Math Lab 13 MS 4

Using Graphical Techniques to Locate Images Formed by Two Lenses

Solving Beam-Expander Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

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TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach students how to apply ray-tracing techniques for locating the image points of rays that have passed through two lenses.
2. Teach students the basic components of a beam-expander system.
3. Teach students how to solve laser beam-width and beam-spread problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete the activities. Students should have read the discussion material and looked at the examples for each activity before coming to this class. You should summarize the main points in each activity, work an example or two, and have the students complete the Practice Exercises for each activity on their own.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell students to read Lab 13*7, "Polarization."

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Math Skills Laboratory

MATH ACTIVITIES

Activity 1: Using Graphical Techniques to Locate Images Formed by Two Lenses

Activity 2: Solving Beam-Expander Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve image-formation problems by ray-tracing through a system of two lenses.***
2. ***Draw a beam-expander optical system.***
3. ***Solve problems involving the beam width and beam spread of laser beams.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

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ACTIVITY 1

Using Graphical Techniques to Locate Images Formed by Two Lenses

MATERIALS

For this activity, you'll need graph paper, a ruler and a calculator.

Often two or more thin lenses are used in combination to produce a desired image. A single-lens camera, used by professional studio photographers, provides an inverted image for viewing when the camera is focused on the subject. The photographer studies the inverted image and, if acceptable, inserts a film plate in the camera and takes the picture.

For an experienced photographer, looking at inverted figures is quite acceptable. For the amateur photographer, viewing an erect image—as it will appear on the film when developed—is more desirable.

Two convex lenses in sequence produce an erect image of an original object. The first lens produces an inverted image of the object. The second lens inverts the first image and thereby produces a final image that is erect (upright).

Depending on the lens placement and object location along an optical axis, the final image may be larger, smaller or the same size as the object. The location of the two lenses along the optical axis, and their focal points, is shown in Figure 1. Each lens has a single focal length, f_1 or f_2.

Example A uses the graphical techniques learned in Math Lab 13MS2. These ray-tracing techniques are now applied to the two-lens system.

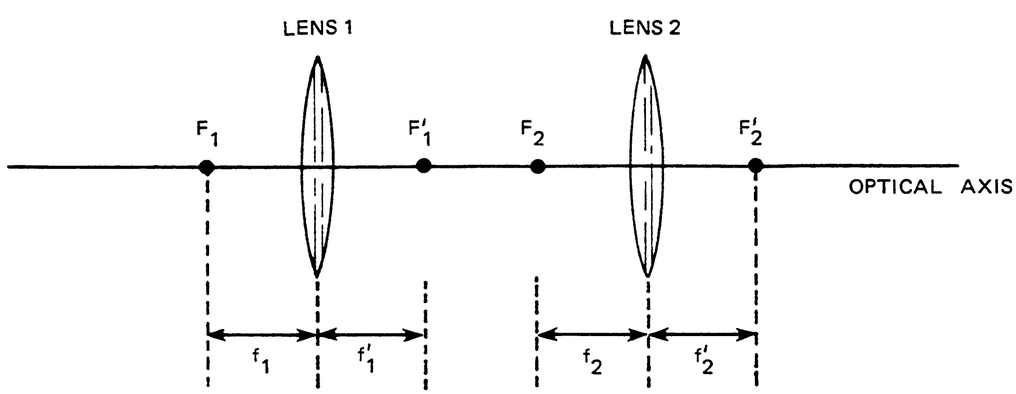

Fig. 1 Arrangement of two convex lenses.

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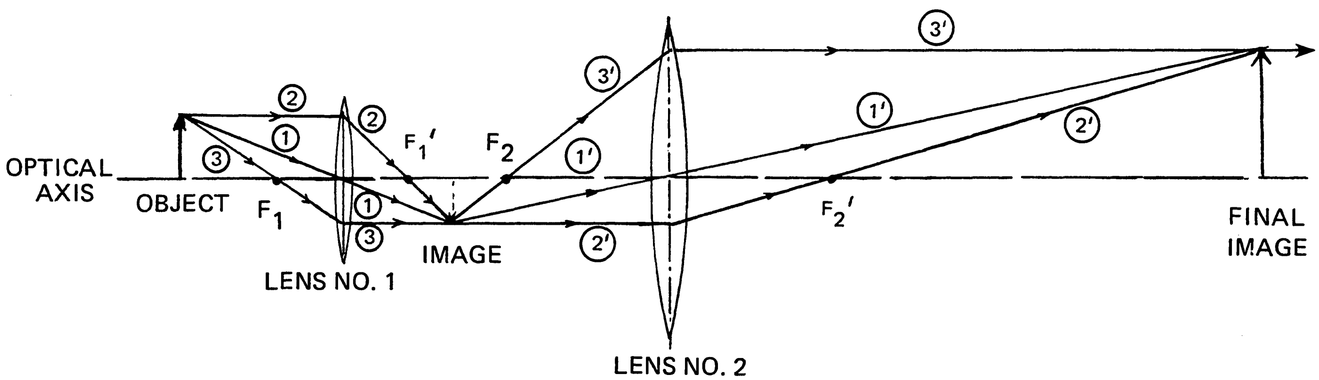
NOTE: Teach Example A. Make a Xerox transparency of the figure that accompanies Example A. Project on screen and carefully trace rays 1, 2 and 3 that form the first image. Have students read the description in the text for each ray, while you trace it, or vice versa. Do the same for rays 1', 2' and 3', to locate the final image. Going through this procedure will help students review the ray tracing techniques they learned earlier and also help them learn how these techniques are applied to multiple-lens systems.

NOTE: In the previous ray-tracing drawings we used only two rays to locate the image. We did that to keep the drawing from becoming cluttered. Tell students that only two of the three rays shown in Example A are needed to locate the image. The third can be used as a check on the accuracy of the drawing made with the other two. Also tell students that any two of the three rays shown can be chosen to locate the image. That is, they can choose 1 and 2, 1 and 3, or 2 and 3.

ANSWERS TO PRACTICE EXERCISE

Activity 1:

Problem: The drawing completed by the students should look like the following sketch:



The final image is upright and larger than the original object.

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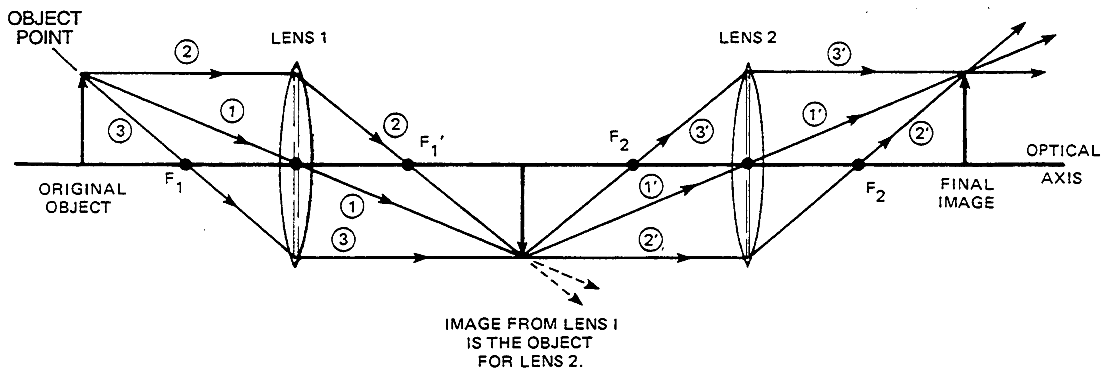
Example A: *Ray-Tracing Through Two Lenses*

Given: An object and two thin convex lenses, located along a common optical axis (see figure), and the following ray-tracing rules:

- ray 1—This ray leaves the object point and passes through the center of the lens. It continues along a straight line with no bending.
- ray 2—This ray leaves the object point parallel to the optical axis. It refracts at the lens and passes through the focal point F′, on the opposite side of the lens.
- ray 3—This ray leaves the object point and passes through the focal point F on the same side of the lens. It refracts at the lens and emerges on the other side as a ray parallel to the optical axis.

Find:
a. The initial image of the object as formed by lens 1.
b. The final image as formed by lens 2.

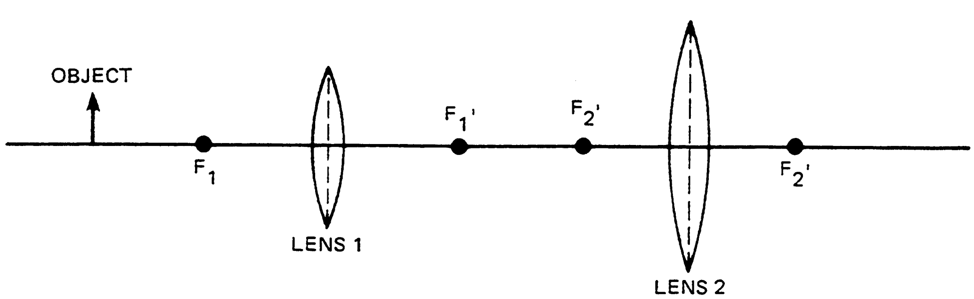
Solution:
a. Apply the ray-tracing rules to lens 1. Label rays 1, 2, and 3 and trace them from the object (arrow tip) to the image formed between the two lenses. See the sketch in this example.
b. Use the image of the arrow tip formed by lens 1 as the *object* for lens 2. Apply the same three ray-tracing rules to lens 2. Label rays 1′, 2′, and 3′ and trace them from the image between the lenses to the final image. See the sketch in this example.



Now work the Practice Exercise. Follow the procedure outlined in Example A.

PRACTICE EXERCISES

Problem: You are given the two-lens system that's drawn here. Copy the drawing on a clean sheet of paper. Starting from the tip of the arrowhead (object), use the graphical technique outlined in Example A to locate the final real image. First, trace the three rays through lens one and locate the image. Then use this image as an object for lens 2 and again trace three rays to the final image. Is the final image upright or inverted? Is it larger or smaller than the original object?



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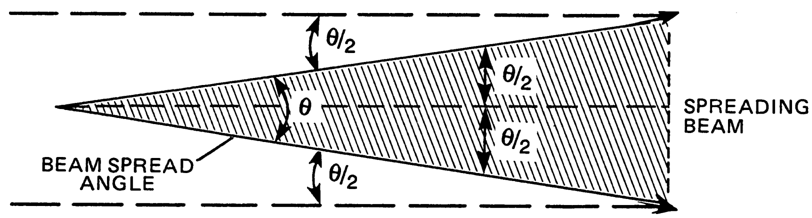
NOTE: See Figure 2--"A Beam-Expander"

Point out that the full beam spread angles are θ_1 at the input and θ_2 at the output. The drawing shows only half-angle spread--1/2 θ_1 at the top for the input beam and 1/2 θ_2 for the output beam. There is an equal spread--1/2 θ_1 and 1/2 θ_2--at the bottom of the input and output beams. In exaggerated form, the full-angle spread and half-angle spread are related as follows:



$\theta/2 + \theta/2 = \theta$

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ACTIVITY 2

Solving Beam-Expander Problems

MATERIALS:

For this activity, you'll need graph paper, a ruler, a protractor and a calculator.

DISCUSSION

In this unit, you learned that lenses are used to make beam-expanders. Such beam-expanders are used in laser systems to reduce the amount of beam spread as the laser beam moves away from the laser. The diagram in Figure 2 shows a beam-expander.

The two lenses, lens 1 of focal length f_1, and lens 2 of focal length f_2, are the important elements of the beam-expander. The input beam of width d_1, and the output beam of width d_2, are indicated.

The half-angle beam spread angles ($1/2\ \theta_1$ and $1/2\ \theta_2$) also are indicated.

The beam widths, beam-spread angles and lens focal lengths are related by the following equations:

$$\frac{d_1}{d_2} = \frac{f_1}{f_2} \text{ and } \frac{d_1}{d_2} = \frac{\theta_2}{\theta_1}$$

where: d_1 = width of unexpanded beam
d_2 = width of expanded beam
f_1 = focal length of input lens
f_2 = focal length of output lens
θ_1 = spread angle of input beam
θ_2 = spread angle of output beam

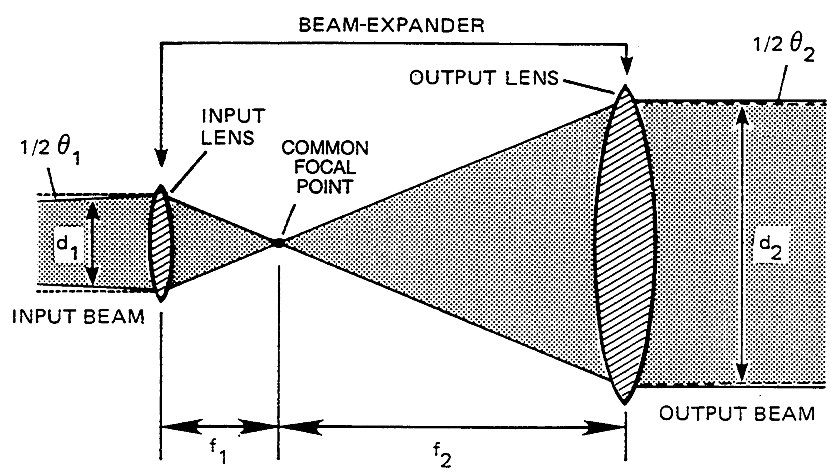

Fig. 2 A beam-expander.

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Example B shows how to use the diameter, focal length and spread-angle ratios to find unknown quantities in beam-expander problems.

Example B: *Calculating Beam Width and Beam Spread for a Laser-Beam Expansion System*

Given: A simple beam-expander similar to that shown in Figure 2. The input lens has a focal length (f_1) = 4 cm, and the output lens has a focal length (f_2) = 20 cm. The input beam has a width (d_1) = 3 mm with a beam-spread angle (θ_1) = 1.0 milliradian.

Find:
a. The width (d_2) of the output beam.
b. The beam spread (θ_2) of the output beam.

Solution: Use the relations $d_1/d_2 = f_1/f_2$ and $d_1/d_2 = \theta_2/\theta_1$ to solve parts "a" and "b."

a. First solve for the expanded beam width (d_2). Use the relationship:

$$\frac{d_1}{d_2} = \frac{f_1}{f_2} \qquad \text{where: } d_1 = 3 \text{ mm} = 0.3 \text{ cm}, \; f_1 = 4 \text{ cm}, \; f_2 = 20 \text{ cm}$$

Isolate d_2 by multiplying both sides of the equation by ($\frac{d_2 f_2}{f_1}$). Cancel like terms. Rearrange the equation.

$$\left(\frac{\cancel{d_2} f_2}{f_1}\right)\frac{d_1}{\cancel{d_2}} = \left(\frac{d_2 \cancel{f_2}}{\cancel{f_1}}\right)\frac{\cancel{f_1}}{\cancel{f_2}}$$

$$d_2 = \frac{f_2}{f_1} d_1$$

Then substitute in values for f_1, f_2, and d_1.

Solve.

$$d_2 = \frac{20 \text{ cm} \times 0.3 \text{ cm}}{4 \text{ cm}} = \left[\frac{20 \times 0.3}{4}\right]\frac{\text{cm} \times \cancel{\text{cm}}}{\cancel{\text{cm}}} = 1.5 \text{ cm}$$

$d_2 = 1.5$ cm, or 15 mm.

The output beam has been expanded five times, from 3 mm to 15 mm.

b. Next solve for the beam-spread angle (θ_2) of the output (expanded) beam. Use the equation:

$$\frac{d_1}{d_2} = \frac{\theta_2}{\theta_1} \qquad \text{where: } d_1 = 3 \text{ mm}, \; d_2 = 15 \text{ mm (from Part a)}, \; \theta_1 = 1.0 \text{ milliradian}$$

Isolate θ_2 by multiplying both sides of the equation by θ_1. Cancel like terms and rearrange the equation.

$$(\theta_1)\frac{d_1}{d_2} = (\cancel{\theta_1})\frac{\theta_2}{\cancel{\theta_1}}$$

$$\theta_2 = \frac{d_1}{d_2}\theta_1$$

Now substitute values for d_1, d_2, θ_1. Solve.

$$\theta_2 = \frac{3 \text{ mm}}{15 \text{ mm}} \times 1.0 \text{ milliradian} = \left[\frac{3 \times 1.0 \text{ cm}}{15}\right]\frac{\cancel{\text{mm}} \times \text{millirad}}{\cancel{\text{mm}}} = 0.2 \text{ millirad}$$

$\theta_2 = 0.2$ milliradian, a little over one-hundredth of a degree.

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ANSWERS TO PRACTICE EXERCISES

Activity 2:

Problem 1: $\frac{d_1}{d_2} = \frac{f_1}{f_2}$

$f_1 = 2.5$ cm
$d_1 = 5$ mm
$d_2 = 20$ mm

Rearrange equation to isolate f_2. Multiply each side by $\frac{f_2 d_2}{d_1}$ and cancel appropriately. You should get:

$$f_2 = \frac{d_2}{d_1} f_1$$

Now substitute in values for d_2, d_1, f_1 and solve.

$$f_2 = \frac{20 \text{ mm}}{5 \text{ mm}} (2.5 \text{ cm}) = \left(\frac{20 \times 2.5}{5}\right) \frac{\cancel{\text{mm}} \cdot \text{cm}}{\cancel{\text{mm}}}$$

$$f_2 = 10 \text{ cm}$$

To achieve a beam expansion from 5 mm to 20 mm, a factor of <u>four</u> increase, the output lens must have a focal length of 10 cm, <u>four</u> times the focal length of the input lens.

Problem 2: $\frac{d_1}{d_2} = \frac{\theta_2}{\theta_1}$

$f_1 = 2.5$ cm
$d_1 = 5$ mm
$f_2 = 10$ cm (from Problem #1)
$d_2 = 20$ mm
$\theta_1 = 1.2$ milliradians

Rearrange equation to isolate θ_2. Multiply each side by $\theta_1/1$. Cancel appropriately. You should get:

$$\theta_2 = \frac{d_1}{d_2} \theta_1$$

Now substitute in values for d_1, d_2, θ_1 and solve.

$$\theta_2 = \frac{5 \text{ mm}}{20 \text{ mm}} (1.2 \times 10^{-3} \text{ rads}) = \left(\frac{5 \times 1.2 \times 10^{-3}}{20}\right) \frac{\cancel{\text{mm}} \cdot \text{rad}}{\cancel{\text{mm}}}$$

$$\theta_2 = 0.3 \times 10^{-3} \text{ rad or } 0.3 \text{ milliradians}$$

The beam spread has been <u>decreased</u> by a factor of <u>four</u>, from 1.2 milliradians to 0.3 milliradians.

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Answer to Practice Exercise #3 follows on Page T-128c.

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ANSWERS TO STUDENT EXERCISES, Continued

Problem 3:

a. First solve for the power in the laser beam.

$P = \frac{\text{Energy}}{\text{time}}$ where: Energy = 20 J
time = 120 μsec

$$P = \frac{20\ \text{J}}{120 \times 10^{-6}\ \text{sec}} = \left(\frac{20}{120 \times 10^{-6}}\right)\frac{\text{J}}{\text{sec}} = \left(\frac{20 \times 10^{6}}{120}\right)\ \text{W}$$

$P = 1.67 \times 10^5$ W

b. $\text{Irradiance} = \frac{\text{Power}}{\text{Target Area}} = \frac{P}{A}$ where: $P = 1.67 \times 10^5$ W
$A = 0.02\ \text{cm}^2$

$$\text{Irradiance} = \frac{1.67 \times 10^5\ \text{W}}{0.02\ \text{cm}^2}$$

$$\text{Irradiance} = \left(\frac{1.67 \times 10^5}{0.02}\right)\ \text{W/cm}^2$$

$\text{Irradiance} = 8.33 \times 10^6\ \text{W/cm}^2$

The irradiance at the focal spot is 8.33 million watts/cm^2!!

c. The beam expansion ratio is equal to d_2/d_1, where d_2 = 15 mm and d_1 = 3 mm.

$$\frac{d_2}{d_1} = \frac{15\ \cancel{\text{mm}}}{3\ \cancel{\text{mm}}} = 5$$

The beam expansion ratio is 5X.

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The output beam has its beam spread ***reduced*** after beam expansion. The beam spread decreased by a factor of five, from 1 milliradian to 0.2 milliradian. The output beam has, therefore, been made more parallel.

If a third lens with short focal length is used with the beam-expander, as shown in Figure 3, the beam can be focused down to a tiny spot. This reduction in area allows the power to be concentrated in a very small region. This makes the beam irradiance (power density) very high.

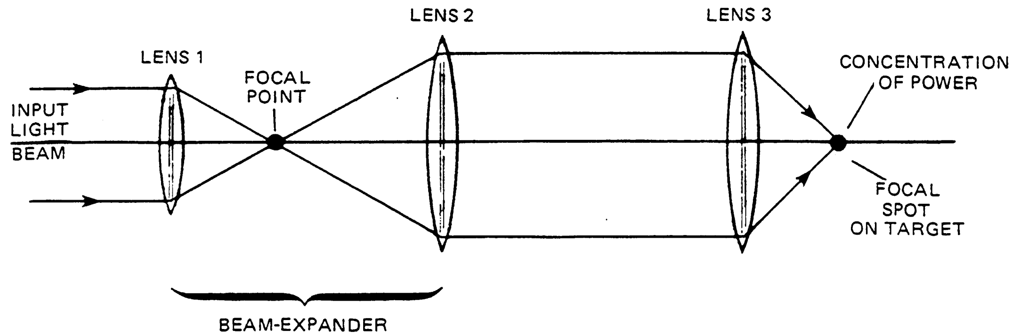

Fig. 3 Lens system to increase power density on target.

A laser beam that might cause only a mild burn if it were allowed to shine on your skin can be used to *burn a hole in steel* when used with the properly designed lens system (similar to that shown in Figure 3).

That's because concentrating the beam power in a small area increases the power density dramatically. And it's **power density on target** that counts when you are using lasers to burn through the target.

Now solve the following beam-expander problems.

PRACTICE EXERCISES

Problem 1: Given: A beam-expander system similar to that shown in Figure 2. Input lens 1 has a focal length of 2.5 cm and is to be used with a laser input beam 5 mm wide. Output lens 2 of the beam-expander expands the output beam to 20 mm.

Find: The focal length of output lens 2.

Solution: ***Hint:*** Use the equation, $d_1/d_2 = f_1/f_2$.

Problem 2: Given: The same conditions in Problem 1. In addition, you know that the beam-spread angle (θ_1) of the input laser beam is 1.2 milliradians.

Find: The beam-spread angle (θ_2) of the laser beam after expansion.

Solution: ***Hint:*** Use the equation, $d_1/d_2 = \theta_2/\theta_1$.

Problem 3: Given: A laser delivers 20 joules of energy to a beam-expander every 120 microseconds. The laser input beam has width of 3 mm. The beam-expander expands the beam to an output width of 15 mm. A third lens focuses the beam down to an area of 0.02 cm^2. See Figure 3. Assume that no energy is lost from the beam as it passes through the beam-expander.

Find:

a. The radiant power delivered to the focal spot of 0.02 cm^2.

b. The irradiance or power density of the beam at the focal spot of 0.02 cm^2.

c. The beam-expansion ratio of this beam-expander.

Solution:

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