Supplemental Experiment 2
Solving Vector Forces Using Trigonometric Functions

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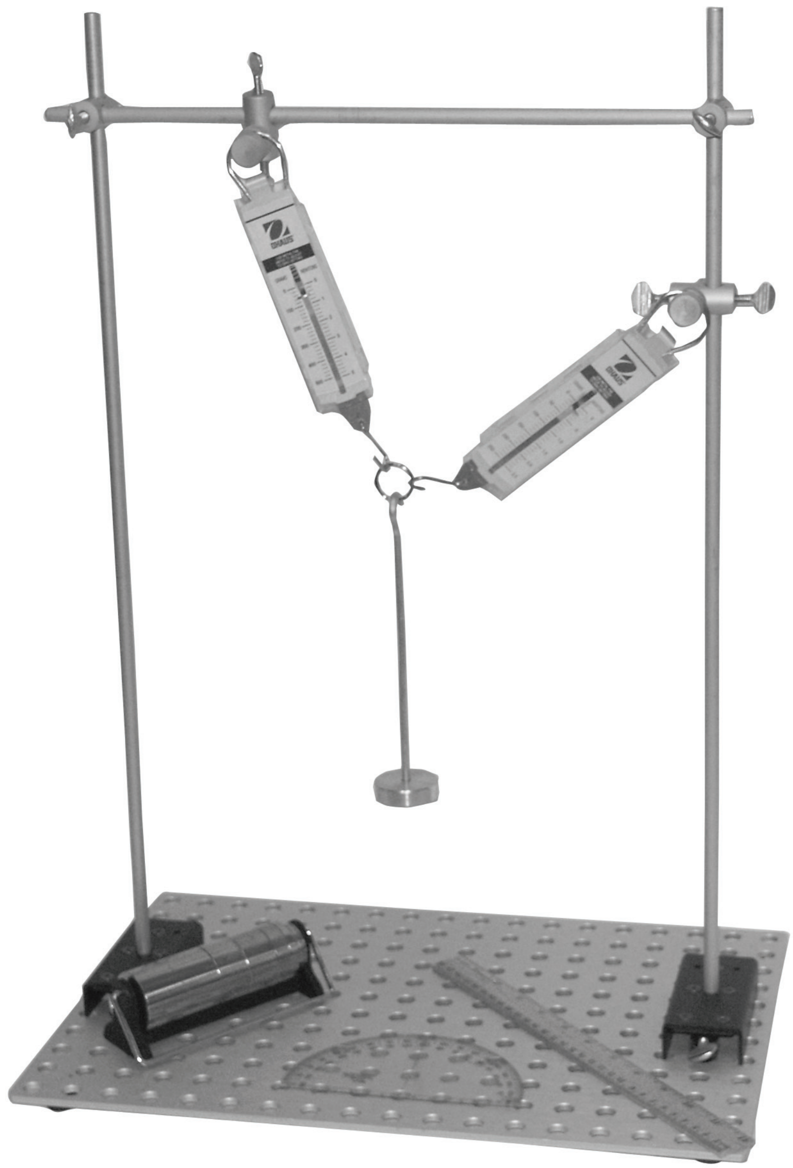
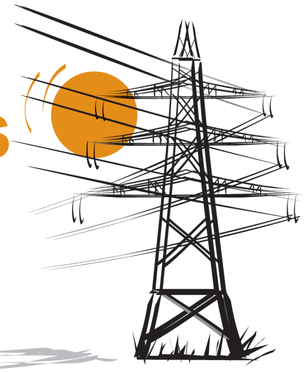


Figure 1
Setup for Supplemental Experiment 2

Supplemental
Experiment 2

Solving Vector Forces Using Trigonometric Functions



Experiment Objectives

- Calculate the theoretical values of distributed forces when a given mass is suspended from multiple points.
- Comprehend how forces work when the system is in equilibrium.
- Use mathematics, specifically, trigonometry, to calculate the vector forces.

Laboratory Proficiencies

- Use a protractor to measure degrees to the nearest degree.
- Use a scientific calculator or a trig table to find the trigonometric function values of a right triangle.

Discussion

In the Measuring Vector Forces experiment, we graphed data to calculate and find the resultant forces. In this experiment, we will be using trigonometry functions to show that we can arrive at the same result using mathematics only. Trigonometry functions are commonly used by architects, physicists, mathematicians, and engineers.

Three common functions in trigonometry are sine, cosine, and tangent. They can be found by using a scientific calculator or trig tables found in a textbook or online. These functions are used with a right triangle.

A right triangle is a triangle in which one angle is 90° and the three sides (base, height, and hypotenuse) satisfy the Pythagorean theorem ($\text{Base}^2 + \text{Height}^2 = \text{Hypotenuse}^2$). See Figure 1. For every triangle, inside angles total to 180°. Therefore, in a right triangle, the two non-right angles must total 90°. The hypotenuse is always the longest side of a right triangle and the hypotenuse is opposite the right angle.

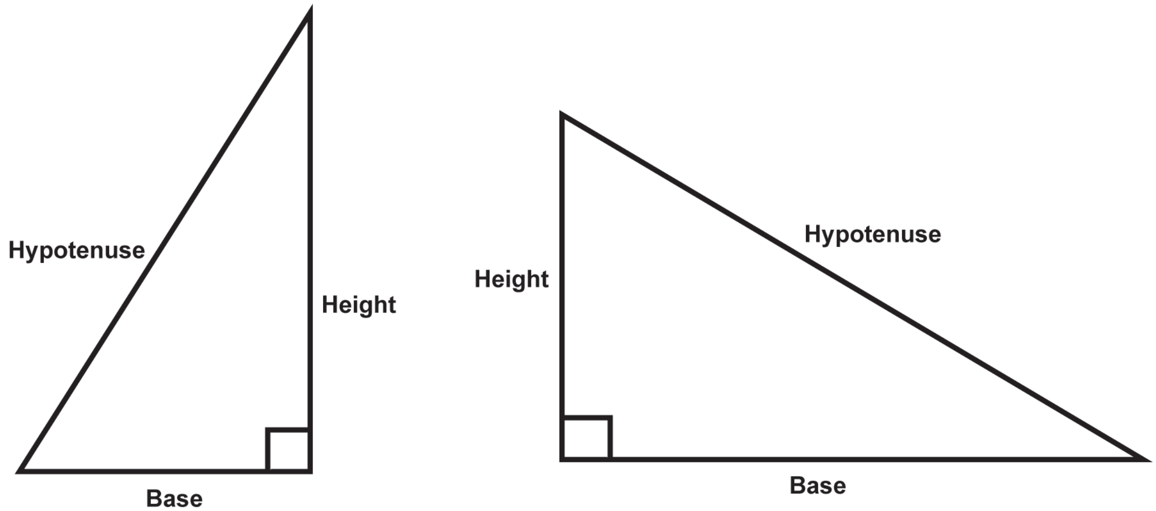


Figure 2
Examples of right triangles

Sine, Cosine, and Tangent Functions

The sin (sine) function finds the value of a side, hypotenuse, or angle as long as the triangle is a right triangle. The sine function is given by:

$$\sin(\text{angle}) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

The cos (cosine) function delivers a similar purpose as the sine function. However, the equation of cosine function would be:

$$\cos(\text{angle}) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Unlike sine and cosine, the tan (tangent) function does not require the hypotenuse. The equation for the tangent function looks like:

$$\tan(\text{angle}) = \frac{\text{opposite side}}{\text{adjacent side}}$$

Opposite and adjacent are relative to the chosen angle. Trig functions do not have units because they are in ratio. The phrase "Oscar Has A Hat On Always" can be used to memorize these three trig functions easily. Figure 3 shows the equations of the basic trigonometric functions along with an easy way to memorize them.

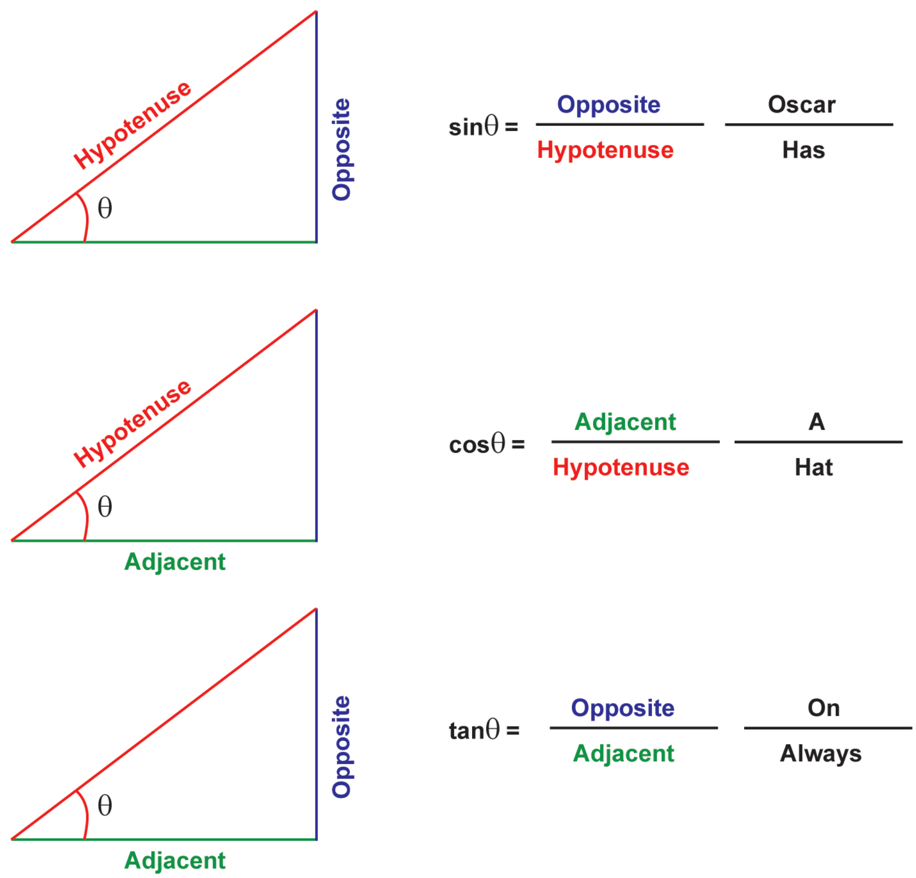
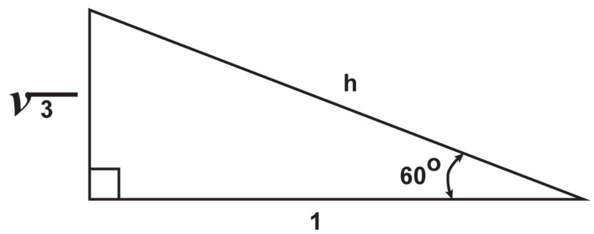


Figure 3
Easy memory trick for trig functions

We can use sine and cosine functions in this experiment to find theoretical values of vector forces. Figure 4 shows separate examples involving the Pythagorean theorem and the sine function giving the same result.



Using the Pythagorean theorem

$$h^2 = (1)^2 + \left(\sqrt{3}\right)^2$$
$$= 1 + 3$$
$$= 4$$
$$h = 2$$

Using the sine function

$$\text{Sin(angle)} = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$
$$\sin\left(60^\circ\right) = \frac{\sqrt{3}}{h}$$
$$h \sin\left(60^\circ\right) = \sqrt{3}$$
$$h = \frac{\sqrt{3}}{\sin\left(60^\circ\right)}$$
$$= \frac{1.732}{0.866}$$
$$= 2$$

Figure 4
Pythagorean theorem *v.* sin function

Equipment and Materials Required

- Calculator (supports trig functions or trig tables)
- Metal O-Ring, 1”
- Protractor
- Ruler, 30 cm
- Small Slotted Weight Set
- Spring Scale, 2.5 Newton
- Spring Scale, 5 Newton
- Support Stand Set (following parts)
  — Long Crossbar
  — Mechanical Breadboard
  — Rod Connectors, 4
  — Scale Hangers, 2
  — Support Rods with Base, 2

Procedure

Lab Setup

The lab setup is shown in Figure 1 at the beginning of the experiment. Refer to this figure and the detailed figures that follow when assembling the equipment.

❑ 1. Mount the two scale hangers. Position one of the scale hangers on the crossbar, 10 cm from the left rod connector. Position the other hanger on the right support rod, 9 cm below the crossbar.

❑ 2. Hang the 5 Newton scale from the scale hanger on the crossbar. Hang the 2.5 Newton scale from the scale hanger on the support rod. Fully extend the tongue of the 5 Newton scale. Use a pencil to draw a line down the middle of the tongue. Repeat for the 2.5 Newton scale.

❑ 3. The spring scales should read zero when hanging vertically. Adjust both scales by sliding the movable indicator plate up or down.

❑ 4. Attach the S-hook from each scale to the 1” O-Ring. Then suspend the 50-gram weight hanger from the ring.

❑ 5. Add 350 grams of mass to the weight hanger for a total hanging mass of 400 grams.

Observations and Data Collection

Enter your answers in the Student Journal.

❑ 1. Read the scales and center the forces in Data Table 1.

❑ 2. Use the protractor to measure the angle of each force with respect to the vertical axis. Position the protractor so that the center mark is aligned with the center of the O-ring. Align the baseline of the protractor along the shaft of the weight hanger.

Label the angle between the vertical axis and the 5 Newton scale as angle *A*. Label the angle between the vertical axis and the 2.5 Newton scale as angle *B*. See Figure 5 on the next page. Enter the angles in Data Table 1.

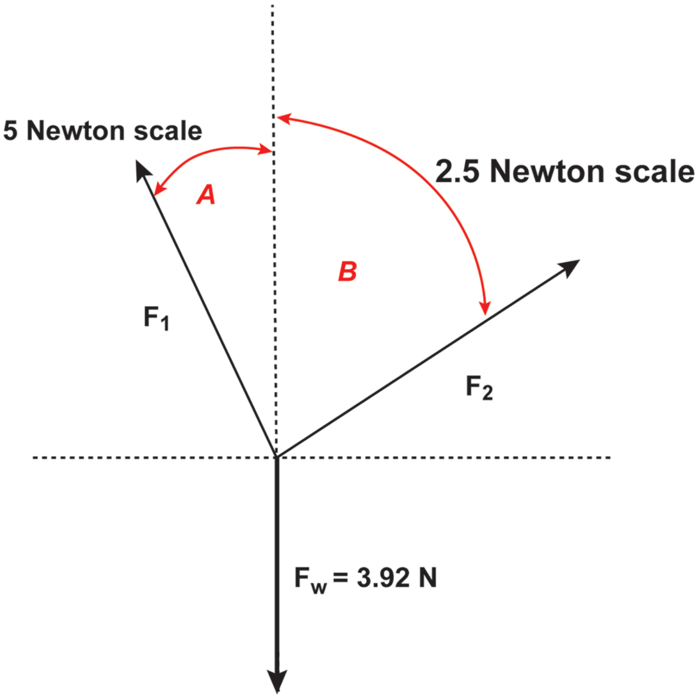


Figure 5
Identifying the measured angles

❑ 3. Subtract angle *A* from 90° and label this value as θ "theta." Subtract angle *B* from 90° and label this value as ϕ "phi." See Figure 6. In Figures 6 and 7 vector F_W has been left out in order to simplify the drawing. Enter the angle sin Data Table 1.

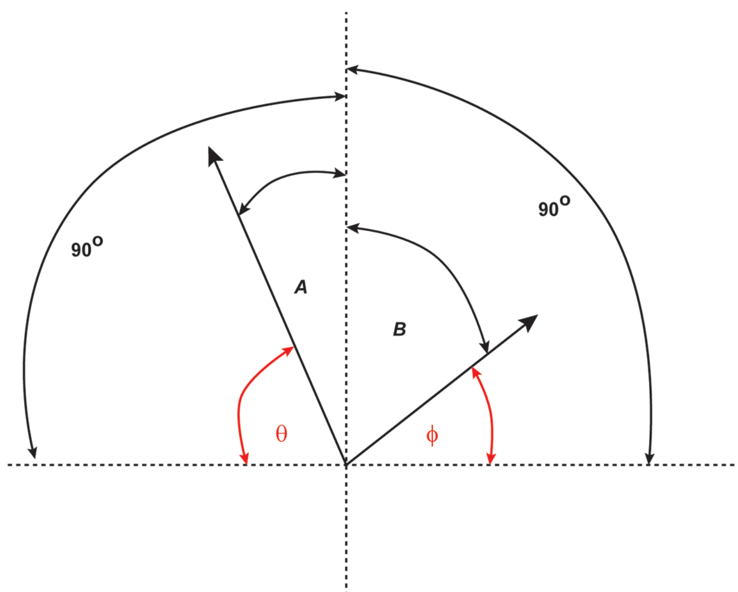


Figure 6
Identifying angles θ and ϕ

Horizontal Balance of Forces

❑ 1. Figure 7 shows the horizontal forces that are derived from the two force vectors.

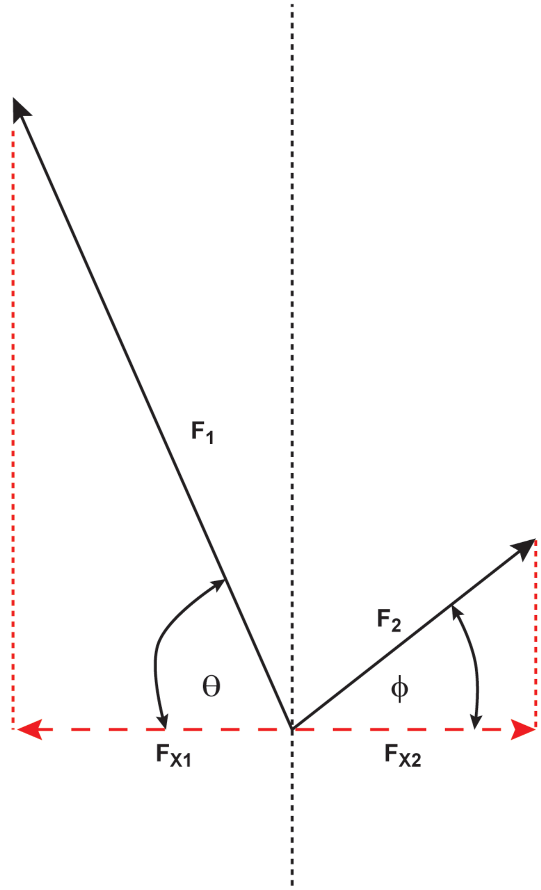


Figure 7
The horizontal forces that are derived from the force vectors

❑ 2. Although in the experiment the horizontal forces cannot be measured directly, they can be calculated mathematically. Enter the values in Data Table 2.

$$\cos(\theta \text{ or } \phi) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{F_{X1}}{F_1} \qquad \cos\phi = \frac{F_{X2}}{F_2}$$

$$F_{X1} = F_1\cos\theta \qquad F_{X2} = F_2\cos\phi$$

- ❑ 3. Since the system is in equilibrium, meaning there is no horizontal movement, the horizontal forces are equal but opposite in direction. Are the forces equal?

Vertical Balance of Forces

- ❑ 1. The vertical component of F_1 is shown in Figure 8.The vertical component of F_1 can be written as:

$$\mathbf{F_{Y1} = F_1 \sin\theta}$$

Enter F_{Y1} in Data Table 3.

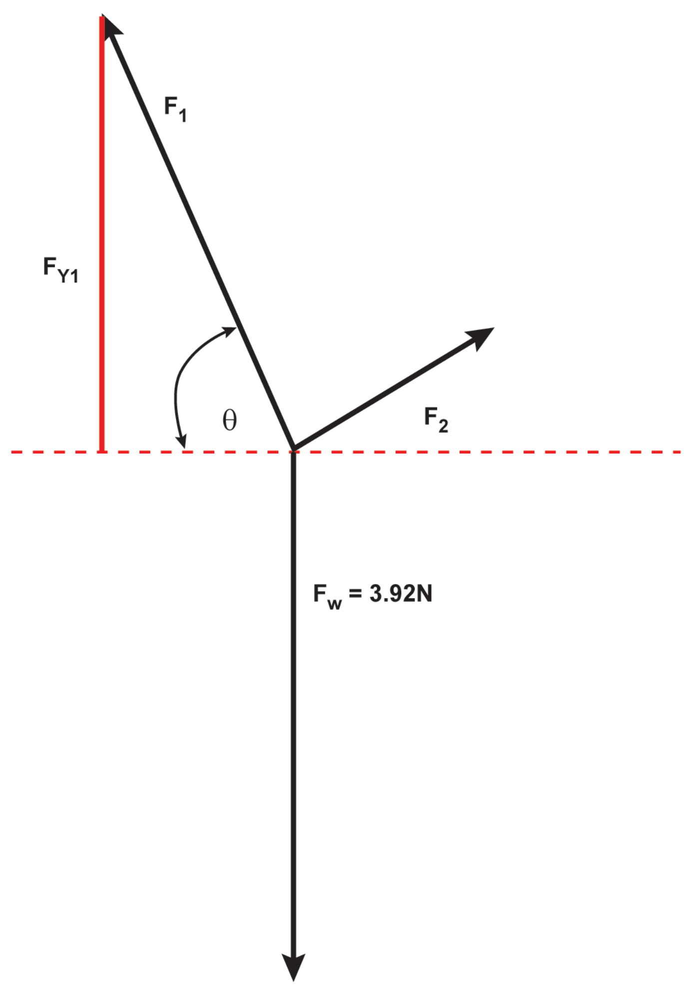


Figure 8
The vertical component of F_1

❑ 2. Figure 9 shows the vertical component of F_2. The vertical component of F_2 can be written as:

$$\mathbf{F_{Y2} = F_2 \sin\phi}$$

Enter F_{Y2} in Data Table 3.

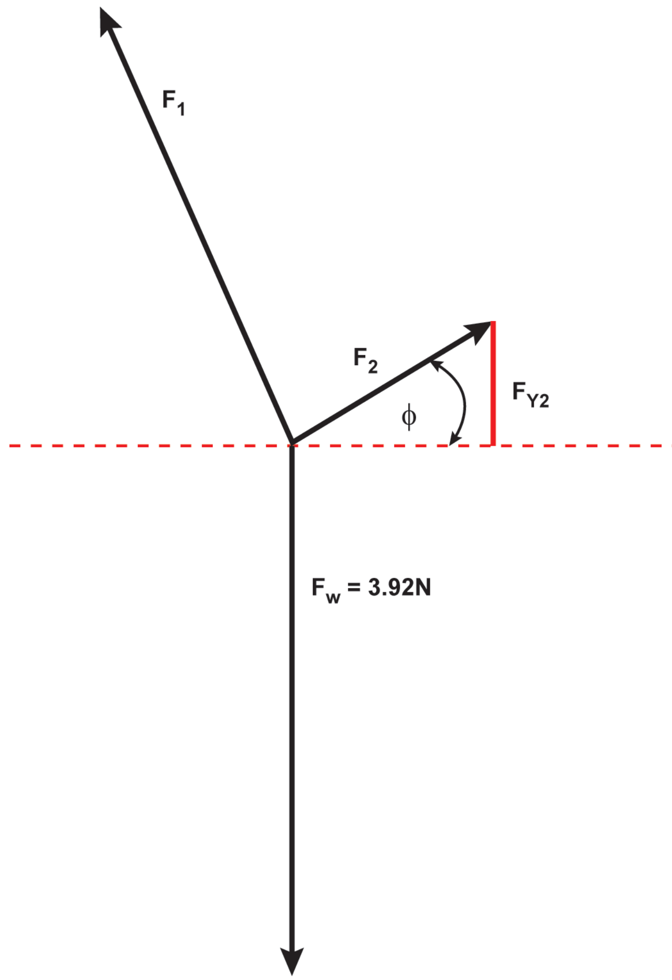


Figure 9
The vertical component of F_2.

❑ 3. Since the vertical components are acting in the same direction, they can be added as shown in Figure 10. The summation can be written as:

$$\mathbf{F_R = F_{Y1} + F_{Y2}}$$

Enter the value in Data Table 4.

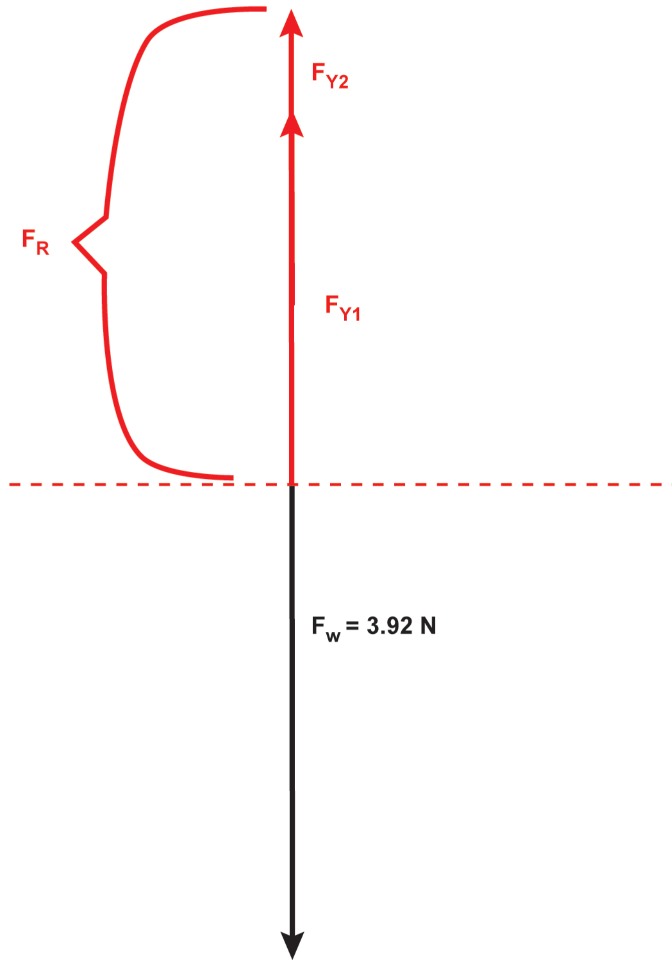


Figure 10
The addition of vectors

- ❑ 4. Convert the mass of the hanging mass to kilograms. Enter the value in Data Table 4.

$$1\ \text{g} = \frac{1\ \text{kg}}{1000\ \text{g}}$$

- ❑ 5. Use the following equation to convert the mass of the hanging weight to units of force in Newtons.

$$F_w = \text{Mass}(\text{kg}) \times \frac{9.81\ \text{N}}{\text{kg}}$$

Enter your answer in Data Table 4.

- ❑ 6. Is F_w equal to F_R? Enter your answer in the Student Journal.

Questions and Interpretations

1. Show mathematically how F_{Y1} is the same as $F_1 \sin\theta$, and F_{Y2} is the same as $F_2 \sin\phi$. Refer to the equations in step 6.

2. If θ "theta" was to equal 47° and ϕ "phi" was to equal 12°, what would the new F_1 and F_2 be? Assume $F_R = 3.92$ N.

3. If we changed the height of 2.5 Newton scale from 9 cm below the crossbar to 3 cm below the crossbar, how would that affect the vector force of the 5 Newton scale (F_1)? Will it increase, decrease, or stay the same? Refer to Figure 5.

4. Where should you position the 2.5 Newton scale in order for the 5 Newton scale to support the whole weight (vertically in line with the weight)?

Notes