Math Lab 7 MS 2

Solving Rotational "Force" Transformer Problems

For best results, print this document front-to-back and place it in a three-ring binder. Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Successfully complete Math Skills Lab 7MS2.
2. Be able to perform the Objectives of Math Skills Lab 7MS2.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete as many activities as time permits. (How much you accomplish will depend on your students' mathematics skills.)
   a. Sum up the explanatory material for "Activity: Solving Rotational Force Transformer Problems."
   b. Then have students complete the Practice Exercises at the end of the activity.
3. Before this class ends, ask your students to read Lab 7M3, "Gears," as homework.

Math Skills Laboratory

Lab 7 M S 2

MATH ACTIVITY

Activity: Solving Rotational "Force" Transformer Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve and interpret rotational force transformer problems for mechanical advantage and efficiency.***
2. ***Distinguish between force, torque and speed mechanical advantages.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Work the problems.***

ACTIVITY

Solving Rotational 'Force' Transformer Problems

Lathes, bandsaws, drill presses, mills and shapers are machine tools. These tools are more useful when their operating speed can be changed.

A common way to change the operating speed of a rotational mechanical device involves using a "stepped-cone" pulley. This pulley system is shown in Figure 1. The term, "stepped-cone pulley," means that the pulley has a cone shape and has different-diameter pulleys (steps) machined into each complete pulley.

If the pulleys are arranged as shown in Figure 1, the same belt can be connected to corresponding pulley steps. Moving the belt from one pair of pulley steps to another changes the operating speed of the device.

The ratio of circumferences (distance around) of the corresponding pulleys is one thing that determines the speed of the driven shaft. The rpm of the drive pulley also determines the operating speed of the device.

During this lab, you'll learn to find the mechanical advantage of different types of rotational mechanical devices. You'll solve technical problems that involve wheel-and-axle force transformers, belt-drive systems and friction-drive systems.

You should understand ratio and proportion before working these problems. You must also know how to rearrange simple formulas. Also, you should make drawings for each problem. Label forces, torques, radii, angular speeds (and so on) for each problem. This will help you solve the problems correctly.

Problem 1: Given: ω_i = 1750 rpm = 183 rad/sec
T_i = 4 lb•ft

a.

(1) $\frac{r_o}{r_i} = \frac{\omega_i}{\omega_o}$ or $\omega_o = (\frac{r_i}{r_o})\ \omega_i$ r_i = 4 in
r_o = 4 in

$\omega_o = \frac{4\ \cancel{in}}{4\ \cancel{in}}$ x 183 rad/sec **Note:** Ratio of **radii** $\frac{r_i}{r_o}$ = 1.

$\omega_o = \omega_i$ = 1750 rpm = 183 rad/sec. (This is the same speed, as we expected, since pulley wheels are the same size.)

(2) r_i = 3 in
r_o = 6 in
$\omega_o = \frac{3\ \cancel{in}}{6\ \cancel{in}}$ x 183 rad/sec **Note:** Ratio of **radii** $\frac{r_i}{r_o} = \frac{1}{2}$.

ω_o = 91.5 rad/sec or 875 rpm.

(3) r_i = 6 in
r_o = 3 in
$\omega_o = \frac{6\ \cancel{in}}{3\ \cancel{in}}$ x 183 rad/sec **Note:** Ratio of **radii** $\frac{r_i}{r_o}$ = 2.

ω_o = 366 rad/sec or 3500 rpm.

b. $\frac{r_o}{r_i} = \frac{T_o}{T_i}$ thus, $T_o = (\frac{r_o}{r_i})T_i$.

Condition number:

(2) r_i = 3" $T_o = (\frac{6}{3})$(4 lb•ft)
r_o = 6" T_o = 8 lb•ft.
T_i = 4 lb•ft

(3) r_i = 6" $T_o = (\frac{3}{6})$(4 lb•ft)
r_o = 3" T_o = 2 lb•ft.
T_i = 4 lb•ft.

The answer to Problem 1 is continued on page T-58c.

Problem 1: Continued

c. $IMA = \frac{r_o}{r_i}$

Condition number:

(2) $IMA = \frac{6 \cancel{in}}{3 \cancel{in}} = 2$ where: $r_o = 6$ in
$r_i = 3$ in

(3) $IMA = \frac{3 \cancel{in}}{6 \cancel{in}} = 0.5$ where: $r_o = 3$ in
$r_i = 6$ in

Problem 1: Given: A stepped-cone drive pulley is mounted on a motor shaft that turns at 1750 rpm (183 rad/sec). The driven stepped-cone pulley is mounted on the driven shaft of a bandsaw. Motor shaft torque is 4 lb·ft under full load. Figure 1 shows the diameter of each step on the pulley. The equations for ideal mechanical advantage are:

$$\text{IMA} = \frac{r_o}{r_i} = \frac{T_o}{T_i} = \frac{\omega_i}{\omega_o}$$

The conditions in Problem 1:

(1) drive belt placed on the 4-inch drive pulley and 4-inch driven pulley.

(2) drive belt placed on the 3-inch drive pulley and 6-inch driven pulley.

(3) drive belt placed on the 6-inch drive pulley and 3-inch driven pulley.

Note: Assume ideal efficiency in the system.

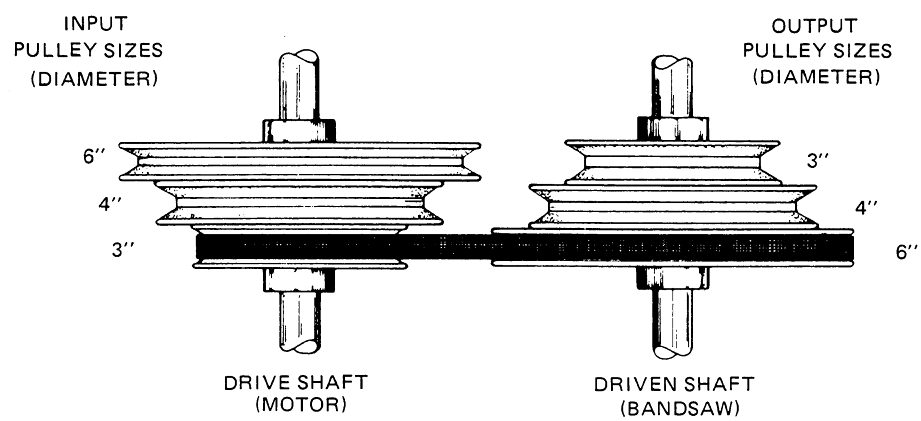


Fig. 1 Stepped-cone pulleys.

Find:

a. The ***angular speed*** of the bandsaw shaft with conditions (1), (2) and (3) above.

b. The bandsaw shaft torque with conditions (2) and (3) above.

c. The ***ideal mechanical advantage*** of the belt-drive system with conditions (2) and (3) above.

Solution:

Business and industrial buildings have doorways through which bulk materials and equipment can be carried in and out. This doorway is called a "loading dock."

Frequently, the loading dock has a "roll-up" door similar to the one shown in Figure 2. When this type of door opens, it rolls up around a shaft that spans the doorway opening. The shaft around which the door wraps is actually an axle. This axle is driven by a wheel. The wheel is actually a sprocket. And the sprocket is driven by a chain.

Pulling on the chain causes the sprocket to rotate. This causes the axle to rotate. The direction in which the chain is pulled determines whether the door wraps around the axle and opens, or unwraps from the axle and closes.

Problem 2: $r_i = \frac{D_i}{2} = \frac{12 \text{ in}}{2} = 6 \text{ in}$ $F_i = 50 \text{ lb}$

$r_o = \frac{D_o}{2} = \frac{1.5 \text{ in}}{2} = 0.75 \text{ in}$ $F_o = 400 \text{ lb}$

$$IMA = \frac{r_i}{r_o} = \frac{6 \cancel{\text{in}}}{0.75 \cancel{\text{in}}}$$

$IMA = 8$

$$AMA = \frac{F_o}{F_i} = \frac{400 \cancel{\text{lb}}}{50 \cancel{\text{lb}}}$$

$AMA = 8.$

Yes, they're equal--because we neglected friction!

b. Work In = Work Out

$F_i \times D_i = F_o \times D_o$

where: $F_i = 50 \text{ lb}$
$F_o = 400 \text{ lb}$
$D_o = 5 \text{ ft}$

$$D_i = \frac{F_o \times D_o}{F_i}$$

$$D_i = \frac{400 \cancel{\text{lb}} \times 5 \text{ ft}}{50 \cancel{\text{lb}}}$$

$D_i = 40$ ft; the chain on the input side must be pulled through 40 feet.

Problem 3: a. $AMA = \frac{F_o}{F_i}$

where: $F_o = 250 \text{ lb}$
$F_i = 50 \text{ lb}$

$$AMA = \frac{250 \cancel{\text{lb}}}{50 \cancel{\text{lb}}}$$

$AMA = 5.$ The IMA also is equal to 5.

b. Work In = Work Out

$F_i \times D_i = F_o \times D_o$

$$D_o = \frac{F_i \times D_i}{F_o}$$

where: $F_i = 50 \text{ lb}$
$F_o = 250 \text{ lb}$
$D_i = 5 \text{ ft}$

$$D_o = \frac{50 \cancel{\text{lb}} \times 5 \text{ ft}}{250 \cancel{\text{lb}}}$$

$D_o = 1$ ft.

The answer to Problem 4 is on page T-59c.

Problem 4: $IMA = \frac{r_i}{r_o} = \frac{diam_i}{diam_o}$ where: $IMA = 5$

$d_o = 1.5$ in

$d_i = IMA \times d_o$

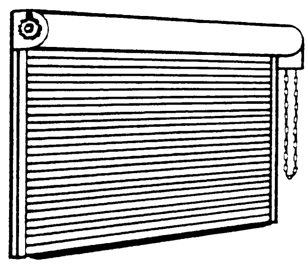
$d_i = 5 \times 1.5$ in

$d_i = 7.5$ in. Minimum diameter of the wheel that will provide an IMA of 5.

Problem 2: Given: A sheet-metal roll-up door weighs 400 lb. The chain-operated input wheel (sprocket) is 12 inches in diameter. The shaft (axle) the door cable wraps around is 1.5 inches in diameter. An input force of 50 pounds is required to open or close the door. The mechanical advantage of a wheel-and-axle force transformer can be determined with the following relationships.

$$\text{IMA} = \frac{r_i}{r_o} \text{ and } \text{AMA} = \frac{F_o}{F_i}$$

Assume ideal efficiency (no friction) for the door mechanism.



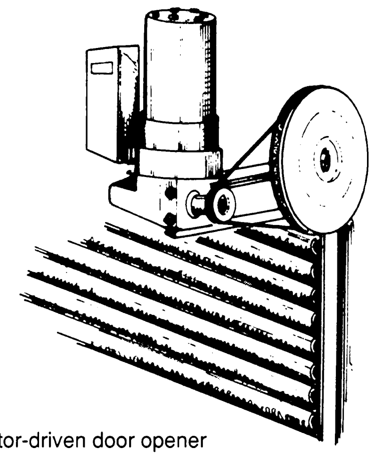


Fig. 2 Roll-up type door.

Find:

a. The ideal and actual mechanical advantages of the wheel and axle used with a sheet-metal roll-up door. Are the ideal and actual mechanical advantages equal?

b. The distance the chain must be pulled to raise a sheet-metal door 5 feet.

Solution:

Problem 3: Given: The sheet-metal door in Problem 2 is replaced with an aluminum roll-up door that weighs 250 pounds.

Find:

a. The mechanical advantage needed to raise the aluminum door if the input force remains 50 pounds. Assume ideal efficiency.

b. The distance the aluminum door will rise when the chain is pulled 5 feet.

Solution:

Problem 4: Given: The conditions in Problem 3.

Find: The minimum wheel (sprocket) diameter that can be used with the aluminum door and provide a mechanical advantage of 5 if the axle diameter remains 1.5 inches.

Note: The ratio of the diameters is the same as the ratio of the radii.

Solution:

Problem 5: Drum diam (d_o) = 8 ft
Drum speed (ω_o) = 99 rpm
Drive wheel (d_i) = 18 in = 1.5 ft
Drive motor speed = 1500 rpm; 1050 rpm under load

a. $IMA = \frac{r_o}{r_i} = \frac{\omega_i}{\omega_o}$ The same for friction drive as for belt and pulley drives.

$\frac{r_o}{r_i} = \frac{\omega_i}{\omega_o}$

$\omega_i = \frac{r_o}{r_i} \times \omega_o$ But $\frac{r_o}{r_i} = \frac{d_o}{d_i}$. Ratio of radii are proportional to ratio of diameters.

$\omega_i = (\frac{d_o}{d_i}) \times \omega_o$; $d_o = 8$ ft; $d_i = 1.5$ ft.

So, $\omega_i = \frac{8 \cancel{ft}}{1.5 \cancel{ft}} \times 99$ rpm.

$\omega_i = 5.3 \times 99$ rpm

$\omega_i = 528$ rpm.

b. The gearbox output shaft speed is the same as the drive-wheel speed (ω_i) in part "a," since the gearbox output shaft is coupled directly to the drum drive-wheel shaft.

$$\text{Speed Reduction} = \frac{\text{Input Speed to Gear Reducer}}{\text{Output Speed from Gear Reducer}}$$

$$\text{Speed Reduction} = \frac{1050 \text{ rpm}}{528 \text{ rpm}} = 1.99.$$

So the speed reduction is roughly 2:1. The gear reducer reduces the speed by a factor of 2.

Problem 6: T_i = 2000 lb•ft
ω_i = 1050 rpm
ω_o = 350 rpm

a. $\frac{\omega_i}{\omega_o} = \frac{1050 \text{ rpm}}{350 \text{ rpm}} = \frac{3}{1} = 3:1$

Therefore, the output speed is 1/3 the input speed.
The speed **reduction** ratio is 3.

b. Use Equations 7 and 9 from Table 7-2. Set AMA equal to IMA.

AMA = IMA (and substitution gives)

$\frac{\omega_i}{\omega_o} = \frac{T_o}{T_i} = \frac{3}{1}$

From the equation, torque is increased when speed is decreased, so:

$IMA = \frac{T_o}{T_i} = \frac{3}{1}.$

c. $IMA = \frac{T_o}{T_i}$. Solve for "T_o."

T_o = IMA x T_i
T_o = 3 x 2000 lb•ft
T_o = 6000 lb•ft.

Problem 7: Given: $N_o = 80$ teeth (Idler gear has 40 teeth, but only
$N_i = 20$ teeth changes direction of rotation. Thus, it can be ignored in this problem.)

Refer to Table 7-3 for correct equations to solve this problem.

a. $IMA = \frac{N_o}{N_i} = \frac{80}{20} = 4$

Speed ratio is the inverse of torque ratio.

Thus, $\frac{N_o}{N_i} = \frac{\omega_i}{\omega_o}$

$\omega_o = \left(\frac{N_i}{N_o}\right)\omega_i = 1/4\ \omega_i$. Speed reduction is 4:1.

b. Solve for "ω_i," given $\omega_o = 450$ rpm; IMA = 4.

$IMA = \frac{\omega_i}{\omega_o}$

$\omega_i = (IMA)(\omega_o) = (4)(450 \text{ rpm}) = 1800$ rpm.

Input speed is 1800 rpm, 4 times output speed of 450 rpm.

c. Torque increases as speed decreases. Since "ω_i" is high, "T_i" is low.

$IMA = \frac{\omega_i}{\omega_o} = \frac{T_o}{T_i} = \frac{N_o}{N_i} = 4$ and $T_o = 2000$ lb•ft.

Method 1--

$T_o = (IMA)(T_i)$

$T_i = \frac{T_o}{IMA} = \frac{2000 \text{ lb•ft}}{4} = 500$ lb•ft.

Method 2--

$\frac{\omega_i}{\omega_o} = \frac{T_o}{T_i}$

$T_i = \left(\frac{\omega_o}{\omega_i}\right) T_o = \left(\frac{450\ \cancel{\text{rpm}}}{1800\ \cancel{\text{rpm}}}\right) 2000$ lb•ft

$T_i = 500$ lb•ft.

Student Challenge

When you mix asphalt, you must dry the gravel used in the mix. Excess water must be removed from the gravel. This makes good adhesion between the asphalt and gravel possible.

Figure 3 shows one type of gravel-drying machine. It operates as a friction-drive rotational transformer. The equations that apply are the same as those in Table 7-2.

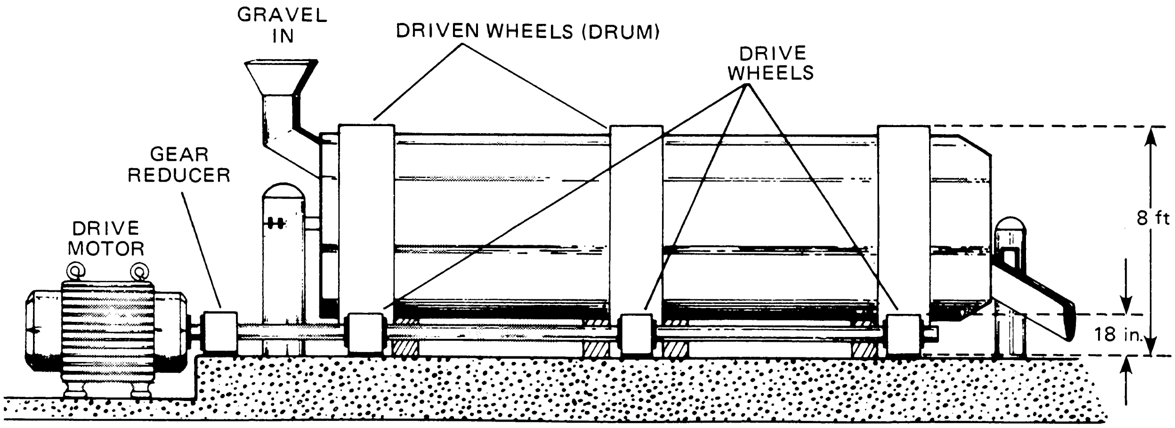


Fig. 3 Gravel dryer.

Problem 5: Given: The dryer drum is 8 feet in diameter. The drum rotates at 99 rpm. The rubber drive wheels are 18 inches in diameter. The drive motor rotates at 1500 rpm. The motor drive is coupled to a gear-type speed reducer. The speed reducer is coupled to the input shaft of the drive wheels.

Find:

a. The speed (rpm) of the drive wheels.

b. If the motor operates at 1050 rpm under load, what's the speed reduction (ω_i to ω_o) of the gear reducer?

Solution:

In industry, most electric motors run at shaft speeds of 1050 rpm or 1725 rpm under full load. To use these motor outputs, most gear-train drive systems are made to reduce—rather than increase—speed. Mechanical devices that do this are called "speed reducers." If a speed reducer links directly to the motor housing, it's called a "motorized speed reducer" or a "gear motor."

Problem 6: Given: A gear motor develops 2000 lb·ft of motor torque. It operates with an output shaft speed of 350 rpm. Input motor speed is 1050 rpm.

Find:

a. The speed reduction ratio of the gear motor.

b. The torque mechanical advantage of the gear motor.

c. The output shaft torque.

Solution:

Problem 7: Given: A gear motor has a gear train. The gear train has a 20-tooth gear on the motor drive shaft. It also has a 40-tooth idler gear. There's an 80-tooth gear on the output shaft.

Find:

a. The speed reduction of the gear motor.

b. The speed of the motor if the output shaft turns 450 rpm.

c. The motor torque if output shaft torque is 2000 lb·ft.

Solution:

Problem 8: Given: $\omega_i = 80$ rpm $\quad d_i = 4"$

$\omega_o = 20$ rpm $\quad d_o = 16"$

a. $IMA = \dfrac{\omega_i}{\omega_o} = \dfrac{d_o}{d_i}$

$IMA = \dfrac{\omega_i}{\omega_o} = \dfrac{80\ \cancel{rpm}}{20\ \cancel{rpm}} = 4$; or $IMA = \dfrac{d_o}{d_i} = \dfrac{16\cancel{"}}{4\cancel{"}} = 4.$

Either equation gives a correct answer of IMA = 4.

b. $T_o = 20$ lb•ft

$IMA = T_o/T_i$ (Rearrange equation to solve for "T_i.")

$T_i = \dfrac{T_o}{IMA} = \dfrac{20 \text{ lb•ft}}{4} = 5$ lb•ft.

c. IMA (input and output reversed) $= \dfrac{1}{\text{(IMA of part "a")}}$

$= \dfrac{1}{4} = 0.25.$

d. If $d_i = d_o$, then $\omega_i = \omega_o$, since $\dfrac{\omega_i}{\omega_o} = \dfrac{d_o}{d_i}$.

Problem 9: Given: $N_i = 16$ (input)

$N_o = 24$ (output)

a. $IMA = \dfrac{N_o}{N_i}$ and since $N_i = 16$ and $N_o = 24$, $IMA = \dfrac{24}{16} = \dfrac{3}{2} = 1.5.$

b. Method 1--

Speed and torque are reciprocals of each other. When one increases, the other decreases. Thus, the combination that produces the highest output speed also produces the lowest output torque. If the 24-tooth gear drives the 16-tooth gear, speed increases. Thus, use the 24-tooth gear as input and the 16-tooth gear as output to produce the lowest torque.

Method 2--

The smallest IMA produced in the gear arrangement leads to the lowest torque output. Therefore,

$IMA = \dfrac{N_o}{N_i}$ $\quad$ ($\dfrac{\text{output}}{\text{input}}$; a ratio)

$\dfrac{24}{16} > \dfrac{16}{24}$. Thus, 16 teeth must be on the output gear and 24 teeth must be on the input gear for the lowest possible torque with this combination.

The answer to Problem 9 is concluded on page T-62.

In industry, machines are more versatile when they can operate at various speeds. You know how stepped-cone pulleys can be used to change the operating speed of a machine. When gears are used to change the speed of a machine, the distance between the driving shaft and the driven shaft is fixed. (It stays the same.)

However, the speed of a gear-driven machine can be changed by installing matched sets of drive and driven gears on the shaft. This is because the gears of these "gear sets" have different diameters. Therefore, different gear sizes can change the shaft speed ratio.

For example, the drive and driven shafts on a machine are to be 4.5 inches apart (center to center). A 3-inch-diameter drive gear and a 6-inch-diameter driven gear would work as one matched set for the machine. A 4-inch-diameter drive gear and a 5-inch-diameter driven gear could be another matched set.

If you add the diameter of each gear in a matched set, you'll notice that the total in this example always equals 9 inches. Why do gears in each matched set have this common total? Because the drive and driven shafts are 4.5 inches apart. And the gears must meet in the middle to mesh.

Problem 8: Given: A matched-gear set on a lathe produces an output shaft speed of 20 rpm when the input shaft speed is 80 rpm. The drive-gear diameter is 4 inches. The driven-gear diameter is 16 inches.

Find:

a. Ideal mechanical advantage (IMA) of the gear set.

b. Input torque when the output torque is 20 lb·ft.

c. Ideal mechanical advantage (IMA) if the input and output gears were exchanged.

d. Output shaft speed if the drive and driven gears were each 10 inches in diameter.

Solution: (***Hint***: Refer to Table 7-3 for formulas to use.)

Problem 9: Given: Matched-gear sets are used to speed up or slow down feed rollers on a printing press. Two gears are used to drive a roller. A 16-tooth gear is on the input shaft. A 24-tooth gear is on the driven-roller shaft.

Find:

a. Ideal mechanical advantage of the gear set.

b. To have the lower output torque on the feed rollers, which gear of a 16-tooth and 24-tooth matched-gear set would you use as the drive gear?

c. If IMA = 1.6, how many teeth would it take on an input gear if the output gear had 32 teeth?

d. The direction the roller turns if the input shaft rotates clockwise and there's an idler gear between the drive and driven gears.

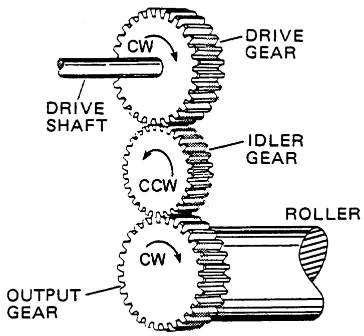
Solution: (***Hint***: Refer to Table 7-3 for formulas to use.)

c. If IMA = 1.6 and IMA = N_o/N_i, and if N_o = 32 teeth--

$$IMA = \frac{N_o}{N_i} \quad \text{(Rearrange to solve for "}N_i\text{.")}$$

$$N_i = \frac{N_o}{IMA} = \frac{32 \text{ teeth}}{1.6} = 20 \text{ teeth}.$$

Therefore, the input gear must have 20 teeth.

d. 

The driven (or output) of the roller will be the same as the input roller (clockwise).

Problem 10: Given: Pitch of worm gear = 1"

a. The IMA of a worm-and-wheel gear train equals the number of teeth on the meshed gear, or:

$$IMA = \frac{N_o}{N_i} = \frac{N_o}{1} \quad \text{(since "}N_i\text{" of worm = 1)}$$

Thus, IMA = 26/1 or 26, if N_o = 26.

b. Since IMA = 26 for this gear train, if the input rpm were 26 rpm, the output gear would have a speed of 1 rpm. In other words, the output shaft turns once for every 26 turns on the input shaft. Thus, 10 turns on the input results in less than 1 on the output. Therefore, Output Turn = 10/26 of 1 revolution = 0.385 rev.

c. Since the pitch of the worm gear is 1 inch, and it takes 26 turns of the worm gear to make one revolution on the output gear, you can guess that the circumference of the output gear is 26 inches. Thus, 10 turns (or revolutions) of the worm gear will mean a movement of 1 inch per revolution--or 10 inches of movement on the rim of the output gear.

d. T_i = 5 lb•ft

$$IMA = \frac{T_o}{T_i} \quad \text{(Assume IMA = 26, from part "a.")}$$

$$T_o = (IMA)(T_i) = (26)(5 \text{ lb•ft}) = 130 \text{ lb•ft}.$$

Problem 10: Given: A worm-gear drive is made so that one full turn of the worm gear (input) turns the other gear by one tooth. The pitch of the worm gear is 1 inch.

Find:

a. Mechanical advantage of this gear system, if the worm gear is meshed with a 26-tooth output gear.

b. Revolutions made by the output shaft when the worm-gear shaft makes 10 revolutions.

c. Distance a point on the rim of the output gear moves during the 10 revolutions described in b above.

d. The output torque, if the input torque on the worm gear is 5 lb·ft.

Solution: