Math Skills Laboratory

Lab 8 MS 2

MATH ACTIVITY

Solving Angular Momentum Problems

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. ***Substitute appropriate numerical values and units in angular momentum equations. Solve the equations for unknown numerical values with the proper units.***
2. ***Use the following equations to solve angular momentum problems.***
   a. $L_{mom} = I \times \omega$
   b. $T \times \Delta t = \Delta(I\omega)$
   c. $Ang\ Imp = T\Delta t$

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

ACTIVITY

Solving Angular Momentum Problems

MATERIALS

For this activity, you'll need a calculator.

In this lab, you'll review basic units. You'll also solve problems that involve angular momentum, angular impulse and changes in angular momentum. The important equations, with typical units, are summarized on the following page.

a. Angular Momentum: $L_{mom} = I\omega$ ***Equation 1***

$$\begin{Bmatrix} \text{Angular} \\ \text{Momentum} \end{Bmatrix} = \begin{Bmatrix} \text{Moment of Inertia} \\ \text{of Object} \end{Bmatrix} \times \begin{Bmatrix} \text{Angular Velocity} \\ \text{of Object} \end{Bmatrix}$$

English Units	*SI Units*
I in *slugs·ft^2	I in kg·m^2
ω in rad/sec	ω in rad/sec
L_{mom} in $\frac{\text{slug·ft}^2}{\text{sec}}$	L_{mom} in $\frac{\text{kg·m}^2}{\text{sec}}$

b. Angular Impulse: Ang Imp = T × Δt ***Equation 2***

$$\text{Angular Impulse} = \begin{Bmatrix} \text{Torque Acting on} \\ \text{Object (or Fluid)} \end{Bmatrix} \times \begin{Bmatrix} \text{Time That} \\ \text{Torque Acts} \end{Bmatrix}$$

English Units	*SI Units*
T in lb·ft	T in N·m
Δ t in sec	Δ t in sec
Ang Imp in (lb·ft) sec	Ang Imp in (N·m) sec

c. Angular Impulse and
Angular Momentum Change: $T \times \Delta t = I \times \Delta\omega$ ***Equation 3***

$$\begin{Bmatrix} \text{Torque Acting} \\ \text{on Object} \\ \text{(or Fluid)} \end{Bmatrix} \times \begin{Bmatrix} \text{Time} \\ \text{During Which} \\ \text{Torque Acts} \end{Bmatrix} = \begin{Bmatrix} \text{Moment of Inertia} \\ \text{of Object} \\ \text{(or Fluid)} \end{Bmatrix} \times \begin{Bmatrix} \text{Change in Angular} \\ \text{Velocity Caused} \\ \text{by the Torque} \end{Bmatrix}$$

English Units	*SI Units*
T in lb·ft	T in N·m
Δ t in sec	Δt in sec
I in slug·ft^2	I in kg·m^2
Δω in rad/sec	Δω in rad/sec

Note: Since $T\Delta t = I\Delta\omega$, it's important to remember that the following relationships exist between units.

$$\text{(lb·ft)·sec} = \frac{\text{slug·ft}^2}{\text{sec}} \quad \text{and} \quad \text{(N·m)·sec} = \frac{\text{kg·m}^2}{\text{sec}}$$

d. Conservation of
Angular Momentum: $L_{mom\ before} = L_{mom\ after}$ ***Equation 4***

$$\begin{Bmatrix} \text{Angular Momentum of Isolated} \\ \text{System BEFORE an Interaction} \end{Bmatrix} = \begin{Bmatrix} \text{Angular Momentum of Same} \\ \text{System AFTER the Interaction} \end{Bmatrix}$$

English Units	*SI Units*
L_{mom} in $\frac{\text{slug·ft}^2}{\text{sec}}$	L_{mom} in $\frac{\text{kg·m}^2}{\text{sec}}$

* The mass of an object in slugs is equal to the weight of the object in pounds divided by 32 ft/sec^2. In units, 1 slug = 1 lb·sec^2/ft.

