Math Lab 1 MS 1
Working with Vectors
Substituting in Formulas

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

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TEACHING PATH - MATH SKILLS LAB - CLASS M

NOTE PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There are two Preparatory Math Skills Labs in a separate book entitled *PT Resource Manual* that contain concepts your students should have mastered before they begin this Math Skills Lab. These two preparatory labs are coded "PMS1" and "PMS2." They are titled "Learning How to Draw and Measure to Scale" and "Learning How to Draw and Measure Angles." Encourage students who need these skills to study the material in PMS1 and PMS2.

RESOURCE MATERIALS

Student Text: Math Skills Lab
Resource Manual: Preparatory Math Skills

CLASS GOALS

1. Teach students how to work with vectors, with emphasis on drawing vectors to scale and on adding two vectors at an angle by the graphical method.
2. Teach students how to substitute numbers in a given formula to solve for an unknown.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to class. (How much is accomplished depends on the math/drawing skills that your students already have.)
    a. Summarize the explanatory material for Activity 1: "Working with Vectors." Then have students complete Practice Exercises given at the end of Activity 1.
    b. Summarize the explanatory material for Activity 2: "Working with Vectors." Then have students complete Practice Exercises given at the end of Activity 2.
    c. Summarize the explanatory material for Activity 3: "Substituting in Formulas." Then have students complete Practice Exercises at the end of Activity 3.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read lab 1M1, "Measuring Forces," as homework.

Math Skills Laboratory

Lab 1 MS 1

MATH ACTIVITIES

Activity 1: Working with Vectors—Part 1

Activity 2: Working with Vectors—Part 2

Activity 3: Substituting in Formulas

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Distinguish between a scalar and a vector quantity.***
2. ***Produce a scale drawing of a given vector at a given angle from a reference system.***
3. ***Determine the actual magnitude and direction of a vector from its given scale drawing.***
4. ***Add vectors that act in the same direction.***
5. ***Add vectors that act in opposite directions.***
6. ***Add vectors that act at any angle with each other.***
7. ***Substitute in the formula for torque ($T = F \times L$) and calculate a torque, given values for the force (F) and the lever arm (L).***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Examples A through F.***
3. ***Work the problems.***

NOTE:

1. Students will need a ruler (English and SI units) and a protractor to carry out the Practice Exercises.
2. Look carefully at the Math Skills Lab Objectives.
3. To summarize the explanatory material--before students practice on the exercises--you may wish to walk them through the examples in this section. (The students should have read through the examples before coming to class. You may hope that the examples will be familiar, even though students may not have understood all of the points.)

NOTE: In Example A, walk the students through Steps 1, 2, 3 and 4. (Some of this material may have already been covered in Class C1.) Emphasize the importance of choosing an appropriate scale. Teach students how to use a protractor.

ACTIVITY 1

Working with Vectors—Part 1

EQUIPMENT

For this activity, you'll need the following:

A ruler (both English and SI)
A protractor

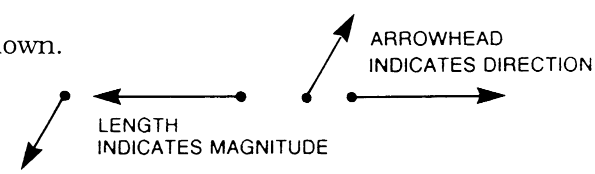
A quantity that's fully specified by measuring its magnitude (size) is called a ***"scalar."*** Examples of scalar quantities are:

(TEMPERATURE)	a. Yesterday was very sunny and ***85°.***
(TIME)	b. It took me ***40 minutes*** to complete my homework.
(LENGTH)	c. Tom is ***6 feet*** tall.
(CAPACITY or VOLUME)	d. This recipe calls for ***3 cups*** of flour.
(AMOUNT OF MONEY)	e. I paid ***$12.00*** for that record album.
(SPEED)	f. I could not go more than ***55 miles per hour.***

A quantity that's fully specified by indicating both its magnitude and direction is called a "vector." Some examples of vector quantities are:

(DISPLACEMENT)	a. He works ***5 miles south*** of here.
(FORCE)	b. Dave had to ***push*** the car for ***1200 feet to the corner gas station.***
(VELOCITY)	c. I was traveling ***40 mph eastward*** on Rt 103 when I saw the accident.

Vectors are represented by arrows, as shown.



The length of the arrow determines the magnitude of the quantity. The head of the arrow points in the direction that the force is acting.

Example A: *Drawing Vectors to Scale*

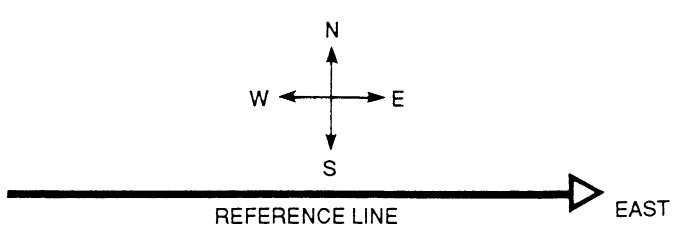
Given: A pencil, protractor and piece of paper.

Find: A scale to draw a 50-lb force at 40° north of east.

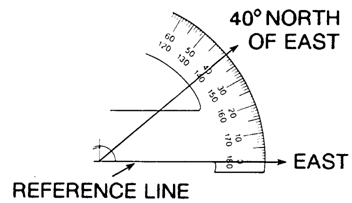
Solution: Step 1: Select a convenient scale, such as 1 cm = 10 lb. (That is, 1 centimeter {cm} on the drawing represents 10 lb of actual magnitude.) We can show the scale by the following representation:

1 cm = 10 lb ***2 cm = 20 lb***

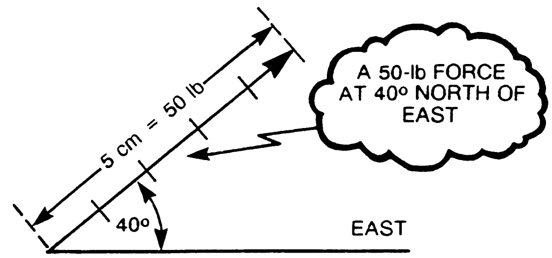
Step 2: Draw the reference line. In this case, a horizontal line is used to designate the east direction.



Step 3: Using a protractor, draw a 40° angle, north of east.



Step 4: The scale indicates that 1 cm = 10 lb. Thus, the scale drawing for the 50-lb force must be 5 cm long. Use the ruler to mark the 5-cm segment on the vector drawn.



Example B: ***Drawing Vectors to Scale***

Given: Reference line, protractor and pencil.

Find: A 15-lb force 70° above the reference line.

Solution: Step 1: Select a convenient scale, such as the following:

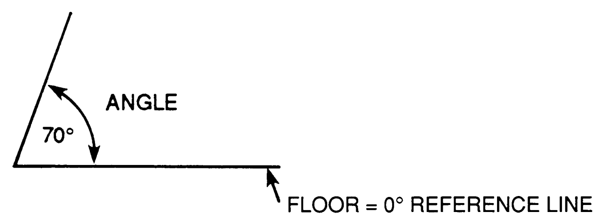
½ inch = 5 lb;

½ inch = 5 lb

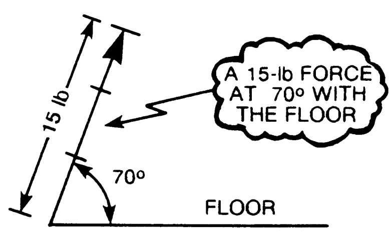
Step 2: Draw the frame of reference. (In this case, a straight line will represent the floor.)

REFERENCE LINE

Step 3: Use a protractor to draw a 70° angle, using the floor as the 0° reference line.



Step 4: Since the scale indicates that ½ inch on the paper represents 5 lb of actual force, the 15-lb force must be represented by a segment 1.5 inches long. Use a ruler to mark the 1.5-inch segment on the vector drawn.*



***NOTE:** Because printing this book changes the size of some illustrations, your ruler may not give you measurements that are exactly the same as the one in your book.

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1

1. The 24-lb force should be shown at an angle of 20° up from a reference line that represents the floor. The length of the arrow drawn will depend on the scale selected by the students. (A reasonable choice might be "1 inch = 12 lb"; for this scale, the arrow should be 2 inches long.)
2. Magnitude of vector A = 50 lb.*

$$(1\,\frac{1}{4}\text{ inches} \times \frac{40\text{ lb}}{1\text{ in}} = 50\text{ lb})$$

Magnitude of vector B = 80 lb.*

$$(2\text{ inches} \times \frac{40\text{ lb}}{\text{in}} = 80\text{ lb})$$

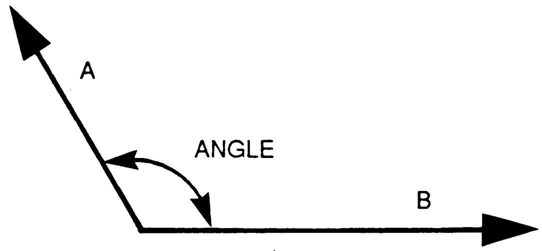
Angle between A and B = 120°.

*Line lengths as originally drawn may change due to copying process which may reduce or lengthen original drawing of the vector. Measure vector carefully and use <u>your</u> measurement with the scale: 1/2 inch = 20 lb.

NOTE: Work quickly through Example C and Example D. Students should understand these readily.

PRACTICE EXERCISES FOR ACTIVITY 1

1. On a separate sheet of paper, make a scale drawing of a 24-lb force acting along a line 20° above the floor.
2. Given the scale drawing below, determine the actual magnitude of vector A and vector B. Measure the angle formed between them.



SCALE: $\frac{1}{2}$ in. = 20 lb

ACTIVITY 2

Working with Vectors—Part 2

Adding two vectors simply means to replace the two vectors with a single one that will accomplish the same result. The single vector is called the ***resultant***. The rules for the addition of vectors can be classified as follows:

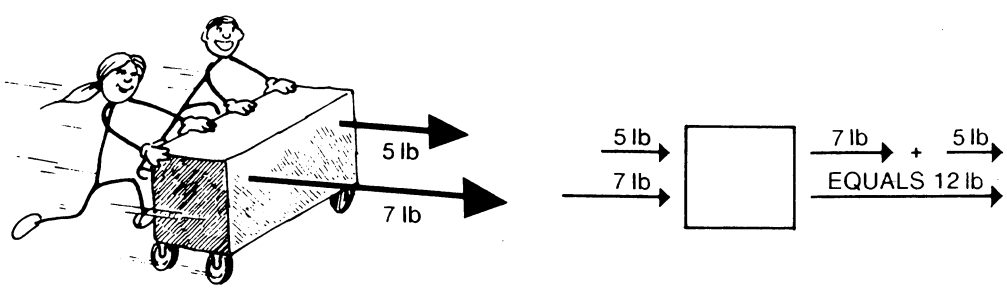
Rule 1: When two (or more) forces are acting at the same point in the same or opposite direction, the resultant can be found by performing a simple arithmetic addition.

Example C: *Adding Vectors in Line*

Given: John and Sue are pushing the cart with a force of 5 lb and 7 lb, respectively, in the same direction (to the right).

Find: The magnitude and direction of the resultant.

Solution: To find the magnitude of the resultant, add 5 lb + 7 lb = 12 lb. Since both vectors were to the right, the resultant must be to the right.



Example D: *Adding Vectors in Line*

Given: John is pushing to the right with 5 lb of force on an object. Sue is pushing to the left with 7 lb of force.

Find: The magnitude and direction of the resultant.

Solution: To find the resultant, we add the forces acting in opposite directions, as shown in the following diagram.

NOTE: Spend time explaining Example E, going through each of the eight steps. This example teaches the students how to add two forces at an angle to find the resultant.

5 lb (right) + 7 lb (left) = 2 lb (left)

Rule 2: When the angle between the vectors is other than 0° (same direction) or 180° (opposite direction), we must use the graphical method of addition (scale drawing method).

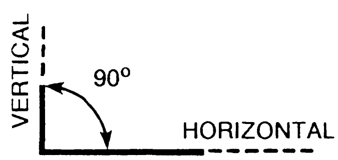
Example E: *Adding Vectors Not in Line*

Given: Two forces, one 3 lb and the other 4 lb, pull on an object. The angle between them is 90°.

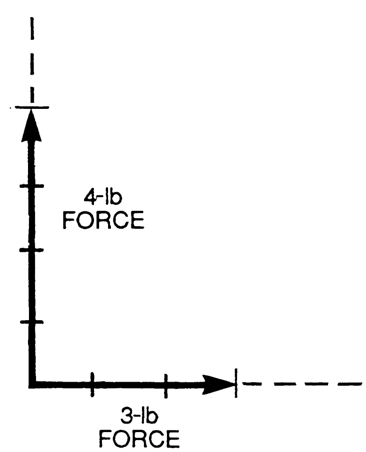
Find: The magnitude and direction of the resultant vector.

Solution: Step 1: Select a convenient scale: 1 cm = 1 lb.

Step 2: Draw an angle of 90°. (The sides of the angle give us the direction of the forces.)



Step 3: Since our scale is 1 cm = 1 lb, the 3-lb force will be 3 cm in length along one of the lines of direction. (It doesn't matter which one, but let's choose the horizontal line.) The 4-lb force will be 4 cm long along the other line of direction (the vertical line). Draw the vectors.



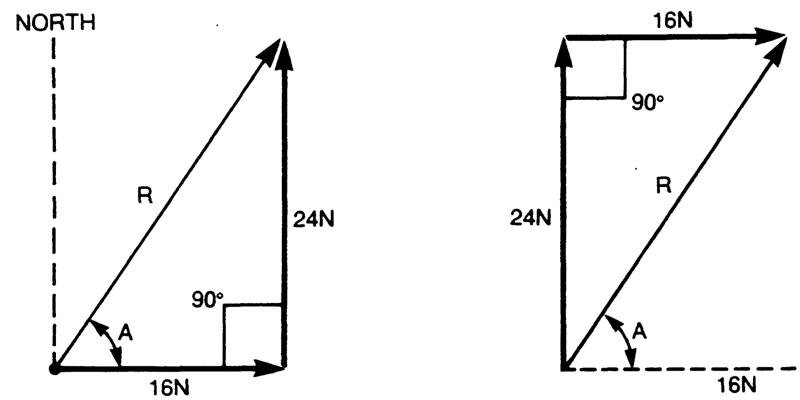
SOLUTIONS FOR PRACTICE EXERCISES, ACTIVITY 2

Case A: 40 N, to the right

Case B: 8 N, to the right

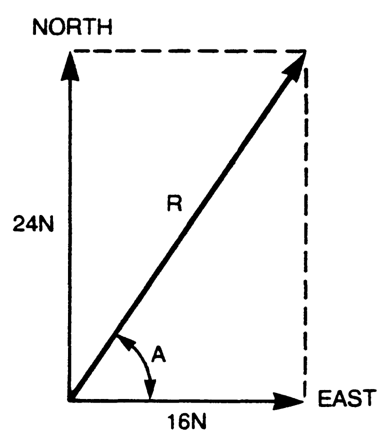
Case C: Scale for drawing is (roughly) 1 inch = 16 N. See comment on page T-26 about copier effect on length of lines.

Solving for **resultant** R by the *tail-to-head method:*



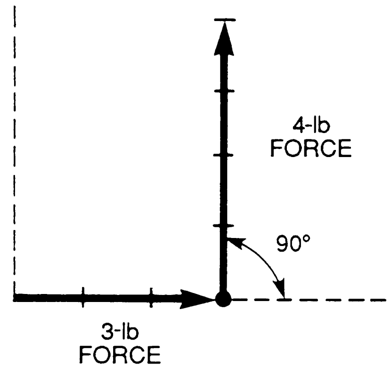
Either drawing gives R ≃ 29 N and angle A ≃ 56°

Solving for **resultant** R by the *parallelogram method.* (See earlier note on page T-13.) The two dashed lines complete the "parallelogram"--in this case a simple rectangle -

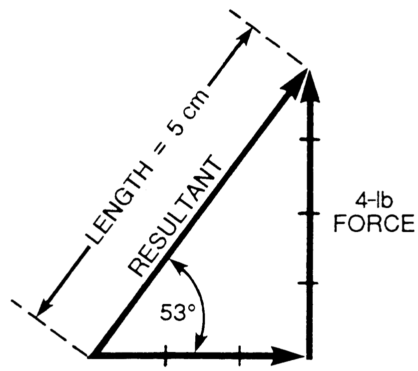


Note that the resultant R obtained by the *parallelogram* method is identical to the resultant R obtained by the *tail-to-head* method.

Step 4: Now redraw the 4-lb force with its tail at the head of the 3-lb force, at 90° to the 3-lb force, as shown.



Step 5: Draw a straight line connecting the tail of the 3-lb force to the head to the 4-lb force. This line is the resultant of the 3-lb and 4-lb force.



Step 6: Lay a ruler along the length of the resultant. Measure its length. In this problem, it should be close to 5 cm. Since the scale we're using is 1 cm = 1 lb, the 5-cm length must represent a 5-lb force. Thus, the magnitude of the resultant vector is equal to 5 pounds.

Step 7: Measure the angle between the 3-lb force and the resultant vector with a protractor. The result is about 53°.

Step 8: The magnitude and direction of the resultant of a 3-lb force and a 4-lb force at right angles to each other is a 5-lb force acting at an angle of 53° "up" from the 3-lb force.

PRACTICE EXERCISE FOR ACTIVITY 2

A force of 16 N and a force of 24 N are acting on an object at the same point. Find the magnitude and direction of the resultant for each of the three cases that follow:

Case A:** **when the two forces act in the same direction to the right.

Case B:** **when the two forces act in opposite directions, 24 N to the right, 16 N to the left.

Case C:** **when the two forces are at right angles to one another, 16 N to the east and 24 N to the north.

NOTE: Work through Example F. Then have students do Problems 1 and 2.

ANSWERS TO PROBLEMS

Problem 1: 1. $T = F \times L$

$T = 20 \text{ lb} \times 1.5 \text{ ft} = 30 \text{ lb}\cdot\text{ft}.$

Problem 2: 2. $T = F \times L$

$T = 80 \text{ lb} \times 0.25 \text{ ft} = 20 \text{ lb}\cdot\text{ft}.$

ACTIVITY 3

Substituting in Formulas

A ***formula*** is an equation that relates certain things to each other. Usually, the formula is expressed in letters and numbers, rather than words. An example of a formula is $T = F \times L$. We have learned that this is an expression for torque (T) in terms of applied force (F) and lever arm (L). There are many other formulas.

Substituting in a formula is straightforward. All you need to do is write down the formula, substitute a given number for its appropriate symbol, and then combine the numbers (multiply, add, subtract, etc.), as the formula indicates. BE SURE TO KEEP TRACK OF THE UNITS THAT GO WITH THE NUMBERS.

Example F: *Torque on a Bolt*

Given: A mechanic tightens a bolt by applying a force of 10 pounds to the end of a wrench that is 24 inches long.

Find: The torque applied to the bolt.

Solution: Step 1: Write down the formula for torque. Identify the symbols.

$T = F \times L$

where: T = torque (lb·ft)
F = force (lb)
L = lever arm (ft)

Step 2: Identify what is given in the problem.

F = 10 lb
L = 2 ft (that is, 24 inches)

Step 3: Substitute in the formula.

$T = F \times L$
$T = 10 \text{ lb} \times 2 \text{ ft}$

Step 4: Complete the indicated multiplication.

$T = 10 \text{ lb} \times 2 \text{ ft}$
$T = (10 \times 2)\ (\text{lb·ft})$
$T = 20 \text{ lb·ft}$*

Thus, the torque wrench applies a force of 20 lb·ft to the bolt.

*Notice that the correct answer contains both a number (20) and a unit (lb·ft).

PRACTICE EXERCISES FOR ACTIVITY 3

Problem 1: How much torque is produced by a torque wrench that tightens a nut when the applied force is 20 lb and the lever arm is 1.5 ft?

Problem 2: Find the torque exerted on the driven gear by the teeth of the driving gear in the diagram shown here.