Math Lab 4 MS 2

Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

Solving Fluid Resistance Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There is a Preparatory Math Skills Lab in a separate book entitled *PT Resource Manual* that contains concepts your students should have mastered before they begin this Math Skills Lab. The preparatory lab is coded and titled PMS13: "Checking Dimensions in Equations." Encourage students who need these skills to review the material in PMS13.

RESOURCE MATERIALS

Student Text: Math Skills Lab

PT Resource Manual

CLASS GOALS

1. Teach your students how to rearrange symbols in resistance equations to isolate certain numbers.
2. Teach your students how to solve fluid resistance problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete as many activities as time permits. (How much is accomplished depends on the math skills that your students already have.)
   a. Summarize the explanatory material for Activity 1: "Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns." Then have students complete Practice Exercises at the end of Activity 1.
   b. Summarize the explanatory materials for Activity 2: "Solving Fluid Resistance Problems." Then have students complete Practice Exercises at the end of Activity 2.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell students to read Lab 4F1, "Measuring Fluid Resistance in Tubes," as homework.

Math Skills Laboratory

Lab 4 MS 2

MATH ACTIVITIES

Activity 1: Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

Activity 2: Solving Fluid Resistance Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Isolate the change in pressure (Δp) or the volume-flow rate (Q_v) when given the equation for fluid resistance, $R_F = {}^{\Delta p}/_{Q_V}$.***
2. ***Substitute appropriate numerical values and units in fluid resistance equations. Solve the equations for a numerical value with proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Work the problems for Activities 1 and 2.***

ANSWERS TO REVIEW OF UNITS

a. lb/in^2

b. N/m^2

c. m^3/sec

d. fluid resistance

ACTIVITY 1

Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

MATERIALS

For this activity, you'll need pencil and paper.

We learned in previous math labs that equations and formulas are used to show a relationship between several physical quantities. Equation 1 is a relationship between (1) fluid resistance, (2) change in pressure within the fluid, and (3) volume-flow rate of the fluid:

$$\text{Fluid Resistance} = \frac{\text{Pressure Difference}}{\text{Volume-flow Rate}}$$

This relationship helps us to find the value of one physical quantity if we know the value and units of the other two physical quantities. The relationship is often written with symbols rather than words, as follows:

$$R_F = \frac{\Delta p}{Q_V} \qquad \textit{Equation 1}$$

where: R_F = resistance within a fluid flowing in a pipe, usually in $\frac{lb/in^2}{gal/min}$ or $\frac{N/m^2}{m^3/sec}$

Δp = pressure difference of a fluid in a pipe or duct, typically in lb/in^2 or N/m^2

Q_V = volume-flow rate of the fluid in a pipe, typically in gal/min or m^3/sec

Fluid resistance units are given above as pressure units divided by volume-flow rate units. Table 1 summarizes the units used with each of the physical quantities given in Equation 1. The table is meant to help you learn the units. Study Table 1.

TABLE 1. RESISTANCE UNITS FOR FLUID RESISTANCE

System of Units	Typical Units for Equation 1: $R_F = \Delta p/Q_v$		
	R_F	Δp	Q_V
English	$\frac{lb/in^2}{gal/min}$	lb/in^2	gal/min
SI	$\frac{N/m^2}{m^3/sec}$	N/m^2	m^3/sec

LET'S REVIEW UNITS!

Before beginning the Practice Exercises, let's review. Answer the following questions on units for fluid resistance. Fill in the blank with the correct word or words.

a. Fluid pressure in English units is usually measured in _____ (lb/in^2; N/m^2).

b. Fluid pressure in SI units is usually measured in _____ (lb/in^2; N/m^2).

c. The unit _____ (gal/min; ft^3/min; m^3/sec; in^3/sec) is NOT a fluid rate in the English system of units.

d. The units $\frac{lb/in^2}{gal/min}$ or $\frac{N/m^2}{m^3/sec}$ are correct when describing a _____ (fluid resistance; drag resistance).

SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: Solution given in text.

Problem 2: Step 1: Isolate "Q_V" by rearranging Equation 1. First write down Equation 1.

$$R_F = \frac{\Delta p}{Q_V}$$

Step 2: Multiply both sides by "Q_V."

$$Q_V \times R_F = \frac{\Delta p}{\cancel{Q_V}} \times \cancel{Q_V}$$ (Cancel "Q_V" on the right side.)

Step 3: Rewrite the equation without the "Q_V's" that canceled.

$$Q_V \times R_F = \Delta p$$

Step 4: Divide both sides by "R_F."

$$\frac{Q_V \times \cancel{R_F}}{\cancel{R_F}} = \frac{\Delta p}{Q_V}$$ (Cancel "R_F" on the left side.)

Step 5: Rewrite the equation without the "R_F" that canceled.

$$Q_V = \frac{\Delta p}{R_F} .$$

The problem has been solved. The equation, $R_F = \Delta p/Q_V$, has been rearranged, and "Q_V" has been isolated.

(See page T-49c for Problem 3 [Student Challenge].)

SOLUTIONS TO STUDENT CHALLENGE FOR ACTIVITY 1, Continued

Problem 3:

English

a. $R_F = \frac{\Delta p}{Q_v}$

$\frac{lb/in^2}{gal/min} = \frac{lb/in^2}{gal/min}$. (Checks!)

b. $R_F = \frac{\Delta p}{Q_v}$

$gal/min = \frac{lb/in^2}{(\frac{lb/in^2}{gal/min})}$

$\frac{gal}{min} = (\frac{lb}{in^2}) \times (\frac{gal/min}{lb/in^2})$

gal/min = gal/min. (Checks!)

$\Delta p = R_F \times Q_v$

$lb/in^2 = \frac{lb/in^2}{gal/min} \times gal/min$

$lb/in^2 = lb/in^2$. (Checks!)

SI

a. $R_F = \frac{\Delta p}{Q_v}$

$\frac{N/m^2}{m^3/sec} = \frac{N/m^2}{m^3/sec}$. (Checks!)

b. $R_F = \frac{\Delta p}{Q_v}$

$m^3/sec = \frac{N/m^2}{(\frac{N/m^2}{m^3/sec})}$

$\frac{m^3}{sec} = (\frac{N}{m^2}) \times (\frac{m^3/sec}{N/m^2})$

$m^3/sec = m^3/sec$. (Checks!)

$\Delta p = R_F \times Q_v$

$N/m^2 = \frac{N/m^2}{m^3/sec} \times m^3/sec$

$N/m^2 = N/m^2$. (Checks!)

SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 2

Problem 4: Use $R_F = \frac{\Delta p}{Q_v}$

where: $Q_v = 550$ gal/min

$\Delta p = 200$ lb/in^2

$R_F = \frac{200 \text{ lb/in}^2}{550 \text{ gal/min}}$

$R_F = 0.36 \frac{lb/in^2}{gal/min}$.

PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: Given: $R_F = {}^{\Delta p}/_{Q_v}$ (Equation 1)

Find: Δp

Solution: (Rearrange equation and isolate Δp.)

Step 1: Isolate Δp by rearranging Equation 1. First, write down Equation 1.

$R_F = {}^{\Delta p}/_{Q_v}$

Step 2: Multiply both sides by Q_V.

$$R_F \times Q_V = \frac{\Delta p}{Q_v} \times Q_V$$ (Cancel Q_V on right side.)

Step 3: Rewrite the equation without the canceled terms.

$R_F \times Q_V = \Delta p$ (Δp is isolated.)

Step 4: Reverse the order of the equation with Δp on the left side.

$\Delta p = R_F \times Q_V$.

The problem has been solved. The equation $R_F = {}^{\Delta p}/_{Q_v}$ has been rearranged, and Δp has been isolated.

Problem 2: Given: $R_F = {}^{\Delta p}/_{Q_v}$ (Equation 1)

Find: Q_V

Solution: (Rearrange the equation. Isolate Q_V.)

Student Challenge

Problem 3: Given: Problems 1 and 2 and the solutions to each problem.

Find: Whether or not the equation is stated correctly by substituting English or SI units for the symbols.

Solution: (***Hint:*** If the equation is stated correctly, the units on the left side of the equation will be equal to the units on the right side.)

ACTIVITY 2

Solving Fluid Resistance Problems

MATERIALS

For this activity, you'll need a hand calculator.

The basic fluid resistance formula used in this activity is $R_F = {}^{\Delta p}/_{Q_v}$. If the equation isn't in a "useful" form, first rearrange the equation to isolate the proper symbol that you're asked to find. Then solve the rearranged equation for the unknown value. Your final answer should include a correct number with the proper units.

PRACTICE EXERCISES FOR ACTIVITY 2

Problem 4: Given: Water flowing through a pipe at 550 gal/min has a pressure difference of 200 lb/in^2 over a distance of one mile.

Find: The resistance (R_F) that causes the pressure difference.

Solution:

SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 2 (continued)

Problem 5: Use $R_F = \frac{\Delta p}{Q_V}$ in the form found in Problem 1.

$\Delta p = R_F \times Q_V$ where: $R_F = 2 \frac{lb/in^2}{gal/min}$

$Q_V = 550$ gal/min

$\Delta p = 2 \frac{lb/in^2}{gal/min} \times 550$ gal/min

$\Delta p = 1100$ lb/in^2.

Problem 6: Use $R_F = \frac{\Delta p}{Q_V}$ where: $\Delta p = 10$ lb/in^2

$Q_V = 3$ gal/min

$R_F = \frac{10 \text{ lb/in}^2}{3 \text{ gal/min}}$

$R_F = 3.3 \frac{lb/in^2}{gal/min}$.

Problem 7: Use $R_F = \frac{\Delta p}{Q_V}$ in the form found in Problem 2.

$Q_V = \frac{\Delta p}{R_F}$ where: $\Delta p = 15$ lb/in^2

$R_F = 0.5 \frac{lb/in^2}{gal/hr}$

$Q_V = (\frac{15}{0.5})(\frac{lb/in^2}{\frac{lb/in^2}{gal/hr}}) = 30 \ (\frac{lb}{in^2} \times \frac{gal/hr}{lb/in^2}) = 30$ gal/hr

$Q_V = 30$ gal/hr. (1 hr = 60 min)

$Q_V = 30 \frac{gal}{hr} \times \frac{1 \text{ hr}}{60 \text{ min}}$

$Q_V = 30/60$ gal/min

$Q_V = 0.5$ gal/min or 0.5 gpm.

(continued on page T-50c)

(See page T-50c for Problems 8 and 9.)

Problem 8: Use $R_F = \frac{\Delta p}{Q_v}$.

a. To find p_{at} at the air tool, we know that the hose loses 1.8 lb/in^2 for every 10 ft of hose.

Pressure at Air Tool [p_{at}] = (Pressure at Receiver Tank) − (Pressure Loss in Hose)

$$p_{at} = 100 \text{ lb/in}^2 - (100 \text{ ft} \times \frac{1.8 \text{ lb/in}^2}{10 \text{ ft}})$$

$$p_{at} = 100 \text{ lb/in}^2 - (\frac{100}{10} \times 1.8 \text{ lb/in}^2)$$

$$p_{at} = 100 \text{ lb/in}^2 - 18 \text{ lb/in}^2$$

$$p_{at} = 82 \text{ lb/in}^2.$$

b. $R_F = \frac{\Delta p}{Q_v}$ where: $\Delta p = 100 \text{ lb/in}^2 - 82 \text{ lb/in}^2$

$\Delta p = 18 \text{ lb/in}^2$

$Q_v = 9 \text{ ft}^3\text{/min}$

$$R_F = \frac{18 \text{ lb/in}^2}{9 \text{ ft}^3\text{/min}}$$

$$R_F = 2 \frac{\text{lb/in}^2}{\text{ft}^3\text{/min}}.$$

Problem 9: Pressure loss in hose at 50 ft is:

$$\Delta p = 50 \text{ ft} \times \frac{1.8 \text{ lb/in}^2}{10 \text{ ft}} = 9 \text{ lb/in}^2$$

$$R_F = \frac{\Delta p}{Q_v}$$

$R_F = \frac{9 \text{ lb/in}^2}{9 \text{ ft}^3\text{/min}} = 1 \frac{\text{lb/in}^2}{\text{ft}^3\text{/min}}$, half as much as for the 100-ft hose.

Problem 5: Given: The resistance (R_F) in a fuel-transfer pipeline is 2 $\frac{lb/in^2}{gal/min}$. The line is 6 inches in diameter. Fuel moves through the line at a volume-flow rate of 550 gal/min.

Find: The pressure difference along the length of the pipeline.

Solution:

Problem 6: Given: The pump in a hydraulic system delivers hydraulic oil to a cylinder at a pressure difference of 10 lb/in^2. The volume-flow rate is 3 gal/min.

Find: The resistance of the hydraulic control system.

Solution:

Problem 7: Given: In designing a circulating water-heating system (hydronic system), a technician uses a copper pipe that has a resistance of 0.5 $\frac{lb/in^2}{gal/hr}$. The pipe operates at a water pressure of 15 lb/in^2 from the circulating pump.

Find: The pump volume-flow rate in gallons per hour (gal/hr) and gallons per minute (gal/min).

Solution:

Student Challenge

Problem 8: Given: An air compressor delivers air to a receiver tank at 100 lb/in^2 (that is, 100 psi). A 100-ft air hose with a pressure drop of 1.8 lb/in^2 for each 10 ft of length is connected to the tank. The air hose operates an air tool.

Find:
a. The pressure at the air tool.
b. The resistance in the air hose if the air tool requires 9 ft^3/min of air to operate.

Solution: (***Hint:*** The pressure drop must be found for the 100 ft of air hose.)

Problem 9: Given: Facts given in Problem 8 and your answers.

Find:
a. The pressure at the air tool if the air hose is shortened to 50 ft.
b. The fluid resistance of the 50-ft air hose.

Solution: