Math Lab 7 MS 3

Solving “Force” Transformer Problems for Fluid Systems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

Solve and interpret force transformer problems for rotational mechanical systems and fluid systems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through Student Exercises. Make sure that students understand the correct answers.
2. Ask students to solve math problems that involve hydraulic jacks and pressure intensifiers.
3. Before the class ends, ask students to read Lab 7F1, "The Hydraulic Jack," as homework.

Problem 1: a. Given: $A_i = 1.76 \text{ in}^2$

$F_i = 50 \text{ lb}$

$F_o = 600 \text{ lb}$

Because of the statement, IMA $= \frac{F_o}{F_i} = \frac{A_o}{A_i}$, you know that we are assuming 100% efficiency, where IMA = AMA.

$$\text{IMA} = \frac{F_o}{F_i} = \frac{A_o}{A_i}\ .$$

Step 1--Solve for IMA.

$$\text{IMA} = \frac{600 \text{ lb}}{50 \text{ lb}} = 12.$$

Step 2--Solve for "A_o."

$\text{IMA} = \frac{A_o}{A_i}$; rearrange equation to isolate "A_o."

$$A_o = (\text{IMA})\ A_i = (12)(1.77 \text{ in}^2) = \underline{21.24 \text{ in}^2}.$$

b. Given: $d_i = 1.5$ inches

$F_i = 50 \text{ lb}$

$F_o = 600 \text{ lb}$

$\text{IMA} = \frac{F_o}{F_i} = 12$, just as in part "a."

$$\text{IMA} = \frac{A_o}{A_i} = \frac{0.7854\ d_o^2}{0.7854\ d_i^2} = \frac{d_o^2}{d_i^2}\ .$$

Thus, IMA $= d_o^2/d_i^2$; isolate "d_o^2" and solve for "d_o."

$$d_o^2 = (d_i^2)(\text{IMA}) = (1.5)^2(12) = 27$$

$$d_o^2 = 27$$

$$d_o = \sqrt{27} = \underline{5.2 \text{ in}}.$$

c. $\text{IMA} = \frac{A_o}{A_i} = \frac{D_i}{D_o}$

$\text{IMA} = \frac{D_i}{D_o}$; rearrange this equation by isolating "D_o."

$D_o = \frac{D_i}{\text{IMA}}$ where: $D_i = 12$ in

IMA = 12

$$D_o = \frac{12 \text{ in}}{12} = 1 \text{ in}.$$

The output piston moves one inch.

Math Skills Laboratory

Lab 7 MS 3

MATH ACTIVITY

Solving "Force" Transformer Problems for Fluid Systems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve and interpret "force" transformer problems for fluid systems.***
2. ***Relate ideal mechanical advantage to ratio of piston face areas (or diameters) on input and output sides of the transformer.***
3. ***Relate actual mechanical advantage to ratio of forces (or pressures) on input and output sides of the transformer.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Work the problems.***

ACTIVITY

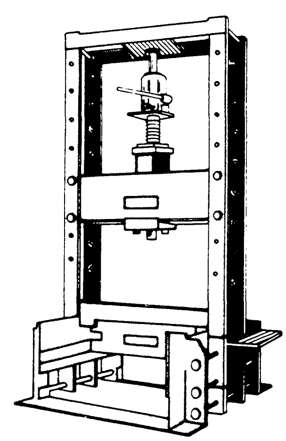
Solving "Force" Transformer Problems for Fluid Systems

In this lab, you'll work problems that involve "force" transformers in fluid systems. The problems involve fluid pressure intensifiers, fluid jacks and cylinders.

To solve these problems, refer to the formulas in Table 7-4, "Force Transformer Formulas for a Hydraulic Jack," and Table 7-5, "Force Transformer Formulas for a Pressure Booster."

Problem 1: Given: Shop Press, Inc., makes presses like the one in this drawing. This press uses a hydraulic jack to apply force to parts that are being put together and taken apart. In fluid systems, pressure is constant throughout the volume containing the fluid. This means that "pressure input" equals "pressure output" in a hydraulic jack. The ideal mechanical advantage of a hydraulic jack can be stated as:

$$IMA = \frac{F_o}{F_i} = \frac{A_o}{A_i}$$



Shop press

Problem 2: a. Given: $A_o = 10\ in^2$

$p_o = 1500$ psi

$F_o = p_o \times A_o = (1500\ lb/in^2)(10\ in^2) = 15{,}000$ lb.

b. Given: $F_o = 15{,}000$ lb

$D_o = 1$ ft

$W_o = F_o \times D_o$

$W_o = (15{,}000\ lb)(1\ ft) = 15{,}000$ ft•lb.

c. Given: MA = 12

$F_o = 15{,}000$ lb

$IMA = \dfrac{F_o}{F_i}$; rearrange to solve for "F_i."

$F_i = \dfrac{F_o}{IMA} = \dfrac{15{,}000\ lb}{12} = 1250$ lb.

d. Given: No losses to resistance. You assume a 100% efficiency. Thus,

$W_o = W_i = 15{,}000$ ft•lb.

Problem 3: Use Table 7-5.

a. $IMA = \dfrac{p_o}{p_i}$; therefore, $p_o = IMA \times p_i$ (assuming there's no resistance).

$p_o = MA_i \times p_i$ where: $MA_i = 20$

$p_o = 20 \times 125$ psi $p_i = 125$ psi

$p_o = 2500$ psi.

b. $IMA = \dfrac{A_i}{A_o}$; therefore $A_o = \dfrac{A_i}{IMA}$.

$A_o = \dfrac{A_i}{IMA}$ where: $A_i = 0.7854\ D_i^2$

$D_i = 3$ in

$IMA = 20$

Substituting in values:

$$A_o = \frac{(0.7854) \times (3\ in)^2}{20}$$

$$A_o = \frac{(0.7854 \times 9\ in^2)}{20}$$

$$A_o = \frac{7.07\ in^2}{20}$$

$A_o = 0.35\ in^2$.

(Suppose the input pump piston of the hydraulic jack shown in the drawing has a surface area of 1.77 in^2.)

Find:
a. Surface area of the output piston face when a 50-pound input force causes the output piston of the jack to raise a 600-pound load.
b. The diameter of the output piston when a 50-pound input force moves a 600-pound load.
c. The distance the load moves if the input piston moves 12 inches, under the conditions of b.

Solution:

Problem 2: Given: Century Construction Company has several hydraulically operated earth-moving machines. Ron Brown works as a hydraulics technician for the company. He's been asked to modify the hydraulic system of one of the machines. He'll replace a small hydraulic cylinder with a larger one. The new cylinder has a usable output piston face area of 10 in^2. The pump on this machine can provide 1500 psi of pressure in the fluid.

Find:
a. Maximum output force that can be exerted by the cylinder.
b. Output work done by the cylinder to move the cylinder rod out 1 ft against a load.
c. Input force supplied by the pump if the system's ideal mechanical advantage is 12.
d. Input work done by the pump (assuming there are no losses due to friction/resistance).

Solution:

Problem 3: Given: Carl Renchler is a machine repair technician for Harris Manufacturing. This company produces copper fittings. These fittings are used by plumbers and makers of heating and cooling equipment. Fitting ends are expanded so that tubes can be eased into the fittings and soldered into place. Fitting ends are expanded in a forming machine that's equipped with an air-to-water pressure booster. The pressure booster is usually operated at 125 psi air pressure. This produces a 20:1 ideal mechanical advantage.

Find:
a. The water pressure applied to the inside surface of the copper fitting.
b. The face area of the output piston if the input piston is 3 inches in diameter.

Solution:

Problem 4: Use Table 7-5.

Given: $d_i = 3.5"$

$d_o = 0.5"$

a. $IMA = \frac{A_i}{A_o} = \frac{0.7854\ d_i^2}{0.7854\ d_o^2} = \frac{d_i^2}{d_o^2}$

Also, $IMA = p_o/p_i$. Since $IMA = d_i^2/d_o^2$ and $IMA = p_o/p_i$, we can write:

$\frac{p_o}{p_i} = \frac{(d_i)^2}{(d_o)^2}$. (Rearrange the equation to solve for "p_o.")

$p_o = (\frac{d_i}{d_o})^2\ p_i = (\frac{3.5}{0.5})^2(100\text{ psi}) = (7)^2(100) = 4900\text{ psi}.$

b. The IMA of the pressure intensifier is:

$$IMA = \frac{A_i}{A_o} = \frac{0.7854\ d_i^2}{0.7854\ d_o^2} = \frac{d_i^2}{d_o^2} = \frac{(3.5)^2}{(0.5)^2} = \frac{12.25}{0.25} = 49.$$

c. Logically, the IMA for a system of force transformers is the product of the individual MAs for each force transformer.

$(IMA_{\#1})(IMA_{\#2})$ = MA of system where: $IMA_{\#1} = 49$

$IMA_{\#2} = 49$

$IMA = (49)(49) = 2401.$

Problem 5: Use Table 7-5.

$IMA = 12$

$p_i = 14.7$ psi

$IMA = p_o/p_i$; rearrange equation to solve for "p_o."

$p_o = (IMA)(p_i) = (12)(14.7\text{ psi}) = 176.4\text{ psi}.$

Problem 6: Use Table 7-5.

Given: $p_i = 50$ psi; $A_o = 1\text{ in}^2$; $F_o = 600$ lb

a. $p_o = \frac{F_o}{A_o} = \frac{600\text{ lb}}{1\text{ in}^2} = 600\text{ psi}$

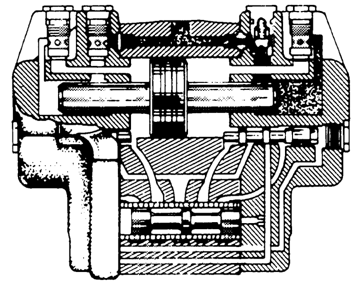
$IMA = \frac{p_o}{p_i}$ where $p_o = 600$ psi and $p_i = 50$ psi

$IMA = \frac{600\ \cancel{psi}}{50\ \cancel{psi}} = 12$

b. $IMA = \frac{A_i}{A_o}$. Therefore, $A_i = IMA \times A_o$

$A_i = IMA \times A_o = (12)(1\text{ in}^2) = 12\text{ in}^2$

Problem 4: Given: Kali-Alto Laboratories employs Kay Lewis as a standards lab technician. Part of Kay's job is to calibrate high-pressure hydraulic gages. An air-to-hydraulic pressure intensifier is used to pressurize the gages being tested and calibrated. The intensifier has a 3.5-inch-diameter input cylinder and a 0.5-inch-diameter output cylinder.



Intensifier

Find:

a. Maximum pressure available to test the gages if the air pressure is 100 psi.

b. Mechanical advantage of the pressure intensifier.

c. Overall mechanical advantage if the output pressure of the intensifier were connected to the input side of an identical pressure intensifier.

Solution:

Problem 5: Given: Power Systems makes a pressure booster that's used on the power brake system of some cars. One side of the pressure booster is linked to the intake manifold of a car's engine. When the engine runs, a vacuum forms on this side of the pressure booster. When the brake pedal is pushed down, atmospheric air pressure acts on the surface of the large piston. This increases the pressure available to push on the piston in the master cylinder of the hydraulic brake system. A research technician for this company might be asked to provide an answer to the following question.

Find: If the atmospheric pressure is 14.7 psi and the mechanical advantage of the booster is 12, what's the pressure applied to the piston of the master cylinder?

Solution:

Problem 6: Given: Williamsport Tower Company constructs and installs girder towers that support high-voltage electric power lines. These towers are very tall. They must be assembled "on site." Individual pieces of the tower are riveted together with a portable air-to-hydraulic rivet gun. The rivet gun is a pressure intensifier that uses a 50-psi air source.

Find:

a. Mechanical advantage of the pressure intensifier when a 1-in^2 output piston pushes on a rivet with 600 lb of force.

b. Area of the piston on the air input side of the pressure intensifier.

Solution:

Problem 7: Given: $A_i = 7\ ft^2$
$A_o = 1\ ft^2$

a. $IMA = \frac{A_o}{A_i} = \frac{1\ \cancel{ft^2}}{7\ \cancel{ft^2}} = \frac{1}{7}$. Force is reduced but piston travel is increased. That's what's needed.

b. The hydraulic lift is like a hydraulic jack. Thus, the same equations apply.

$IMA = \frac{F_o}{F_i}$, and $F_o = IMA \times F_i$.

where: $IMA = \frac{1}{7}$ (from part "a" above)

The input force (F_i) comes from the basic equation "Force = Pressure × Area." So:

$F_i = p_i \times A_i$. Change pressure in psi to lb./ft^2; then find "F_i."

$$F_i = \left[\left(100\ \frac{lb}{\cancel{in^2}}\right)\left(\frac{144\ \cancel{in^2}}{1\ \cancel{ft^2}}\right)\right](7\ \cancel{ft^2}) = 100{,}800\ lb.$$

$$F_o = \frac{1}{7} \times F_i = \frac{1}{7}(100{,}800\ lb) = 14{,}400\ lb\ (7.2\ tons)$$

NOTE: You don't have to worry about the area of the hydraulic tank (7 ft^2) or calculating the total force F_i on the top surface of the hydraulic fluid (100,800 lb). Since we are given that 100 psi of pressure is applied to the hydraulic tank, we know that the same pressure is transmitted unchanged everywhere throughout the fluid. Thus, there is a pressure of 100 psi pushing up on the 1 ft^2 area of the movable cylinder piston. Then, quite directly, you can calculate

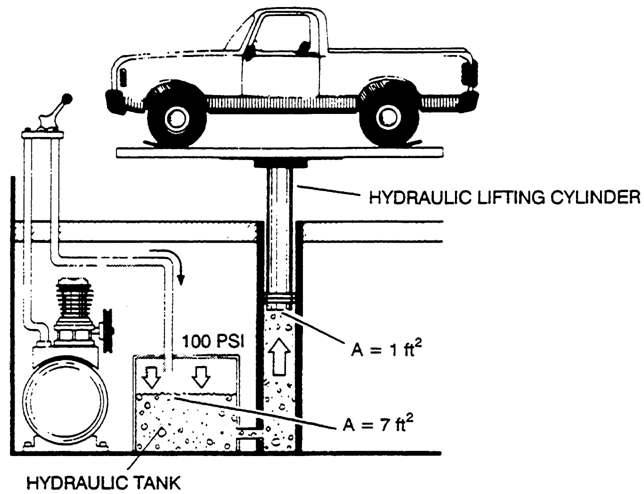
$$F_o = P_o \times A_o \text{ where } P_o = 100\ psi \text{ and } A_o = 1\ ft^2 = 144\ in^2.$$

$$F_o = 100\ \frac{lb}{\cancel{in^2}} \times 144\ \cancel{in^2} = 14{,}400\ lb.$$

This is the same answer obtained above.

c. Since $IMA = \frac{D_i}{D_o} = \frac{1}{7}$, if $D_i = 1$, then $D_o = 7$ ft. That, of course, is what is desired. The hydraulic lift raises the car 7 feet for every 1 foot of fall of the hydraulic fluid level in the large tank.

Problem 7: Given: The sketch with this problem shows a type of air-over-hydraulic lift used by automotive garages. In this example, the air compressor is connected to a hydraulic tank that has a cross-sectional area of 7 ft^2. The tank is connected to a hydraulic lifting cylinder. The cylinder piston has 1 ft^2 of face area in contact with the oil. (Note: Air pressure above fluid in hydraulic tank serves the same function as the input piston in a hydraulic jack.)



Find:

a. The ideal mechanical advantage of this device.

b. The force applied to a load by the movable piston if 100 psi of air pressure is applied to oil in the tank. Remember: 1 ft^2 = 144 in^2.

c. How far up the top of the lifting piston a load moves if the oil level in the large tank moves down one foot.

Solution: