***BEFORE collision*** = P_{mom} ***AFTER collision***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

ACTIVITY

Solving Linear Momentum Problems

MATERIALS

For this activity, you'll need a calculator.

In this lab, you'll review basic units. You'll also solve problems that involve linear momentum, impulse and changes in momentum. The important equations, with typical units, are summarized.

a. Linear Momentum: $P_{mom} = mv$ *Equation 1*

$$\left\{\begin{matrix}\text{Linear}\\ \text{Momentum}\end{matrix}\right\} = \left\{\begin{matrix}\text{Mass of Object}\\ \text{(or Fluid)}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Velocity of Object}\\ \text{(or Fluid)}\end{matrix}\right\}$$

English Units	*SI Units*
m in slugs *	m in kg
v in ft/sec	v in m/sec
P_{mom} in $\frac{\text{slug·ft}}{\text{sec}}$	P_{mom} in $\frac{\text{kg·m}}{\text{sec}}$

* The mass of an object in slugs is equal to the weight of the object in pounds divided by 32 ft/sec^2. In units, 1 slug = 1 lb·sec^2/ft.

b. Impulse: Imp = F × Δt *Equation 2*

$$\text{Impulse} = \left\{\begin{matrix}\text{Force Acting on} \\ \text{Object (or Fluid)}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Time That} \\ \text{Force Acts}\end{matrix}\right\}$$

English Units	*SI Units*
F in lb	F in N
Δt in sec	Δt in sec
Imp in lb·sec	Imp in N·sec

c. Impulse and Momentum Change: F × Δt = m × Δv *Equation 3*

$$\left\{\begin{matrix}\text{Force Acting} \\ \text{on Object}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Time During} \\ \text{Which Force Acts}\end{matrix}\right\} = \left\{\begin{matrix}\text{Mass of} \\ \text{Object}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Change in Velocity} \\ \text{Caused by the Force}\end{matrix}\right\}$$

English Units	*SI Units*
F in lb	F in N
Δt in sec	Δt in sec
m in slugs (same as $\frac{\text{lb}\cdot\text{sec}^2}{\text{ft}}$)	m in kg
Δv in ft/sec	Δv in $\frac{\text{m}}{\text{sec}}$

Note: Since $F\Delta t = m\Delta v$, it's important to remember that the following relationships exist between units.

$$\text{lb}\cdot\text{sec} = \frac{\text{slug}\cdot\text{ft}}{\text{sec}} \quad \text{and} \quad \text{N}\cdot\text{sec} = \frac{\text{kg}\cdot\text{m}}{\text{sec}}$$

d. Conservation of Linear Momentum: $P_{mom\ before} = P_{mom\ after}$ *Equation 4*

$$\left\{\begin{matrix}\text{Linear Momentum of Isolated} \\ \text{System BEFORE an Interaction}\end{matrix}\right\} = \left\{\begin{matrix}\text{Linear Momentum of Same} \\ \text{System AFTER the Interaction}\end{matrix}\right\}$$

English Units	*SI Units*
P_{mom} in $\frac{\text{slug}\cdot\text{ft}}{\text{sec}}$	P_{mom} in $\frac{\text{kg}\cdot\text{m}}{\text{sec}}$

e. Summary of Units

Quantity	*Symbol*	*English Units*	*SI Units*
Mass	m	slug or $\frac{\text{lb}\cdot\text{sec}^2}{\text{ft}}$	kg
Velocity (speed)	v	ft/sec	m/sec
Change in velocity	Δv	ft/sec	m/sec
Force	F	lb	N
Time interval	Δt	sec	sec
Linear momentum	mv	$\frac{\text{slug}\cdot\text{ft}}{\text{sec}}$	$\frac{\text{kg}\cdot\text{m}}{\text{sec}}$
Change in linear momentum	Δmv	$\frac{\text{slug}\cdot\text{ft}}{\text{sec}}$	$\frac{\text{kg}\cdot\text{m}}{\text{sec}}$
Impulse	FΔt	lb·sec	N·sec

LET'S REVIEW UNITS

Before studying the Practice Exercises and solving the Problems, answer the following questions. They will check your understanding of the previous discussion about units. Fill in the blanks with the correct word or words.

a. The unit for linear momentum in the English system is ____ .

b. The unit for linear momentum in SI is ____ .

c. The slug is an English unit for ____ (mass; weight).

d. The unit for impulse in the English system is ____ .

e. The unit for impulse in the SI system is ____ .

f. The equation, $F\Delta t = m\Delta v$, tells us that the units for impulse and change in momentum are equivalent. That means that one lb·sec is equivalent to ____ .

g. Based on Question "f" above, we can also say that 1 N·sec is equivalent to ____ .

h. In the English system, the unit for mass is obtained by dividing the weight in pounds by g (32 ft/sec²). This gives the unit $\frac{\text{lb}\cdot\text{sec}^2}{\text{ft}}$ or ____ .

PRACTICE EXERCISES

Example 1: *Linear Momentum*

Given: A 20-lb hydraulic cylinder rod moves at a speed of 3 ft/sec during a certain instant of its motion.

Find: The linear momentum of the cylinder rod at this instant.

Solution: Use Equation 1.

$$P_{mom} = m \times v$$

$$\text{where: } m = \frac{w}{g} = \frac{20 \text{ lb}}{32 \text{ ft/sec}^2} = 0.625 \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}} = 0.625 \text{ slug}$$

$$v = 3 \text{ ft/sec}$$

$$P_{mom} = m \times v$$
$$P_{mom} = (0.625 \text{ slug})(3 \text{ ft/sec})$$
$$P_{mom} = (0.625 \times 3)(\text{slug} \times \text{ft/sec})$$
$$P_{mom} = 1.875 \text{ slug}\cdot\text{ft/sec}$$

Example 2: *Linear Impulse and Change in Momentum*

Given: The hydraulic cylinder and rod of Example 1. The pressure in the hydraulic system causes a 7.5-lb force to act on the rod for 0.5 sec. During this time, the 20-lb hydraulic cylinder rod speeds up from zero to 6 ft/sec.

Find:
a. Change in linear momentum of the rod.
b. Impulse produced by the applied force on the rod.

Solution: a. Change in the linear momentum of the rod.

$$\Delta P_{mom} = \Delta(mv) = m\Delta v$$

where: $m = 0.625$ slug (from Example 1)

$$\Delta v = v_{final} - v_{initial} = 6 \text{ ft/sec} - 0 \text{ ft/sec} = 6 \text{ ft/sec}$$

$$\Delta P_{mom} = m\Delta v$$
$$\Delta P_{mom} = 0.625 \text{ slug} \times 6 \text{ ft/sec}$$
$$\Delta P_{mom} = (0.625 \times 6)(\text{slug} \times \text{ft/sec})$$
$$\Delta P_{mom} = 3.75 \text{ slug}\cdot\text{ft/sec}$$

b. The impulse produced by the applied force

$$IMP = F\Delta t$$

where: $F = 7.5$ lb

$\Delta t = 0.5$ sec

$$IMP = 7.5 \text{ lb} \times 0.5 \text{ sec}$$
$$IMP = (7.5 \times 0.5)(\text{lb} \times \text{sec})$$
$$IMP = 3.75 \text{ lb·sec}$$

Note: Since we know that $F\Delta t = m\Delta v$, the answers to Parts "a" and "b" should be the same. They are! But they're different units. However, we know that 1 slug·ft/sec is the same as 1 lb·sec. Recall that we "proved" this in Table 8.1.

PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: Given: A car with a mass of 1500 kg is moving along a straight road at a speed of 50 km/hr.

Find: The linear momentum of the car. (Units should be in $\frac{\text{kg·m}}{\text{sec}}$.)

Solution:

Problem 2: Given: A man who weighs 200 lb is thrown forward from his seat at a speed of 10 ft/sec when the bus he's on stops suddenly.

Find: The momentum of the man. (Units should be in slug·ft/sec.)

Solution:

Problem 3: Given: The anvil on an electric nail driver has a mass of 0.3 kg. It has a linear momentum of 10.17 kg·m/sec as it strikes the nail.

Find: The velocity (speed) of the anvil in m/sec as it hits the nail.

Solution:

Problem 4: Given: The same nail driver as in Problem 3. The anvil starts at an initial speed of zero. It accelerates (speeds up) to 33.9 m/sec in 0.2 second. The acceleration is caused by a constant impulse force acting on the anvil.

Find: The force applied to the anvil.

Solution: (***Hint:*** Rearrange the equation, $F\Delta t = m\Delta v$, to isolate F. Then solve the new equation.)

Problem 5: Given: A forge hammer in a metal-shaping shop strikes a piece of metal with a force of 6 tons. The hammer is in contact with the metal for ⅛ sec (0.125 sec). The hammer weighs 2400 lb.

Find: The impulse delivered by the forge hammer.

Solution:

Problem 6: Sally Johnson of Johnson's Auto Sales is asked if installing shock-absorbing bumpers has any real advantage. To answer the question, she looks up the data on shock-absorbing bumpers. She finds that such bumpers extend the stopping time from 0.35 sec to 0.78 sec in a 5-mph collision. She then illustrates the forces involved with the following problem.

Given: When a vehicle that weighs 4000 lb backs up into a tree at 5 mph (7.3 ft/sec), it stops in 0.35 sec. A shock-absorbing bumper extends this time to 0.78 sec.

Find: The force required to stop the car with and without shock-absorbing bumpers. (What's the difference in the two forces?)

Solution:

Problem 7: Given: In spring tryouts for the football team, the coach has two new players try to tackle Juan, a 200-lb halfback who runs at 8 ft/sec. The first player, Alex, weighs 100 lb and runs at 16 ft/sec. The second player, Grover, weighs 300 lb and runs at 5.3 ft/sec.

Find: The player who would be more effective in stopping Juan in a head-on tackle. Explain your answer.

Solution:

Student Challenge

Problem 8: Given: An empty truck of mass m_o moves with a speed v_o. Its momentum is $P_{mom} = m_o v_o$. Suppose the empty truck is loaded until its mass is double its empty mass. Suppose its speed is also doubled.

Find:

a. The truck's new momentum is
   (1) the same as before, $m_o v_o$.
   (2) $2\ m_o v_o$.
   (3) $4\ m_o v_o$.
   (4) None of the above.

b. Use the equation "$P_{mom} = mv$" to explain your answer.

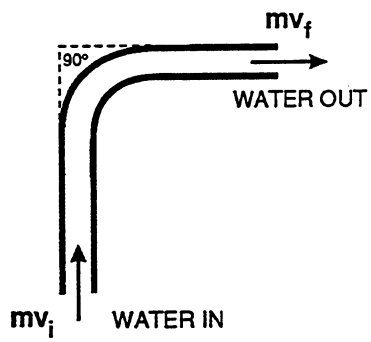
Solution:

Problem 9: Given: The law of conservation of linear momentum states that linear momentum is conserved for an isolated system when a 200-lb woman steps from a 60-ton yacht to the dock. When she does so, the only movement seen is the woman stepping on the dock. The yacht hardly moves. If the same woman steps from a 100-lb fiberglass boat to the dock, she may fall into the water if the boat isn't tied to the dock.

Find: Why the two situations are different. Explain by using the law of conservation of momentum.

Solution:

Problem 10: Given: Water moves through a pipe, as shown in the diagram. A 10-kg mass of water moves through a pipe at a velocity of 3 m/sec. The water is turned 90° from its original path by the 90° elbow. After the change in direction, neither the mass of the water nor the magnitude of the velocity has changed. Therefore, $mv_i = mv_f$. A vector diagram would be drawn in the following fashion:



Note: The two ***vector*** momentum quantities, $m\vec{v}_i$ and $m\vec{v}_f$, are ***not*** equal, since one ($m\vec{v}_i$) points up and the other ($m\vec{v}_f$) points to the right. But their ***magnitudes are equal***, so that $mv_i = mv_f$.

Find:

a. Length of the vector $m\vec{v}_i$. Plot it on graph paper with the proper direction. Choose a convenient scale.

b. Length of the vector $m\vec{v}_f$. Plot it on graph paper with the proper direction. Use the same scale.

c. Complete the vector triangle. Measure $\vec{F}\Delta t$ (or $m\Delta\vec{v}$).

d. If $\Delta t = 1$ sec, find the impulse force that caused the water to turn.

Solution: