# Math Lab 3 MS 2
# Using Power-of-Ten Numbers
# in Fluid Rate Problems
# Solving Fluid Rate Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

# TEACHING PATH - MATH SKILLS LAB - CLASS M

## PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There are Preparatory Math Skills Labs, located in a separate book entitled *PT Resource Manual*, that contain concepts your students should have mastered before they begin this Math Skills lab. These labs are coded and titled--

PMS9: "Writing Decimal Numbers as Numbers and Writing Power-of-Ten Numbers as Decimal Numbers"

PMS10: "Writing Decimal Numbers as Power-of-Ten Numbers in "Scientific Notation"

PMS11: "Conducting Mathematical Operations with Numbers Expressed in Power-of-Ten Form"

Encourage students to refer to these preparatory labs if they need assistance in sharpening the skills listed.

## RESOURCE MATERIALS

Student Text: Math Skills Lab
*PT Resource Manual*

## CLASS GOALS

1. Given addition or subtraction problems containing numbers in powers-of-ten notation, solve the problems.
2. Given multiplication or division problems containing numbers in exponential form, solve the problems.

## CLASS ACTIVITIES

1. Take five or ten minutes to go through Student Exercises. Make sure that your students understand the correct answers.
2. Complete as many activities as time permits. Students should already have read the Discussion material and looked at the examples for each activity before coming to this class. You should summarize the main points in each activity, work an example or two, and have the students complete the Practice Exercises for each activity on their own. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 3F1, "Measuring Liquid-flow Rate in a Channel," as homework.

# Math Skills Laboratory

Lab 3 MS 2

### *MATH ACTIVITIES*

*Activity 1: Using Power-of-ten Numbers in Fluid Rate Problems*
*Activity 2: Solving Fluid Rate Problems*

### *MATH SKILLS LAB OBJECTIVES*

***When you complete these activities, you should be able to do the following:***

1. ***Use power-of-ten notation, where appropriate, to express large numbers in fluid rate problems. Multiply and divide numbers in power-of-ten notation.***
2. ***Given the equation for volume-flow rate, $Q_v = V/t$, rearrange the equation to isolate V or t. Solve for volume-flow rate ($Q_v$), volume (V), or elapsed time (t).***
3. ***Given the equation for mass-flow rate, $Q_m = m/t$, rearrange the equation to isolate m or t. Solve for mass-flow rate ($Q_m$), mass (m) or elapsed time (t).***
4. ***Substitute appropriate numerical values and units in fluid rate equations. Solve the equations for the unknown variable.***

### *LEARNING PATH*

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems for Activities 1 and 2.***

*ACTIVITY 1:*

# Using Power-of-ten Numbers in Fluid Rate Problems

***EQUIPMENT***

For this activity, you'll need a hand calculator.

This activity will give you practice in rearranging equations to isolate a symbol. It also provides practice in multiplying and dividing numbers expressed in power-of-ten notation. Examples A and B demonstrate how power-of-ten numbers can be used to simplify calculations in problems that involve large numbers.

**Example A: *Volume-flow Rate of a Supertanker Pump***

Given: A supertanker loaded with $1.5 \times 10^6$ gallons of crude oil is to unload at an offshore discharge buoy. The ship's pumps can discharge the oil in 30 hours.

Find: The volume-flow rate of crude oil through the discharge pipeline in gallons per minute.

Solution: Step 1: Volume-flow rate $= \dfrac{\text{Volume of fluid moved}}{\text{Time of movement}}$

In symbols: $Q_V = \dfrac{V}{t}$

where: $Q_V$ = volume-flow rate (gal/min, $ft^3$/sec, liters/sec, $m^3$/sec)
$V$ = fluid volume moved (gal, $ft^3$, liters, $m^3$)
$t$ = elapsed time of movement (hr, min, sec)

Step 2: Start with the basic equation. Identify the known values.

$$Q_V = \frac{V}{t}$$

where: $V = 1.5 \times 10^6$ gal of crude oil
$t = 30 \text{ hr} = 1800 \text{ min} = 1.8 \times 10^3 \text{ min}$

$$Q_V = \frac{1.5 \times 10^6 \text{ gal}}{1.8 \times 10^3 \text{ min}}$$

Reminder: To divide power-of-ten numbers, the first factors of each number (1.5 and 1.8) are divided as usual. The exponents of 10 (6 and 3) are subtracted algebraically. The result is the quotient of the first two factors times (×) a power of ten, as follows:

Step 3:

$$Q_V = \left(\frac{1.5}{1.8} \times 10^{6-3}\right) \frac{\text{gal}}{\text{min}}$$

$$Q_V = 0.833 \times 10^3 \text{ gal/min}$$

$$Q_V = 833 \text{ gal/min}$$

**Example B: *Volume of Fluid Moved by a Supertanker Pump***

Given: The buoy attendant who unloads supertankers knows that a pump can unload tankers at a rate of 800 gal/min. His flowmeter breaks during the unloading of a tanker. He knows it took 20 hours to unload.

Find: The number of gallons of oil pumped into storage tanks.

Solution: Find the volume (V, number of gallons) of oil pumped.

Step 1: The volume-flow rate equation is:

$$Q_V = \frac{V}{t}$$

Since the unknown we want to solve for is V, first rearrange the equation to isolate V.

## SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 1

1. $Q_V = \dfrac{V}{t}$ where: $V = 1.8 \times 10^6$ gal

$t = 35 \text{ hr} = 35 \text{ hr} \times \dfrac{60 \text{ min}}{1 \text{ hr}} = 2100 \text{ min}$

$t = 2.1 \times 10^3 \text{ min}$

$$Q_V = \frac{1.8 \times 10^6 \text{ gal}}{2.1 \times 10^3 \text{ min}}$$

$$Q_V = \frac{1.8}{2.1} \times 10^{6-3} \frac{\text{gal}}{\text{min}}$$

$$Q_V = 0.857 \times 10^3 \text{ gal/min}$$

$$Q_V = 857 \text{ gal/min.}$$

2. $Q_V = \dfrac{V}{t}$ Solve for V.

$V = Q_V \times t$ where: $Q_V = 850 \dfrac{\text{gal}}{\text{min}} = 8.5 \times 10^2 \dfrac{\text{gal}}{\text{min}}$

$t = 25 \text{ hrs} = 25 \text{ hr} \times 60 \dfrac{\text{min}}{\text{hr}}$

$t = 1500 \text{ min} = 1.5 \times 10^3 \text{ min}$

$$V = 8.5 \times 10^2 \frac{\text{gal}}{\text{min}} \times 1.5 \times 10^3 \text{ min}$$

$$V = 8.5 \times 1.5 \times 10^{2+3} \frac{\text{gal}}{\text{min}} \cdot \text{min}$$

$$V = 12.75 \times 10^5 \text{ gal}$$

$$V = 1,257,000 \text{ gal.}$$

3. Challenge Problem

$Q_V = \dfrac{V}{t}$ Solve for t. where: $V = 2.0 \times 10^6$ gal

$Q_V = 800 \dfrac{\text{gal}}{\text{min}} = 8 \times 10^2 \dfrac{\text{gal}}{\text{min}}$

$$t = \frac{V}{Q_V}$$

$$t = \frac{2 \times 10^6 \text{ gal}}{8 \times 10^2 \frac{\text{gal}}{\text{min}}} = \frac{2}{8} \times 10^{6-2} \frac{\text{gal}}{\frac{\text{gal}}{\text{min}}}$$

$$t = 0.25 \times 10^4 \text{ min}$$

$$t = 2500 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} = 41.7 \text{ hr}$$

$$t = 41.7 \text{ hr.}$$

Step 2: Isolate V by multiplying both sides by t.

$$Q_V \times t = \frac{V}{\cancel{t}} \times \cancel{t}$$ (The variable t cancels on the right side.)

Step 3: Rewrite the equation with t on the right side removed.

$$Q_V \times t = V$$ (V is isolated.)

Step 4: Reverse the order of the equation so that V is on the left. V has been isolated. Now solve.

$V = Q_V \times t$

where: $Q_V$ = 800 gal/min, or $8 \times 10^2$ gal/min
$t$ = 20 hr = $1.2 \times 10^3$ min

Substitute numerical values for $Q_V$ and t in the equation, $V = Q_V \times t$.

$V = (8 \times 10^2 \text{ gal/min}) \times (1.2 \times 10^3 \text{ min.})$

Reminder: To multiply power-of-ten numbers, the first factors of each number (8 and 1.2) are multiplied as usual. The exponents of 10 (2 and 3) are added algebraically. The result is a product of the first two factors times (×) a power of 10.

Step 5: $$V = \left[(8 \times 1.2) \times 10^{(2+3)}\right]\left[\frac{\text{gal}}{\cancel{\text{min}}} \times \cancel{\text{min}}\right]$$

$V = 9.6 \times 10^5$ gal (or 960,000 gal) of oil stored in 20 hours

---

***PRACTICE EXERCISES FOR ACTIVITY 1***

***Problem 1:*** A supertanker carries $1.8 \times 10^6$ gallons of crude oil. The oil is unloaded at an offshore discharge buoy in 35 hours. Find the volume-flow rate ($Q_V$) in gallons per minute. (***Hint:*** Use the equation, $Q_V = V/t$. Use power-of-ten notation.)

***Problem 2:*** A supertanker unloads oil at the rate of 850 gallons per minute. It takes 25 hours to unload. How many gallons of oil are unloaded in 25 hours? (***Hint:*** Start with the equation $Q_V = V/t$. Isolate the symbol, "V." Then solve for volume [V] in gallons. Use power-of-ten notation.)

***Student Challenge***

***Problem 3:*** A supertanker unloads $2.0 \times 10^6$ gallons of crude oil at a rate of 800 gal/min. How long does it take to unload the tanker—in hours? (***Hint:*** Start with the equation $Q_V = V/t$. Isolate the symbol, "t." Solve for time [t] in minutes. Convert minutes to hours. Use power-of-ten notation.)

## Activity 2: Solving Fluid-rate Problems

***EQUIPMENT***

For this activity, you'll need a hand calculator.

In this activity, you'll solve fluid-rate problems that involve both a volume-flow rate and a mass-flow rate. The equation for volume-flow rate is:

$$\text{Volume-flow Rate} = \frac{\text{Volume of Fluid Moved}}{\text{Time of Movement}}$$ ***Equation 1***

## SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 2

**Problem 4:**

$Q_V = \frac{V}{t}$ (Equation 2) where: $V = 3.6 \times 10^3$ liters
$t = 10$ hr $= 600$ min
$= 6 \times 10^2$ min

$$Q_V = \frac{3.6 \times 10^3 \text{ liter}}{6 \times 10^2 \text{ min}}$$

$Q_V = 0.6 \times 10^{3-2}$ liter/min
$Q_V = 0.6 \times 10^1$ liter/min
$Q_V = 6$ liter/min.

**Problem 5:**

$Q_m = \frac{m}{t}$ (Equation 4) where: $m = 250$ kg $= 2.5 \times 10^2$ kg
$t = 8$ hr $= 480$ min
$t = 4.8 \times 10^2$ min

$$Q_m = \frac{2.5 \times 10^2 \text{ kg}}{4.8 \times 10^2 \text{ min}}$$

$Q_m = 0.52 \times 10^{2-2}$ kg/min
$Q_m = 0.52 \times 10^0$ kg/min ($10^0 = 1$)
$Q_m = 0.52$ kg/min.

**Problem 6:**

$Q_m = \frac{m}{t}$ (Equation 4) where: $Q_m = 168 \frac{\text{kg}}{\text{hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} = 2.8 \frac{\text{kg}}{\text{min}}$
$t = 5$ min

Solve for m.
$m = Q_m \times t$

$$m = 2.8 \frac{\text{kg}}{\text{min}} \times 5 \text{ min}$$

$m = 14.0$ kg.

or, in symbol form:

$$Q_V = \frac{V}{t} \qquad \textit{Equation 2}$$

where: $Q_V$ = volume-flow rate (gal/min, $ft^3/min$, liters/min, $m^3/sec$)
$V$ = volume of fluid moved (gal, $ft^3$, liters, $m^3$)
$t$ = time of movement (hours, minutes, seconds)

The equation for mass-flow rate is:

$$\text{Mass-flow Rate} = \frac{\text{Mass of Fluid Moved}}{\text{Time of Movement}} \qquad \textit{Equation 3}$$

or, in symbol form:

$$Q_m = \frac{m}{t} \qquad \textit{Equation 4}$$

where: $Q_m$ = mass-flow rate (kg/min, gm/sec)
$m$ = mass (kg, gm)
$t$ = time of movement (hours, minutes, seconds)

### *PRACTICE EXERCISES FOR ACTIVITY 2*

Use either Equation 2 or 4 to solve the following problems:

***Problem 4:*** Given: The hydraulic control pump on a construction bucket loader pumps $3.6 \times 10^3$ liters of fluid through the closed system during a 10-hour work period at a constant pressure.

Find: The pump's volume-flow rate in liters per minute. (***Hint:*** Use Equation 2. Use power-of-ten notation. Be careful with units.)

Solution:

***Problem 5:*** Given: A propane tank used to supply propane gas to a boiler will empty 250 kg of fuel into the boiler firebox in 8 hr.

Find: The mass-flow rate of the gas between the tank and boiler, in kg/hr.

Solution:

***Problem 6:*** Given: An air motor used to operate a stirrer that blends chemical dyes uses air at a rate of 168 kg/hr.

Find: The air mass consumed (used) by the air motor in 5 minutes of operation. (***Hint:*** Isolate "m" in the equation $Q_m = m/t$. Convert 5 minutes to hours. Then substitute numbers in the rearranged equation with "m" isolated, and solve for "m" in kg.)

Solution:

## STUDENT CHALLENGE ANSWERS

**Problem 7:** $Q_m = \frac{m}{t}$ (Equation 4) where: $m = 116.8$ kg
$Q_m = 14.6$ kg/min

Solve for t.

$t = \frac{m}{Q_m}$

$t = \frac{116.8 \text{ kg}}{14.6 \text{ kg/min}}$

$t = 8$ min. (Eight minutes is within the exchange time of 6-10 minutes)

**Problem 8:** $Q_V = \frac{V}{t}$ (Equation 2) where $Q_V = 210 \frac{ft^3}{min}$
$t = 1$ hr $= 60$ min

Solve for V.

$V = Q_V \times t$

$V = 210 \frac{ft^3}{min} \times 60 \text{ min}$

$V = (210 \times 60) \frac{ft^3}{min} \cdot min$

$V = 12{,}600 \text{ ft}^3$ or $12.6 \times 10^3 \text{ ft}^3$.

Two approaches may be used:

$1 \text{ ft}^3{}_{air} = 80.5 \times 10^{-3}$ lb mass

$m = V \times 80.5 \times 10^{-3}$ lb

$m = 12.6 \times 10^3 \text{ ft}^3 \times \frac{80.5 \times 10^{-3} \text{ lb}}{1 \text{ ft}^3}$

$m = (12.6 \times 80.5) \times 10^{3-3} \text{ ft}^3 \frac{\text{lb}}{\text{ft}^3}$

$m = 1014$ lb mass.

or

$1 \text{ ft}^3{}_{air} = 3.6 \times 10^{-2}$ kg

$m = V \times \frac{3.6 \times 10^{-2} \text{ kg}}{1 \text{ ft}^3}$

$m = (12.6 \times 3.6) \times 10^{3-2} \frac{\text{ft}^3 \cdot \text{kg}}{\text{ft}^3}$

$m = 45.36 \times 10^1$ kg $(10^1 = 10)$

$m = 453.6$ kg.

***Student Challenge***

***Problem 7:*** Given: For health reasons, air-conditioning is designed to circulate or exchange air in a room every 6 to 10 minutes. An air conditioner with a mass-flow rate of 14.6 kg/min circulates the air in a classroom of size 6 m × 6 m × 2.5 m (about 20 ft × 20 ft × 8 ft). The room holds 116.8 kg of air. Dry air at 20°C (68°F) has a mass of 1.2 kg per $m^3$.

Find: Does this air conditioner meet the design requirement to exchange the mass of air in the room every 6 to 10 minutes? (***Hint:*** Isolate "t" in the fluid rate equation, $Q_m = m/t$. Then solve for t.)

Solution:

***Problem 8:*** Given: An air compressor is rated at "210 cfm, free air." The symbol, "cfm," means "cubic feet per minute." The words "free air" mean air at atmospheric pressure. The air compressor operates by taking in air and compressing it to higher pressures.

Find: The mass of air taken in and compressed by the air compressor while operating for one hour at its rated value. (***Hint:*** One cubic foot of air has a mass of $80.5 \times 10^{-3}$ lb or $3.6 \times 10^{-2}$ kg.)

Solution: