Math Lab 2 MS 2
Rearranging Symbols in Equations to Solve for Unknowns
Solving Fluid Work Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB.

There is a Preparatory Math Skill Lab available in a separate book entitled *PT Resource Manual* that contains concepts your students should have mastered before they begin this Math Skills Lab. This preparatory lab is coded PMS6 and is titled "Area and Volume Measurement." Encourage students who need these skills to study the material in PMS6.

RESOURCE MATERIALS

Student Text: Math Skills Lab
PT Resource Manual
Hand calculator

CLASS GOALS

1. Given an equation with three symbols--such as $W = F \times D$, $W = T \times \theta$ or $W = p \times (\Delta V)$--rearrange the equation to isolate any one of the symbols.
2. Teach students how to substitute numbers in a fluid work formula to solve for an unknown.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete the activities. Students already should have read the discussion material and looked at the examples for each activity before coming to class. (How much is accomplished depends on the math skills that your students already have.)
   a. Summarize for your students the explanatory material for Activity 1: "Rearranging Symbols in Equations to Solve for Unknowns." Then have students complete the Practice Exercises given at the end of Activity 1.
   b. Summarize the explanatory material for Activity 2: "Solving Fluid Work Problems." Then have students complete the Practice Exercises given at the end of Activity 2.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 2F1, "Work Done by a Piston," as homework.

Math Skills Laboratory

Lab 2 MS 2

MATH ACTIVITIES

Activity 1: Rearranging Symbols in Equations to Solve for Unknowns

Activity 2: Solving Fluid Work Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Given the equation for linear work, $W = F \times D$, rearrange the equation. Solve for force (F) or distance (D).***
2. ***Given the equation for rotational work, $W = T \times \theta$, rearrange the equation. Solve for torque (T) or rotational angle (θ).***
3. ***Given the equation for fluid work, $W = p \times (\Delta V)$, rearrange the Equation. Solve for pressure (p) or volume moved (ΔV).***
4. ***Substitute appropriate numerical values and units in work equations. Solve the equations.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Examples A and B in Activity 1.***
3. ***Work the problems for Activities 1 and 2.***

NOTE: Work through Example A on the chalkboard with your class. Even though this is simple algebra, don't identify it as algebra. You can show your students that isolating the variable by using a series of logical steps achieves the same result as the so-called "magic triangle" method-- 

. Be sure to emphasize that the logical procedure in Example A is a more "powerful" method--and the logical procedure will work when there are more than three symbols in a formula. The "magic triangle" works only for formulas with three symbols.

ACTIVITY 1

Rearranging Symbols in Equations to Solve for Unknowns

A. Equations and formulas are used to express a relationship between several physical quantities. Equation 1 relates the concept of work, force and distance, and expresses this relationship as follows:

$$\text{Work} = \text{Force Applied} \times \text{Distance Moved} \qquad \textit{Equation 1}$$

This equation gives you a way to determine the value of one physical quantity if you know the numerical values of the other two. Equations often are written with symbols rather than words. Thus, to simplify Equation 1, use the following symbols:

$$W = F \times D \qquad \textit{Equation 2}$$

where: W = work (newton·meters or foot·pounds)
F = force (newtons or pounds)
D = distance (meters or feet)

B. A technician often must rearrange symbols in an equation to obtain a different form of the same equation. Thus, the equation $p = {}^{F}/_{A}$ can be rearranged to yield $F = p \times A$ or $A = {}^{F}/_{p}$. Each of the three forms is correct. Sometimes, one's more useful than the others. It depends on what you want to find.

C. To "solve" an equation means to ***isolate*** one symbol in the equation. To do this, you perform *identical* mathematical operations on both sides of the equation. ***Any operation done on one side of the equation is always done on the other side.*** Therefore, care must be taken to do the following:

- ***Add or subtract the same quantity on both sides of an equation.***
- ***Multiply or divide both sides by the same quantity.***

Study Examples A and B that follow. They show how to isolate a certain symbol by rearranging the given equation.

Example A: ***Rearranging Symbols in W = F × D to Isolate F***

Given: $W = F \times D$

Find: F

Solution:

Step 1: In rearranging the equation $W = F \times D$ to isolate or "solve" for F, start with the given equation, $W = F \times D$.

Step 2: Divide both sides by D to isolate F.

$$\frac{W}{D} = \frac{F \times \cancel{D}}{\cancel{D}}$$ (The D's cancel out on the right side.)

Step 3: Rewrite the equation with the D's on the right side removed.

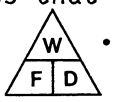
$$\frac{W}{D} = F$$ (F has been isolated.)

Step 4: Reverse the order of the equation so that F is on the left.

$$F = \frac{W}{D}$$

The equation $W = F \times D$ has been "solved" for F. The symbol, "F," has been isolated.

The correct equation is $F = \frac{W}{D}$

NOTE: Work through Example B on chalkboard. Show students that result is the same as that obtained from "magic triangle"-- 

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ANSWERS TO PRACTICE EXERCISES, ACTIVITY 1

Equation 1: Given W = F x D, find D.

$W = F \times D$

$\frac{W}{F} = \frac{\cancel{F} \times D}{\cancel{F}}$ (Divide each side by F. Cancel F's.)

$\frac{W}{F} = D$ (Write equation with F's canceled. "D" has been isolated.)

$D = \frac{W}{F}$. (Rearrange with isolated symbol [D] on the left side.)

Equation 2: Given W = T x θ, find T.

$W = T \times \theta$

$\frac{W}{\theta} = \frac{T \times \cancel{\theta}}{\cancel{\theta}}$ (Divide each side of equation by same quantity [θ] and cancel θ's.)

$\frac{W}{\theta} = T$ (Write equation with θ's canceled. "T" has been isolated.)

$T = \frac{W}{\theta}$. (Rearrange with isolated symbol "T" on the left side.)

ANSWERS TO PRACTICE EXERCISES, ACTIVITY 1, Continued

Equation 3: Given $W = (\Delta p) \times V$, find Δp.

$W = (\Delta p) \times V$

$\frac{W}{V} = \frac{(\Delta p) \times \cancel{V}}{\cancel{V}}$ (Divide each side of the equation by same quantity [V]. Cancel V's.)

$\frac{W}{V} = (\Delta p)$ (Write equation with V's canceled. "Δp" has been isolated.)

$\Delta p = \frac{W}{V}$ (Rearrange with isolated symbol [Δp] on the left side.)

Equation 4: Given $W = p \times (\Delta V)$, find ΔV.

$W = p \times (\Delta V)$

$\frac{W}{p} = \frac{\cancel{p} \times (\Delta V)}{\cancel{p}}$ (Divide each side of the equation by same quantity [p]. Cancel p's.)

$\frac{W}{p} = (\Delta V)$ (Write equation with p's canceled. "ΔV" has been isolated.)

$(\Delta V) = \frac{W}{p}$. (Rearrange with isolated symbol [ΔV] on the left side.)

ANSWERS TO PROBLEMS, ACTIVITY 2

Problem 1: W = p x ΔV. Solve for p.

$$p = \frac{W}{\Delta V}$$

where: $W = 25{,}000\ J = 25{,}000\ N \cdot m$

$\Delta V = 30\ m^3$

$$p = \frac{25{,}000\ N \cdot m}{30\ m^3}$$

$$p = 833.3\ N/m^2.$$

Problem 2: W = p x ΔV. Solve for W.

$$W = p \times \Delta V$$

where: $p = 17.4\ lb/ft^2$

$\Delta V = 1059\ ft^3$

$$W = 17.4\ lb/ft^2 \times 1059\ ft^3$$

$$W = (17.4 \times 1059)\left(\frac{lb}{ft^2} \times ft^3\right)$$

$$W = 18{,}426.6\ ft \cdot lb.$$

NOTE: Problems 1 and 2 describe the same situation, but in different units. Problem 1 is in SI units. Problem 2 is in English units. The work is equal in both problems, as are the pressure and volume displaced.

Example B: *Rearranging Symbols in $W = T \times \theta$ to Isolate θ*

Given: $W = T \times \theta$

Find: θ

Solution: Step 1: To solve for θ, start with the given equation, $W = T \times \theta$.

Step 2: Divide both sides by T to isolate θ.

$$\frac{W}{T} = \frac{\cancel{T} \times \theta}{\cancel{T}}$$ (The T's cancel out on the right side.)

Step 3: Rewrite the equation with the T's on the right side removed.

$$\frac{W}{T} = \theta$$ (The symbol "θ," has been isolated.)

Step 4: Rearrange with isolated symbol on the left side.

$$\theta = \frac{W}{T}$$

The equation $W = T \times \theta$ has been "solved" for θ. The symbol "θ" has been isolated.

The correct equation is $\theta = \frac{W}{T}$

PRACTICE EXERCISES FOR ACTIVITY 1

Use Examples A and B as a guide. Solve the following four equations by isolating the symbol indicated.

Equation 1: Given: $W = F \times D$
Find: D
Solution:

Equation 2: Given: $W = T \times \theta$
Find: T
Solution:

Equation 3: Given: $W = (\Delta p) \times V$
Find: Δp
Solution:

Equation 4: Given: $W = p \times (\Delta V)$
Find: ΔV
Solution:

ACTIVITY 2

Solving Fluid Work Problems

In Activity 1, you rearranged both mechanical and fluid work equations to isolate an unknown quantity. In this activity, you'll be given numerical values and units to substitute in certain equations. This will enable you to solve problems and obtain correct answers. The *correct* answer contains ***both*** the *correct numerical value* and *proper units.*

Problem 1: Given: A pump under constant pressure uses 25,000 joules of energy (work) to move 30 m^3 of water.
Find: The pump pressure required to do the work.
Solution:

Problem 2: Given: A pump doing work at a constant pressure of 17.4 lb/ft^2 moves 1059 ft^3 of water.
Find: The work done in moving the water.
Solution:

ANSWERS TO PROBLEMS, ACTIVITY 2, Continued

Problem 3: $V = \Delta p \times V$. Solve for W.

Step 1: $\Delta p = \rho_w \times h$

$\Delta p = 62.4\ lb/ft^3 \times 100\ ft$

$\Delta p = 62.4 \times 100)(\frac{lb}{ft^3} \times \cancel{ft})$

$\Delta p = 6240\ lb/ft^2.$

where: $\rho_w = 62.4\ lb/ft^3$

$h = 90\ ft + 10\ ft$

$h = 100\ ft$

$V = 600\ ft^3$

Step 2: $W = \Delta p \times V$

$W = 6240\ \frac{lb}{ft^2} \times 600\ ft^3$

$W = (6240 \times 600)(\frac{lb}{\cancel{ft^2}} \times \cancel{ft^3})$

$W = 3{,}744{,}000\ ft \cdot lb$

$W = 37.44 \times 10^5\ ft \cdot lb.$

Problem 4: $W = p \times \Delta V$ and $p = F/A$. Solve for W (part a) and F (part b).

a. $W = p \times \Delta V$

$W = (1500\ \frac{N}{m^2})(0.006\ m^3)$

$W = (1500 \times 0.006)\ (\frac{N}{\cancel{m^2}} \times \cancel{m^3})$

$W = 9.0\ N \cdot m.$

where: $p = 1500\ N/m^2$

$\Delta V = 0.006\ m^3$

b. $p = \frac{F}{A}$ So $F = pA.$

$F = pA = 1500\ N/m^2 \times 0.03\ m^2$

$F = (1500 \times 0.03)(\frac{N}{\cancel{m^2}} \times \cancel{m^2})$

$F = 45\ N.$

where: $p = 1500\ N/m^2$

$A = 0.03\ m^2$

ANSWERS TO PROBLEMS, ACTIVITY 2, Continued

Problem 5: $W = p \times \Delta V$. Solve for W.

$W = p \times \Delta V$ where: $p = 0.3\ lb/in^2 = 43.2\ lb/ft^2$

$W = 43.2 \frac{lb}{ft^2} \times 2400\ ft^3$ $\Delta V = (20\ ft \times 15\ ft \times 8\ ft) = 2400\ ft^3$

$W = (43.2 \times 2400)(\frac{lb}{\cancel{ft^2}} \times \cancel{ft^3})$

$W = 103{,}680\ ft \cdot lb.$

ANSWER TO STUDENT CHALLENGE PROBLEM

Problem 6: $p = F/A$; $A = \pi/4\ d^2$, and $W = p \times \Delta V$.

a. $p = F/A$ Solve for A. where: $F = 70.6$ lb

$$A = \frac{F}{p} = \frac{70.6\ \text{lb}}{100\ \frac{\text{lb}}{\text{in}^2}} \qquad p = 100\ \frac{\text{lb}}{\text{in}^2}$$

$A = 0.706\ \text{in}^2$.

Solving for d^2 from $A = (\pi/4)d^2$

$$d^2 = \frac{4\ A}{\pi} = \frac{4 \times 0.706\ \text{in}^2}{3.14}$$

$d^2 = 0.899\ \text{in}^2$

$d = \sqrt{0.899\ \text{in}^2} = 0.948$ in

b. $\Delta V = A \times$ length

$\Delta V = 0.706\ \text{in}^2 \times 10$ in

$\Delta V = 7.06\ \text{in}^3$

c. $W = p \times \Delta V$ where: $p = 100$ psi $= 100\ \text{lb/in}^2$

$\Delta V = 7.06\ \text{in}^3$

$$W = 100\ \frac{\text{lb}}{\text{in}^2} \times 7.06\ \text{in}^3$$

$$W = (100 \times 7.06)\left(\frac{\text{lb}}{\cancel{\text{in}^2}} \times \cancel{\text{in}^3}\right)$$

$W = 706$ in·lb.

or

$$W = (706\ \cancel{\text{in}}\cdot\text{lb})\ \frac{(\text{ft})}{12\ \cancel{\text{in}}} = 58.8\ \text{ft}\cdot\text{lb}$$

Problem 3: Given: An underground pump lifts water 90 ft to the surface and into the top of a 600-ft^3 storage tank whose top is 10 ft above the surface of the ground. The weight density of water is $\rho_w = 62.4\ lb/ft^3$.

Find: The work done by the pump to fill the tank.
Hint: First, find Δp (pressure difference). Then find the work done.

Solution:

Problem 4: Given: A piston in a hydraulic cylinder is displaced by a constant pressure of 1500 N/m^2. The hydraulic cylinder volume through which the piston moves is 0.006 m^3 when the piston is moved 20 cm.

Find: a. The work required to move the piston 20 cm.
b. The total force on the piston if the piston face is 0.03 m^2.

Solution:

Problem 5: Given: An air-conditioner fan provides an air pressure of 0.3 lb/in^2. That's the same as 43.2 lb/ft^2. The air conditioner is in a room that's 20 ft long, 15 ft wide and 8 ft high.

Find: How much work does the fan do each time it fills the room with cool air?

Solution:

Student Challenge

Problem 6: Given: A pressure of 100 lb/in^2 is applied to the face of a hydraulic cylinder piston rod. This causes a force of 70.6 lb on the piston face.

Find: a. The diameter of the piston face.
b. The volume of fluid moved when the piston moves 10 inches.
c. The work done when the piston moves 10 inches.

Solution: