Math Lab 5 MS 3

Reviewing Examples of Electrical Potential Energy Problems

Solving Practical Problems That Involve Energy and Work in Electrical Energy Systems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach your students how to rearrange symbols in electrical energy equations to solve for the unknown quantity.
2. Teach your students how to solve electrical energy problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete as many activities as time permits.
   a. Summarize the explanatory material for Activity 1: "Reviewing Examples of Electrical Potential Energy Problems." Then have your students review the Practice Exercises for Activity 1.
   b. Summarize the explanatory material for Activity 2: "Solving Practical Problems That Involve Energy and Work in Electrical Energy Systems." Then have your students complete the Practice Exercises at the end of Activity 2.
3. Supervise student progress. Help your students obtain the correct answers.
4. Before the class ends, ask your students to read Lab 5E1, "Energy Stored in a Capacitor," as homework.

Math Skills Laboratory

Lab 5 M S 3

MATH ACTIVITIES

Activity 1: ***Reviewing Examples of Electrical Potential Energy Problems***

Activity 2: ***Solving Practical Problems That Involve Energy and Work in Electrical Energy Systems***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Given the equation for potential energy stored in a capacitor, $E_P = \frac{1}{2} CV^2$, rearrange the equation to solve for capacitance (C), or voltage (V).***
2. ***Given the equation for potential energy stored in an inductor, $E_P = \frac{1}{2} LI^2$, rearrange the equation to solve for inductance (L), or current (I).***
3. ***Substitute correct numerical values and units in energy equations. Solve the equations for a numerical value with the proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Examples A and B and Table 1.***
3. ***Work the problems.***

ACTIVITY 1

Reviewing Examples of Electrical Potential Energy Problems

MATERIALS

For this activity, you'll need a calculator.

During classroom discussions, you learned methods of storing potential energy in capacitors and inductors. You also learned that stored potential energy can be changed to kinetic energy to do work. Activity 1 of this Math Skills Lab explains methods that you can use to solve technical problems. In Activity 2, you'll solve problems similar to those a technician might have to solve.

SOLUTIONS TO REVIEW OF UNITS

a. joules

b. farads

c. henries

d. aren't

e. $\frac{V \cdot sec}{amp}$

f. $\frac{coul}{V}$

Let's take a look at the relationship of the various physical quantities in the two electrical potential energy equations.

$$\text{Potential Energy of a Capacitor} = \tfrac{1}{2} \times \text{Capacitance} \times (\text{Voltage})^2$$

The relationship often is written with symbols rather than words, as follows:

$$E_P = \tfrac{1}{2} CV^2 \qquad \textit{Equation 1}$$

where: E_P = potential energy stored (joules or newton·meters)
C = capacitance (farads)
V = voltage to charge the capacitor (volts)

$$\text{Potential Energy of an Inductor} = \tfrac{1}{2} \times \text{Inductance} \times (\text{Current})^2$$

The relationship often is written with symbols rather than words, as follows:

$$E_P = \tfrac{1}{2} LI^2 \qquad \textit{Equation 2}$$

where: E_P = potential energy of the inductor (newton·meters or joules)
L = inductance (henries)
I = current (amperes)

Table 1 sums up the units for each physical quantity given in Equations 1 and 2. Notice that the units for electrical potential energy are all SI units.

TABLE 1. KINETIC ENERGY UNITS

		System of Units: English	System of Units: SI
Equation 1:	E_P	—	newton·meter or joule
$E_P = \tfrac{1}{2} CV^2$	C	—	farad or coulomb/volt
	V	—	volt
Equation 2:	E_P	—	newton·meter or joule
$E_P = \tfrac{1}{2} LI^2$	L	—	henry or $\frac{\text{volt·sec}}{\text{ampere}}$
	I	—	ampere

LET'S REVIEW UNITS!

Use Table 1 to answer the following questions. Fill in the blanks with the correct word or words.

a. Electrical potential energy is measured in ______ (joules, henries).
b. Capacitance is measured in ______ (henries, farads).
c. Inductance is measured in ______ (henries, farads).
d. English units ______ (are, aren't) used in electrical energy problems.
e. One henry is equal to a ______ $\left(\frac{\text{volt·second}}{\text{ampere}}, \frac{\text{coulomb}}{\text{second}}\right)$.
f. One farad is equal to a ______ $\left(\frac{\text{volt·second}}{\text{ampere}}, \frac{\text{coulomb}}{\text{volt}}\right)$.

Example A: *Potential Energy Stored in a Capacitor*

The quantity of charge that builds up on either conducting plate of a capacitor when the voltage across the capacitor is one volt is a measure of ***capacitance***. The unit of capacitance is the farad. The equation is $C = q/V$.

Given: Twenty volts are applied across a capacitor rated at 1.5 μF to charge it.

Find:
a. Charge stored on the plates of the capacitor.
b. Potential energy stored by the capacitor.
c. What happens to the potential energy when the plates are shorted together.

Solution:
a. The charge stored can be found from the relationship:

$C = q/V$ by isolating q and rearranging to read

$q = C \times V$, where $C = 1.5\ \mu F = 1.5 \times 10^{-6}$ F and $V = 20$ V

$q = 1.5 \times 10^{-6}\ F \times 20\ V$

$q = (1.5 \times 20 \times 10^{-6})(F \cdot V)$ (1 coulomb = 1 farad × 1 volt)

$q = 30 \times 10^{-6}$ coulombs.

b. Use Equation 1 to find the electrical potential energy stored by the capacitor.

$E_P = \frac{1}{2} CV^2$, where $C = 1.5\ \mu F = 1.5 \times 10^{-6}$ F and $V = 20$ V

$E_P = (0.5)(1.5 \times 10^{-6}\ F)(20\ V)^2$

$E_P = (0.5 \times 1.5 \times 400 \times 10^{-6})\ F \cdot V^2$

$E_P = 300 \times 10^{-6} \frac{\text{coul}}{\cancel{V}} \times V^{\cancel{2}}$ (1 farad = 1 coulomb/volt)

$E_P = 300 \times 10^{-6}$ coul·V (1 joule = 1 coulomb·volt)

$E_P = 300 \times 10^{-6}$ J (or 300 microjoules)

c. When the capacitor is shorted, current flows in the shorting wire until the charges are neutralized. This current produces heat energy. A spark may also occur, producing light energy. Thus most of the energy is dissipated (is lost) as heat energy—and some as light energy.

Example B: *Potential Energy Stored in an Inductor*

Given: An inductor has an inductance of 16 henries. It draws 30 amperes of current.

Find: The amount of energy stored by the inductor.

Solution: Use Equation 2 to find the potential energy stored in the inductor.

$E_P = \frac{1}{2} LI^2$, where $L = 16$ H and $I = 30$ A

$E_P = (0.5)(16\ H)(30\ A)^2$

$E_P = (0.5 \times 16 \times 900)(H \cdot A^2)$ (1 H = 1 V·sec/A)

$E_P = 7200 \left(\frac{V \cdot sec}{\cancel{A}}\right)(A^{\cancel{2}})$ (Cancel A's.)

$E_P = 7200$ V·sec·A (1 A = 1 coul/sec)

$E_P = 7200\ V \cdot \cancel{sec} \cdot \frac{\text{coul}}{\cancel{sec}}$ (Cancel sec.)

$E_P = 7200$ V·coul

$E_P = 7200$ J (1 joule = 1 volt·coulomb)

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2

Problem 1:

$E_p = {}^1/_2\ CV^2$, where $C = 320 \times 10^{-6}F$, $V = 115\ V$

$E_p = (0.5)(320 \times 10^{-6}F)(115\ V)^2 = 2.116\ FV^2$

$E_p = 2.116\ FV^2$

$E_p = 2.116\ J$ (since $1\ F = 1\ \frac{coul}{V}$ and $1\ J = 1\ coul{\bullet}V$)

Problem 2:

a. $E_p = {}^1/_2\ CV^2$, where $E_p = 0.039 \times 10^{-3}\ J$, $V = 115\ V$

$$C = \frac{2\ E_p}{V^2} = \frac{2 \times 0.039 \times 10^{-3}\ J}{(115\ V)^2} = 5.9 \times 10^{-9}\frac{J}{V^2}$$

$C = 5.9 \times 10^{-9}F$ (since $1\ J = 1\ coul{\bullet}V$ and $1\ \frac{coul}{V} = 1\ F$)

b. $C = {}^q/_v$, so $q = CV$, where $C = 0.006 \times 10^{-6}F$, $V = 115\ V$

$q = CV = (0.006 \times 10^{-6}F)(115\ V) = 0.69 \times 10^{-6}\ F{\bullet}V$

$q = 0.69 \times 10^{-6}\ coul$ (since $1\ F{\bullet}V = 1\ coul$)

or $q = 0.69\ \mu coul$

Problem 3:

$E_p = 1/2\ LI^2$ where: $L = 3\ henries = 3\ H$; $I = 150\ mA = 0.15\ A$

$E_p = (0.5)(3\ H)(0.15\ A)^2$

$E_p = (0.5)(3)(0.0225)\ H{\bullet}A^2$ ($1\ H = 1\ V{\bullet}sec/A$)

$E_p = 0.03375\ (\frac{V{\bullet}sec}{A})\ A^2$ (Cancel A units.)

$E_p = 33.75 \times 10^{-3}\ V{\bullet}sec{\bullet}A$ ($1\ A = 1\ coul/sec$)

$E_p = 33.75 \times 10^{-3}\ V{\bullet}sec\ (\frac{coul}{sec})$ (Cancel sec.)

$E_p = 33.75 \times 10^{-3}\ V{\bullet}coul = 33.75 \times 10^{-3}\ J.$ ($1\ J = 1\ V{\bullet}coul$)

ACTIVITY 2

Solving Problems That Involve Energy and Work in Electrical Energy Systems

MATERIALS

For this lab, you'll need a calculator.

Problem 1: Air compressors often must start under load. Therefore, the drive motor must develop a high starting torque. John Riley is replacing the drive motor of an air compressor. He's chosen a ⅓-hp capacitor-start motor. The motor data plate contains the following information.

Given: Motor rated at: 250 W (⅓ hp) @ 1750 rpm; 115 V; 60 Hz; TYPE: capacitor-start; capacitor: 320 μF MODEL: 226-250; Serial #22341

Find: The potential energy stored in the capacitor.

Solution: (***Hint:*** Use Equation 1, $E_P = \frac{1}{2} CV^2$.)

Check the units of the solution against the units in Table 1. Are the units correct? For each problem in this math lab, you should compare the units of the solution to the units in Table 1. Do this. It will help you make sure the solution units are correct.

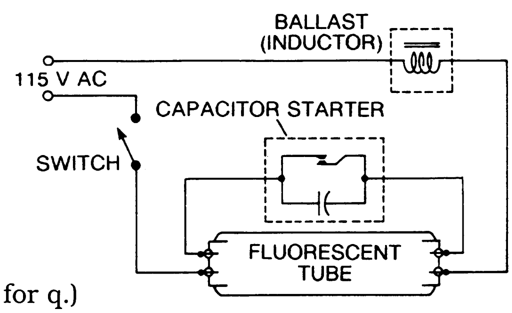
Problem 2: Given: Kathy Rodriguez works as a design technician at a company that manufactures capacitor starters for electric fluorescent light tubes. She knows that at 115 volts, the starter (capacitor) stores 0.039×10^{-3} joules of energy. This is sufficient to light a fluorescent tube rated at 40 watts or less.

Find:

a. The capacitance of the capacitor.

b. The amount of charge stored in the capacitor.

Solution: (***Hint:*** Rearrange the equation, $E_P = \frac{1}{2} CV^2$, to solve for C. Then use the equation, $C = q/V$, to solve for q.)



Problem 3: Given: In the fluorescent lamp assembly described in Problem 2 above, a ballast stores and releases the energy it takes to maintain conduction in the fluorescent tube as the AC voltage increases and decreases. The inductor rating is 3 henries at 150 milliamperes.

Find: The amount of energy stored by the inductor.

Solution: (***Hint:*** Use Equation 2, $E_P = \frac{1}{2} LI^2$.)

Problem 4: $E_p = \frac{1}{2} LI^2$, where $E_p = 3 \times 10^4$ J and I = 150 A

$$L = \frac{2\,E_p}{I^2} = \frac{2\,(3 \times 10^4\ \text{J})}{(150\ \text{A})^2} = 2.67 \frac{\text{J}}{\text{A}^2}$$

But 1 J = 1 V•coul and 1 coul = 1 A•sec

$$\therefore\ L = 267\ \frac{\text{V}\bullet\text{sec}}{\text{A}} = 2.67\ \text{H} \quad (\text{since } 1\ \text{H} = \frac{1\ \text{V}\bullet\text{sec}}{\text{A}})$$

Problem 5: $E_p = 1/2\ CV^2$

where: $E_p = 4 \times 10^3$ J
$C = C_{TOT} = C_1 + C_2$
$C_1 = 0.2$ F
$C_2 = ?$
$V = 110$ V

$$C_{TOT} = \frac{2\,E_p}{V^2}$$

$$C_1 + C_2 = \frac{2\,E_p}{V^2}$$

$$C_2 = \frac{2\,E_p}{V^2} - C_1$$

$$C_2 = \frac{2 \times 4 \times 10^3\ \text{J}}{(110\ \text{V})^2} - 0.2\ \text{F}$$

$$C_2 = \frac{8 \times 10^3}{12{,}100}\ \frac{\text{J}}{\text{V}^2} - 0.2\ \text{F}$$

$$C_2 = 0.66\ \frac{\text{coul}\bullet\text{V}}{\text{V}^2} - 0.2\ \text{F} \qquad (1\ \text{J} = 1\ \text{coul}\bullet\text{V})$$

$$C_2 = 0.66\ \text{F} - 0.2\ \text{F} \qquad (1\ \text{F} = 1\ \text{coul/V})$$

$$C_2 = 0.46\ \text{F}.$$

Problem 6: $C = q/V$

Rearrange to isolate "q."

$q = CV$

$q_1 = (C_1)(V) = (0.2\ \text{F})(110\ \text{V}) = (0.2)(110)\ \text{F}\bullet\text{V} = 22$ coul

$q_2 = (C_2)(V) = (0.46\ \text{F})(110\ \text{V}) = (0.46)(110)\ \text{F}\bullet\text{V} = 50.6$ coul

$q_{TOT} = q_1 + q_2 = 22$ coul $+ 50.6$ coul $= 72.6$ coul.

Problem 4: Given: The Acme Gear Company uses an induction heater to surface-harden small gears. The gear is placed inside an induction coil. The coil is supplied with high-frequency alternating current that heats the gear by using the rapidly changing magnetic field. The inductor coil stores 30,000 joules of energy at 150 amperes of current flow.

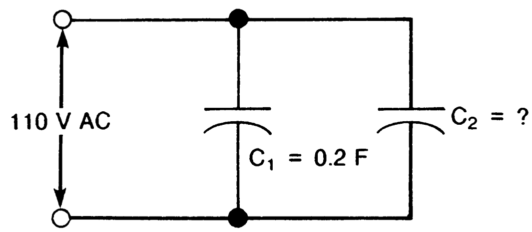
Find: The inductance of the coil (inductor).

Solution: (***Hint:*** Use Equation 2, $E_P = \frac{1}{2} LI^2$. Rearrange to isolate L.)

Student Challenge

The following problems review concepts you studied earlier. You may want to refer to the equations given in Table 1.

Problem 5: Given: An electromechanical technician at Garrett's Manufacturing Company knows that two capacitors, when connected in parallel, are to store 4,000 joules of potential energy when the applied voltage is 110 volts. See the circuit diagram below.



Find: The capacitance that must be placed in parallel with a 0.2-farad capacitor to obtain the 4000 joules of potential energy.

Solution: (***Hint:*** Capacitors in parallel are ***additive***. Therefore, $C_{TOT} = C_1 + C_2$; use the basic equation $E_P = \frac{1}{2} C_{TOT}V^2$.)

Problem 6: Given: Problem 5 and its solution.

Find: Total charge stored on the plates of the two capacitors.

Solution: (***Hint:*** Find the charge stored on each capacitor from $q = CV$. Then add separate charges together to get the total.)