PREPARATORY MATH SKILLS LAB

Lab PMS 13

MATH ACTIVITY

Checking Dimensions in Equations

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to check the dimensions for each item, when given an equation with several terms. Then you should be able to check the equation as a whole.

Operations done with units are similar to operations done with numbers. Therefore, if the units that accompany a numerical answer aren't correct after you solve a problem, some error has been made in the problem-solving process. To make this point more clear, look at the following equation:

$$S = vt + \tfrac{1}{2} at^2$$

where: S = distance
v = speed (distance/time)
a = acceleration (distance/time^2)
t = time

This equation involves solving for distance (S). Therefore, each term, such as "vt" and "$\frac{1}{2}$ at^2" on the right side of the equation, must also have units of "distance." Let's show this by first substituting the correct units for v, t and a on the right side, then canceling units where appropriate.

$$S = vt + \tfrac{1}{2} at^2 \qquad \text{(Ignore constants, such as } \tfrac{1}{2}\text{, that carry no units.)}$$

$$S = \left[\frac{\text{distance}}{\cancel{\text{time}}} \times \cancel{\text{time}}\right] + \left[\frac{\text{distance}}{\cancel{(\text{time})^2}} \times \cancel{(\text{time})^2}\right] \qquad \text{(Cancel time units.)}$$

$$S = \text{distance} + \text{distance}$$

$$S = \text{distance}$$

The equation is correct. That's because a ***distance*** plus a ***distance*** does equal a ***distance***, and we know that S should be a distance unit.

Now let's substitute SI units in the equation and show the same thing. Use t in sec, v in meters/sec, and a in meters/sec^2. Then:

$$S = vt + \tfrac{1}{2} at^2$$

$$S = \left[\frac{\text{meters}}{\cancel{\text{sec}}} \times \cancel{\text{sec}}\right] + \left[\frac{\text{meters}}{\cancel{\text{sec}^2}} \times \cancel{\text{sec}^2}\right] \qquad \text{(Cancel sec units.)}$$

$$S = \text{meters} + \text{meters}$$

$$S = \text{meters}$$

The equation is dimensionally correct. Since the left side S is a distance, it should have "distance" units. It does, since the right side is in meters, a distance unit.

PRACTICE EXERCISES

Problem 1: Given the equation: $v_f = v_i + at$

where: v_f = speed (ft/sec or m/sec)
v_i = initial speed (ft/sec or m/sec)
a = acceleration (ft/sec^2 or m/sec^2)
t = time (sec)

Substitute the proper units for v_i, a and t in first the English system, and then the SI system. Show that each term has dimensions of ft/sec or m/sec, hence speed—the correct units for v_f.

Problem 2: Given the equation: $R_T = \dfrac{\ell}{kA}$

where: R_T = thermal resistance in $\dfrac{C^\circ}{\text{cal/hr}}$ or $\dfrac{F^\circ}{\text{Btu/hr}}$

ℓ = thickness in cm or inches

k = conductivity constant in $\dfrac{\text{cal}\cdot\text{cm}}{\text{hr}\cdot\text{cm}^2\cdot C^\circ}$ or $\dfrac{\text{Btu}\cdot\text{in.}}{\text{hr}\cdot\text{ft}^2\cdot F^\circ}$

A = area in cm^2 or ft^2

When rearranged to solve for ℓ, the previous equation is written as follows:

$$\ell = R_T \times k \times A$$

Substitute the proper units for R_T, k, and A into the equation. Show that the terms on the right side reduce to cm, or inches, as they should, since ℓ on the left side is a "thickness" unit.

Problem 3: Given the equation: $R_E = \dfrac{\rho \ell}{A}$

where: R_E = electrical resistance in ohms (Ω)
ρ = electrical resistivity in ohm·cm
ℓ = length in cm
A = cross-sectional area in cm^2

Substitute the proper units for ρ, ℓ and A into the equation. Show that the terms on the right reduce to ohms (Ω). This is the correct unit for the resistance R_E on the left side.

