Math Lab 10 MS 4

Solving Energy-Conversion Problems for Thermal Energy Convertors

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

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TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Review procedures for converting units in one system to the units of another system.
2. Teach students how to calculate the efficiency of a thermal energy convertor.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to this class. (How much students accomplish depends on the math skills your students already have.) Summarize the explanatory material for Activity 1: "Solving Energy-Conversion Problems for Thermal Energy Convertors." Then have students complete the Practice Exercises given at the end of Activity 1.
3. Supervise student progress. Have students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 10*7, "Multiple Energy-Conversion Systems."

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Math Skills Laboratory

Lab 10 MS4

MATH ACTIVITY

Activity: ***Solving Energy-Conversion Problems for Thermal Energy Convertors***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve energy-conversion problems for thermal energy convertors.***
2. ***Substitute correct numerical values and units in appropriate equations. Solve equations for an unknown numerical value with the proper units.***
3. ***Find the efficiency of thermal energy convertors.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

ACTIVITY

Solving Energy-Conversion Problems for Thermal Energy Convertors

MATERIALS

For this activity, you'll need a calculator.

In this Math Skills Lab, you'll review the important formulas for work and energy. You'll review the units used to measure work, energy and power. You'll also solve problems that involve thermal energy convertors.

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NOTE: You and your students may all be getting tired of seeing Tables 1, 2, and 3, since they're the same tables used in the three previous math labs. Review these tables with student <u>only</u> if students have shown weaknesses in using equations and units in the previous math and hands-on labs. Remember, these tables are included primarily as a handy reference for students to use when they're working through the lab exercises--and they are found in the *Student Resource Book*.

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LET'S REVIEW FORMULAS AND UNITS!

Examine Tables 1, 2 and 3. These are the same tables included in Math Skills Labs 10MS1, 10MS2 and 10MS3.

Table 1 lists the important formulas for work and energy that you've already studied. Table 2 sums up common units that you'll use in energy-conversion problems. Table 3 lists some conversions between units.

You'll find these conversions helpful when you solve the problems in this activity.

TABLE 1. REVIEW OF BASIC FORMULAS FOR WORK AND ENERGY

Energy System	Formulas	Definition of Symbols
MECHANICAL		
Linear Work	$W = F \times D$	F = applied force D = distance moved
Rotational Work	$W = T \times \theta$	T = torque applied θ = angular distance rotated
Gravitational Potential Energy	$E_p = w \times h$	w = weight h = height raised above reference level
Elastic Potential Energy	$E_p = \frac{1}{2} kd^2$	k = force or spring constant d = distance spring is stretched or compressed
Linear Kinetic Energy	$E_k = \frac{1}{2} mv^2$	m = mass v = speed
Rotational Kinetic Energy	$E_k = \frac{1}{2} I\omega^2$	I = moment of inertia ω = angular speed
FLUID		
Fluid Work	$W = p \times (\Delta V)$; $W = (\Delta p) \times V$	p; Δp = pressure or pressure difference ΔV; V = volume of fluid moved
Fluid Kinetic Energy	$E_k = \frac{1}{2} (\rho V)v^2$	ρ = mass density V = volume v = speed
ELECTRICAL		
Electrical Work	$W = q \times \Delta V$	q = electrical charge moved ΔV = voltage difference
Electrical Energy	$E_{elec} = P_{elec} \times t$	P_{elec} = electrical power used t = time power is used
THERMAL		
Thermal Energy	$H = mc\Delta T$	m = mass c = specific heat ΔT = temperature difference

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TABLE 2. SUMMARY OF COMMON UNITS USED IN ENERGY CONVERSION CALCULATIONS

Quantity	Symbol	English Unit	SI Unit
Mass	m	slug or (lb·sec^2/ft)	kg
Weight	w	lb	N
Force	F	lb	N
Work	W	ft·lb	N·m or J
Kinetic Energy	E_k	ft·lb	N·m or J
Potential Energy	E_p	ft·lb	N·m or J
Power	P	(ft·lb)/sec; hp	(N·m)/sec or watt
Spring Constant	k	lb/ft; oz/in.	N/m; kgf/m *
Angular Displacement	θ	rad	rad
Angular Speed	ω	rad/sec	rad/sec
Linear Speed	v	ft/sec	m/sec
Mass Density	ρ	slug/ft^3; lbm/ft^3 **	kg/m^3
Moment of Inertia	I	slug·ft^2	kg/m^2
Pressure or Pressure Difference	p; Δp	lb/ft^2; oz/in^2	N/m^2
Electrical Charge	q	—	coulomb (C)
Torque	T	lb·ft; lb·in.	N·m
Fluid Volume	V; ΔV	ft^3	m^3
Electrical Current	I	—	ampere (A)
Electrical Voltage	V; ΔV	—	volt (V)
Electrical Resistance	R	—	ohm (Ω)
Heat Energy	H	Btu	cal
Temperature	T	°F	°C
Temperature Difference	ΔT	F°	C°
Specific Heat	c	$\frac{\text{Btu}}{\text{lb·F°}}$	$\frac{\text{kcal}}{\text{kg·C°}}$

* kgf ⇒ kilogram force
** lbm ⇒ pound mass

TABLE 3. USEFUL CONVERSIONS FOR UNITS

1 slug	=	32 pound mass (32 lbm)
1 kilogram force (kgf)	=	9.8 newtons
1 newton·meter	=	1 joule
1 volt·coulomb	=	1 joule
1 ampere·second	=	1 coulomb
1 radian	=	57.3°
1 horsepower	=	550 ft·lb/sec
1 horsepower	=	746 watts
1 watt	=	1 newton·meter per second
1 watt	=	1 joule per second
1 watt	=	1 volt·ampere
1 watt·sec	=	1 joule = 0.7376 ft·lb
1 kilowatt·hour (kWh)	=	3.6×10^6 joules
1 calorie	=	3.086 foot·pounds
1 calorie	=	4.184 joules
1 Btu	=	778 foot·pounds
1 Btu	=	1055 joules

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SOLUTIONS TO EQUATIONS/UNITS QUESTIONS

1. $H = mc\Delta T$
2. $W = q \times \Delta V$
3. 1 V•coul = 1 joule
4. 1 watt•sec = 1 joule (since 1W = 1 J/sec)
5. $E_{elec} = P_{elec} \times t$
6. 1 Btu = 778 ft•lb so 2 Btu = 1556 ft•lb

7. $\text{Eff }(\%) = \dfrac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = 1400$ ft•lb
$E_{in} = 2$ Btu $= 1556$ ft•lb

$$\text{Eff }(\%) = \frac{1400\ \cancel{\text{ft•lb}}}{1556\ \cancel{\text{ft•lb}}} \times 100\% = \left(\frac{1400}{1556}\right) \times 100\% = 0.8997 \times 100\%$$

Eff (%) = 89.97%, or about 90%.

8. (a) 1 cal = 4.184 J (See Table 3.)
(b) 1 kWh = 3.6×10^6 J (See Table 3.)
(c) $0.9 \times 10^6 \text{ cal} = 0.9 \times 10^6\ \cancel{\text{cal}} \times \dfrac{4.184 \text{ J}}{1\ \cancel{\text{cal}}} = 3.7656 \times 10^6 \text{ J}$

9. $\text{Eff }(\%) = \dfrac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$
$E_{in} = 0.9 \times 10^6 \text{ cal} = 3.77 \times 10^6 \text{ J}$

$$\text{Eff }(\%) = \frac{3.6 \times 10^6\ \cancel{\text{J}}}{3.77 \times 10^6\ \cancel{\text{J}}} \times 100\% = \left(\frac{3.6}{3.77} \times 10^{6-6}\right) \times 100\%$$

Eff (%) = 0.955 x 100%
Eff (%) = 95.5% (about 95%).

10. See Table 3.
1 Btu = 1054 J.
But 1 J = 1 N•m.
Therefore, 1 Btu = 1054 N•m.

11. $\text{Eff }(\%) = \dfrac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = 1000$ N•m
$E_{in} = 1$ Btu $= 1054$ N•m

$$\text{Eff }(\%) = \frac{1000\ \cancel{\text{N•m}}}{1054\ \cancel{\text{N•m}}} \times 100\% = \left(\frac{1000}{1054}\right) \times 100\%$$

Eff (%) = 0.9487 x 100%
Eff (%) = 94.87% (about 95%).

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Review the contents of Tables 1, 2 and 3. Use these tables to help you answer the following questions.

1. Consider a body of mass m and specific heat c that undergoes an increase in temperature of ΔT. The formula for the amount of heat energy absorbed by this body is ____.
2. The formula for electrical work is ____.
3. One volt·coulomb of work equals one ____ of work.
4. One watt·sec of electrical energy is equal to one ____.
5. The formula for electrical energy used, given the electrical power and time of operation, is ____.
6. A thermal energy convertor changes 2 Btu of thermal energy into 1400 ft·lb of rotational mechanical energy. Two Btu of energy equals ____ ft·lb of energy.
7. The percent efficiency of the thermal energy convertor in Question 6 is ____ %.
8. A thermal energy convertor changes 0.9×10^6 calories of thermal energy into 1 kWh of electrical energy.
   a. One calorie equals ____ joules.
   b. One kWh equals ____ joules.
   c. 0.9×10^6 calories equals ____ joules.
9. The percent efficiency of the thermal energy convertor in Question 8 is ____ %.
10. A thermal energy convertor changes 1 Btu of thermal energy into 1000 N·m of fluid energy. One Btu équals ____ N·m.
11. The percent efficiency of the thermal energy in Question 10 is ____ %.

EXAMPLES OF ENERGY-CONVERSION PROBLEMS.

Study these two examples of energy-conversion problems. Then solve the exercises that follow.

Example A: *Efficiency of a Fuel-Oil-Fired Boiler*

Given: An oil-fired boiler rated at 250 hp output uses 354 Btu of heat energy each second at the input while burning the oil. The output energy is changed to fluid energy in steam that will eventually be used to run a turbine.

Find: The boiler efficiency.

Solution: The formula for efficiency is

$$\text{Eff}(\%) = \frac{P_{OUT}}{P_{IN}} \times 100\% \qquad \text{where: } P_{OUT} = \text{Boiler output power} = 250\text{ hp}, \quad P_{IN} = \text{Boiler input power} = 354\text{ Btu/sec}$$

Since P_{OUT} and P_{IN} are in different units, we must convert them to the *same form* before finding the efficiency. Let's convert P_{IN} from Btu/sec to hp. From Table 3 we get the conversions used below.

$$P_{IN} = \frac{354\text{ Btu}}{\text{sec}} \times \frac{1055\text{ J}}{1\text{ Btu}} \times \frac{1\text{ watt}}{1\text{ J/sec}} \times \frac{1\text{ hp}}{746\text{ watts}}$$

$$P_{IN} = \left[\frac{354 \times 1055 \times 1 \times 1}{1 \times 1 \times 746}\right]\left[\frac{\cancel{\text{Btu}} \times \cancel{\text{J}} \times \cancel{\text{watt}} \times \cancel{\text{sec}} \times \text{hp}}{\cancel{\text{sec}} \times \cancel{\text{Btu}} \times \cancel{\text{J}} \times \cancel{\text{watt}}}\right] \qquad \text{(Cancel units.)}$$

$$P_{IN} = 501\text{ hp (rounded)}$$

So, P_{IN} = 354 Btu/sec = 501 hp. Now find efficiency of the boiler.

$$\text{Eff}(\%) = \frac{250\,\cancel{\text{hp}}}{501\,\cancel{\text{hp}}} \times 100\% \qquad \text{(Cancel units.)}$$

$$\text{Eff}(\%) = 0.499 \times 100\%$$

$$\text{Eff}(\%) = 50\%\text{ (rounded)}$$

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SOLUTIONS TO PRACTICE EXERCISES

Problem 1: a. $\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = P_{out} \times t$

$E_{in} = 1.1 \times 10^6$ J

$E_{out} = P_{out} \times t = 360\ \text{kW} \times 1\ \text{sec}$

$= 360\ \text{kW}\cdot\text{sec} = 360 \times 10^3\ \text{watt}\cdot\text{sec}$

$E_{out} = 360 \times 10^3\ \text{J}$ (since 1 watt•sec = 1 J)

$$\text{Eff (\%)} = \frac{360 \times 10^3\ \cancel{\text{J}}}{1.1 \times 10^6\ \cancel{\text{J}}} \times 100\% = \left(\frac{360}{1.1} \times 10^{3-6} \times 100\%\right)$$

$$= (327.3 \times 10^{-3} \times 10^2)\%$$

$\text{Eff (\%)} = 327.3 \times 10^{-1}\% = 32.7\%.$

b. Energy lost $= (100\% - 32.7\%) \times E_{in}$

Energy lost $= (67.3\%) \times 1.1 \times 10^6$ J

Energy lost $= (0.6733 \times 1.1 \times 10^6)\ \text{J} = 7.4 \times 10^5$ J.

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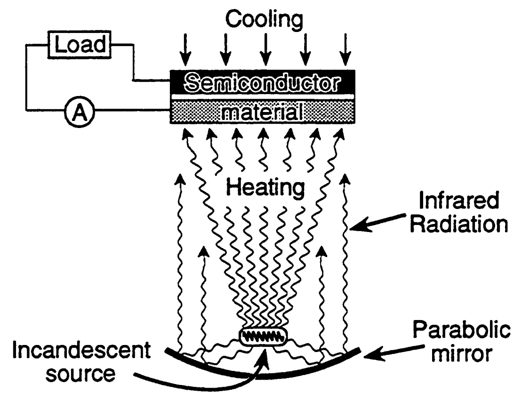
Example B: *Thermo-Photo-Voltaic (TPV) Energy Convertor*

Given: A TPV energy convertor uses heat energy to produce electrical energy. In one particular application, the infrared radiation from an incandescent filament is focused onto the 1.8-cm^2 sensitive area of a TPV semiconducting material. The resulting thermal energy is changed by the semiconductor material to 51.9 millijoules of electrical energy each second. The efficiency of the TPV energy convertor is known to be 35%.



Find: The input thermal energy in calories produced each second as a result of the radiation from the incandescent source striking the semiconductor material.

Solution: The efficiency of this energy convertor is given by the equation:

$$\text{Eff}(\%) = \frac{E_{OUT}}{E_{IN}} \times 100\%$$

where: Eff(%) = 35%
E_{OUT} = 51.9×10^{-3} J each second
E_{IN} = unknown input thermal energy

Solve for E_{IN}.

$$E_{IN} = \frac{E_{OUT}}{\text{Eff}(\%)} \times 100\%$$

Substitute the known values to find E_{IN} in joules.

$$E_{IN} = \frac{51.9 \times 10^{-3}\text{ J}}{35\cancel{\%}} \times 100\cancel{\%}$$ (Cancel % symbols.)

E_{IN} = 0.148 J (rounded)

Then convert E_{IN} to calories.

$$E_{IN} = 0.148\cancel{\text{J}} \times \frac{1\text{ cal}}{4.184\cancel{\text{J}}}$$ (Cancel units.)

E_{IN} = 0.0354 cal (rounded)

The thermal energy that falls on the sensitive area of the semiconductor material each second equals 0.0354 calorie.

PRACTICE EXERCISES

Problem 1: Given: The diesel engine in an 18-wheel tractor-trailer produces 482 hp (that is, 360 kW of power). The dashboard tachometer shows that the engine is operating at 2100 rpm. The output power is stored as rotational mechanical energy in the engine flywheel. It's used to do work to run the tractor-trailer. The fuel burned each second within the engine provides 1.1×10^6 J of thermal input energy. It's converted to mechanical energy by the engine.

Find:

a. The diesel-engine efficiency in changing thermal input energy to mechanical output energy.

b. The amount of thermal energy lost as waste heat through the cooling system and exhaust.

Solution:

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SOLUTIONS TO PRACTICE EXERCISES, Continued

Problem 2: $\text{Eff}\ (\%) = \frac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = P_{out} \times t$, E_{in} = unknown, Eff (%) = 25%

Solve for E_{in}.

$$E_{in} = \frac{E_{out}}{\text{Eff}\ (\%)} \times 100\%$$

$$E_{out} = P_{out} \times t = 9.33 \text{ megawatt} \times 1 \text{ sec}$$
$$= 9.33 \times 10^6 \text{ watt}\cdot\text{sec} = 9.33 \times 10^6 \text{ J}$$

$$E_{in} = \frac{9.33 \times 10^6 \text{ J}}{25\%} \times 100\% = \left(\frac{9.33 \times 10^6}{25} \times 100\right)\text{J}$$

$$E_{in} = (0.373 \times 10^6 \times 100) \text{ J}$$

$$E_{in} = 37.3 \times 10^6 \text{ J} = 37.3 \text{ megawatts}$$

Convert "E_{in}" from "J" to "Btu."

$$E_{in} = 37.3 \times 10^6 \text{ J} \times \frac{1 \text{ Btu}}{1054 \text{ J}} = \left(\frac{37.3 \times 10^6}{1054}\right) \text{Btu} \frac{\cancel{\text{J}}}{\cancel{\text{J}}}$$

$$E_{in} = 35{,}389 \text{ Btu}.$$

Problem 3: Since inputs and outputs are given as units of power, use "Eff" formula for power.

$\text{Eff}\ (\%) = \frac{P_{out}}{P_{in}} \times 100\%$ where: P_{out} = 10 watts, P_{in} = 3 cal/sec

$$P_{in} = 3 \frac{\text{cal}}{\text{sec}} \times 4.184 \frac{\text{J}}{\text{cal}} = (3 \times 4.184) \frac{\cancel{\text{cal}}\cdot\text{J}}{\text{sec}\cdot\cancel{\text{cal}}}$$

$$P_{in} = 12.55 \text{ J/sec} = 12.55 \text{ watts}$$

$$\text{Eff}\ (\%) = \frac{10 \cancel{\text{ watts}}}{12.55 \cancel{\text{ watts}}} \times 100\% = \left(\frac{10}{12.55} \times 100\%\right) = 79.7\%$$

$$\text{Eff}\ (\%) = 79.7\%.$$

Solutions to Problems 4 and 5 are continued on page T-130c.

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(Solutions to Problems 4 and 5 continued on T-130c.)

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Problem 4: $\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\%$ where: $E_{in} = 60 \times 10^{-3}$ Btu
$E_{out} = 40$ watt•sec

Change Btu to watt•sec. Use Table 3.

$$E_{in} = 60 \times 10^{-3} \text{ Btu} \times \frac{1054 \text{ J}}{1 \text{ Btu}} \times \frac{1 \text{ watt•sec}}{1 \text{ J}}$$

$$E_{in} = (60 \times 10^{-3} \times 1054) \frac{\cancel{\text{Btu•J}}\ \text{watt•sec}}{\cancel{\text{Btu•J}}}$$

$E_{in} = 63.24$ watt•sec.

$$\text{Eff (\%)} = \frac{40\ \cancel{\text{watt•sec}}}{63.24\ \cancel{\text{watt•sec}}} \times 100\% = \left(\frac{40}{63.24} \times 100\%\right)$$

Eff (%) = 0.6325 x 100%

Eff (%) = 63.25% or about 63%.

Problem 5: $\text{Eff (\%)} = \frac{E_{out}}{E_{in}} \times 100\%$ where: $E_{out} = P_{out} \times t = 40$ watt•sec
E_{in} = unknown
Eff (%) = 40%

Solve for "E_{in}" by rearranging the equation.

$$E_{in} = \frac{E_{out}}{\text{Eff (\%)}} \times 100\%$$

$$E_{in} = \frac{40 \text{ watt•sec}}{40\%} \times 100\% = \left(\frac{40}{40\cancel{\%}} \times 100\cancel{\%}\right) \text{ watt•sec}$$

$E_{in} = 100$ watt•sec.

Recognize that 1 watt•sec = 1J.
Then E_{in} = 100 watt•sec = 100 J.

Convert 100 J to Btu.

$$E_{in} = 100 \text{ J} \times \frac{1 \text{ Btu}}{1054 \text{ J}} = \left(\frac{100 \times 1}{1054}\right) \frac{\cancel{\text{J}}\text{•Btu}}{\cancel{\text{J}}}$$

$E_{in} = 0.095$ Btu.

The oil lamp had to supply about 1/10 of a Btu each second to operate the radio.

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Problem 2: Given: A power-plant, low-speed, diesel engine operates at 110 rpm. It delivers 12,500 hp (that is, 9.33 megawatts) each second to the shaft of the power-plant generator. The efficiency of this type of diesel engine is in a range of 22% to 30%.

Find: The amount of heat energy in Btu converted to mechanical energy at a 25% efficiency.

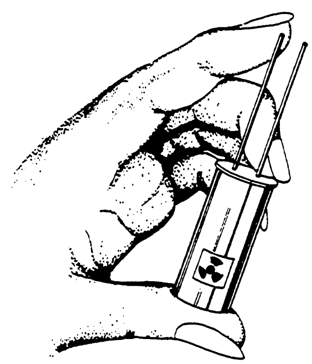
Solution:

Problem 3: Given: A thermoelectric generator is used to provide power for a weather satellite. It uses enriched uranium to provide heat energy and a silicon-germanium thermopile to convert the heat energy into electrical energy. The uranium provides heat energy equal to 3 calories each second. The thermopile generates 10 watts of power from the heat energy provided.

Find: The efficiency of the thermoelectric generator.

Solution:

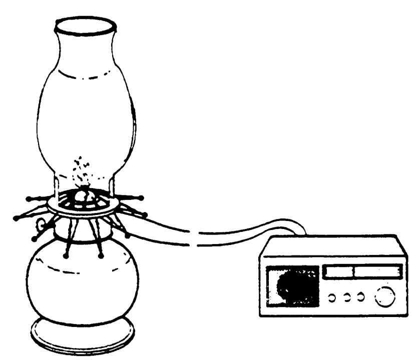
Problem 4: Given: An implantable, heart pacemaker uses a nuclear isotope battery as a heat source for the warm junction of a thermopile. The other junction uses the body temperature of 98°F as the cold junction. Five such batteries provide 60×10^{-3} Btu of thermal energy each second to the thermopile. The thermopile provides 40 watt·sec of energy to operate the pacemaker.



Find: The efficiency of the pacemaker power-supply unit.

Solution:

Problem 5: Given: In the 1930s and 1940s, rural Russians used an ordinary oil lamp to provide light. At the same time, an array of thermocouples (a thermopile) provided power to operate a radio, as shown in the drawing. The radio needed 40 watts of power to operate. The thermoelectric generator was 40% efficient.



Find: The heat energy in Btu the oil lamp had to provide each second to the hot junction of the thermopile to operate the radio.

Solution:

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