e. Summary of Units

Quantity	Symbol	English Units	SI Units
Moment of inertia	I	$slug{\cdot}ft^2$	$kg{\cdot}m^2$
Angular velocity (speed)	ω	rad/sec	rad/sec
Change in angular velocity	$\Delta\omega$	rad/sec	rad/sec
Torque	T	lb·ft	N·m
Time interval	Δt	sec	sec
Angular momentum	$I \times \omega$	$\frac{slug{\cdot}ft^2}{sec}$	$\frac{kg{\cdot}m^2}{sec}$
Change in angular momentum	$I \times \Delta\omega$	$\frac{slug{\cdot}ft^2}{sec}$	$\frac{kg{\cdot}m^2}{sec}$
Angular impulse	$T \times \Delta t$	(lb·ft)·sec	(N·m)·sec

LET'S REVIEW UNITS!

Before studying the ***Practice Exercises*** and solving the ***Problems,*** answer the following questions to check your understanding of the units given above. Fill in the blanks with the correct word or words.

a. The units for angular momentum in the English system are ____.
b. The units for angular momentum in SI are ____.
c. The slug is an English unit for ____ (mass; weight).
d. The units for angular impulse in the English system are ____.
e. The units for angular impulse in SI are ____.
f. The equation $T\Delta t = I\Delta\omega$ tells us that the units for angular impulse and change in angular momentum are equivalent. That means that 1 (lb·ft)·sec is equivalent to ____.
g. Based on Question f above, we can also say that 1 (N·m)·sec is equivalent to ____.

PRACTICE EXERCISES

Example 1: *Angular Momentum*

Given: An energy-storing flywheel (disk shape) on a punch press has a mass of 300 kg. It has a radius of 0.8 m. At full speed, the flywheel turns at 250 rpm.

Find: The angular momentum of the flywheel at full speed.

Solution: Use Equation 1 for the angular momentum of an object rotating about its own axis.

$$L_{mom} = I \times \omega$$

where: $I = \frac{1}{2} mr^2$ for a disk

$$m = 300 \text{ kg}$$
$$r = 0.8 \text{ m}$$
$$\omega = 250 \text{ rpm} = 250 \text{ rev/min}$$

Substitute for I and ω in the equation and solve.

$$L_{mom} = \left[\frac{1}{2} mr^2\right] \times \omega$$

$$L_{mom} = \left[0.5 \times 300 \text{ kg} \times (0.8 \text{ m})^2\right] \times \left[250 \frac{\cancel{rev}}{\cancel{min}} \times \frac{6.28 \text{ rad}}{1 \cancel{rev}} \times \frac{1 \cancel{min}}{60 \text{ sec}}\right]$$

$$L_{mom} = 96 \text{ kg}{\cdot}\text{m}^2 \times 26.17 \text{ rad/sec}$$

$$L_{mom} = 2512 \text{ kg}{\cdot}\text{m}^2/\text{sec}$$

At full speed, the flywheel has an angular momentum of 2512 $kg{\cdot}m^2/sec$.

Example 2: *Angular Impulse and Change in Angular Momentum*

Given: The punch press in Example 1. When the operator engages the punch to shape or punch out a part, the energy-storing flywheel slows from 250 rpm (26.17 rad/sec) to 150 rpm (15.7 rad/sec) during the 6 seconds the punch is engaged.

Find:
a. The change in angular momentum.
b. The angular impulse.

Solution:
a. The change in angular momentum $\Delta(I\omega)$ comes from the change in angular velocity $\Delta\omega$. Therefore the change in angular momentum = $I\Delta\omega$, or

$$\Delta(I\omega) = I\Delta\omega$$

where:
$I = \frac{1}{2}mr^2$ for a disk = 96 kg·m² (from Example 1)
$\Delta\omega$ = change in angular velocity, $\omega_f - \omega_i$
ω_f = 15.7 rad/sec, the final angular velocity
ω_i = 26.17 rad/sec, the initial angular velocity

First determine the change in angular velocity, $\Delta\omega$.

$$\Delta\omega = \omega_f - \omega_i$$
$$\Delta\omega = 15.7 \text{ rad/sec} - 26.17 \text{ rad/sec}$$
$$\Delta\omega = -10.47 \text{ rad/sec}$$

Ignore the minus sign. It simply tells us that the flywheel is *slowing down*. Substitute the values for I and $\Delta\omega$ into the equation.

$$\Delta(I\omega) = I\Delta\omega$$
$$\Delta(I\omega) = 96 \text{ kg}\cdot\text{m}^2 \times (10.47 \text{ rad/sec})$$
$$\Delta(I\omega) = (96 \times 10.47)(\text{kg}\cdot\text{m}^2 \times \text{rad/sec})$$
$$\Delta(I\omega) = 1005 \text{ kg}\cdot\text{m}^2/\text{sec}$$

b. From the equation, $T \times \Delta t = \Delta(I\omega)$, we know that the angular impulse $T \times \Delta t$ is equal to the change in angular momentum $\Delta(I\omega)$. Since we calculated $\Delta(I\omega)$ in Part a above, we have the answer to Part b.

Angular Impulse $T\Delta t = \Delta(I\omega)$
Angular Impulse $T\Delta t$ = 1005 kg·m²/sec, or 1005 (N·m)·sec

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Problem 1: Given: A bullet fired from a rifle is given a rotational motion of 70 rad/sec by the spiral grooves in the rifle bore. The moment of inertia of the bullet is 1.15 g·cm².

Find: The angular momentum of the bullet.

Solution:

Problem 2: Given: The 120-lb drive shaft of a gravel conveyor has a diameter of 3 inches. It has an angular momentum of 0.455 slug·ft²/sec when rotating at 150 rpm (15.7 rad/sec).

Find:
a. The moment of inertia I of the drive shaft by solving for I in the equation, $L_{mom} = I \times \omega$.
b. The moment of inertia I of the drive shaft, calculated using the equation for a cylinder revolving about its center axis, $I = \frac{1}{2}mr^2$. Remember: $m = \frac{w}{g}$ and 1 slug = 1 lb·sec²/ft.

Solution:

Problem 3: Given: A 10-g bullet fired from a rifle has a rotational motion of 50 rad/sec. It has an angular momentum of 57.6 $g \cdot cm^2/sec$. The 155-grain (10-g) bullet is a 38-caliber (0.38 inch in diameter) bullet. The radius of the bullet is 0.48 cm.

Find:

a. The moment of inertia I of the bullet using $I = \frac{1}{2} mr^2$ —the formula for a solid cylinder. (Assume that the bullet shape approximates that of a solid cylinder.)

b. The moment of inertia I of the bullet by solving for I in the equation, $L_{mom} = I\,\omega$.

Solution:

Problem 4: Given: An air cylinder operates a ball-type water valve on a pollution-control scrubber. The valve handle is a 3-inch lever arm. In 2 seconds, it must turn 90° (1.57 rad) from fully open to fully closed. The angular momentum of the valve goes from zero to 50 $slug \cdot ft^2/sec$ when the force is applied.

Find: The force F applied by the air cylinder to close the valve.

Solution: [***Hint:*** Use the equation, $T\Delta t = I\Delta\omega$. Substitute $F \times \ell$ for T. Rearrange the equation to isolate F. Also remember that 50 $slug \cdot ft^2/sec$ = 50 (lb·ft)·sec.]

Problem 5: Given: To balance an automobile wheel without removing the wheel from the car, Ron jacks up the car wheel and uses a portable wheel-spinning machine. This machine applies a 40-lb force to the tread part of the 11-in.-radius wheel and tire for 20 seconds.

Find: The angular impulse given to the wheel.

Solution: (***Hint:*** Use the equations, Ang Imp = $T\Delta t$ and $T = F\ell$.)

Problem 6: Given: Turbochargers are used to increase the horsepower in large diesel engines. The engine exhaust gas is used to spin a turbine with blades of radius 7.5 cm in the intake air passage of the engine. The tips of the intake turbine blades apply a 30-N force to the free air entering the engine. This gives the air an angular momentum around the inside perimeter of the turbocharger intake housing.

Find: The angular impulse delivered by the blades to the air in one second. (Use the equations, Ang Imp = $T\Delta t$ and $T = F\ell$.)

Solution: