Math Lab 2 MS 1
Measuring Angles in Radians
Solving Mechanical Work Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

© 2012, CORD

TEACHING PATH - MATH SKILLS LAB - CLASS M

PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB.

There are two preparatory Math Skills Labs available in a separate book entitled *PT Resource Manual* that contain concepts your students should have mastered before they begin this Math Skills Lab. These two preparatory labs are coded PMS5 and PMS8. They are titled "Learning How to Measure Angles in Radians" and "Substituting in Formulas." Encourage students who need these skills to study the material in PMS5 and PMS8.

RESOURCE MATERIALS

Student Text: Math Skills Lab

PT Resource Manual

Protractor and a hand calculator

CLASS GOALS

1. Teach students how to change angles in degrees to angles in radians.
2. Teach students how to substitute numbers in a given formula and solve mechanical work problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete as many activities as time permits. (How much is accomplished depends on the math/drawing skills that your students already have.)
   a. Summarize the explanatory material for Activity 1: "Measuring Angles in Radians." Then have students complete the Practice Exercises that are given at the end of Activity 1.
   b. Summarize the explanatory material for Activity 2: "Solving Mechanical Work Problems." Then have students complete the Practice Exercises that are given at the end of Activity 2.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 2M1, "Work Done By Pulleys," as homework.

© 2012, CORD

Math Skills Laboratory

Lab 2 MS 1

MATH ACTIVITIES

Activity 1: Measuring Angles in Radians

Activity 2: Solving Mechanical Work Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Given an angle in degrees, find its value in radians.***
2. ***Given an angle in revolutions, find its value in radians.***
3. ***Solve mechanical work problems.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study examples in paragraphs C, D and E of Activity 1.***
3. ***Work the problems for Activities 1 and 2.***

ACTIVITY 1

Measuring Angles in Radians

MATERIALS

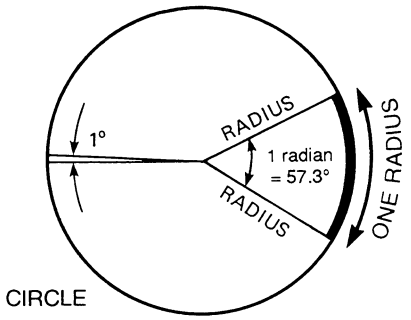
For this activity, you'll need a protractor and a hand calculator.

A. The ***radian*** is a unit of angle measure. It's like a degree, only much larger. It's equal to 57.3°. The radian is defined in terms of a part of a circle. An angle of one radian cuts off an arc on the circle equal to the radius of the circle, as shown on the next page. For comparison, the size of one degree is also shown.

© 2012, CORD

NOTE: The *PT Resource Manual*, available to you and your students, includes a "Preparatory Math Skills Lab" that deals with angle measure in radians. If some of your students have trouble with this math lab, you may want to have them read the Preparatory Math Skills Lab on the same subject. They can do that by themselves and gain practice and insight on the measure of angles in degrees, radians and revolutions.

© 2012, CORD



B. In terms of revolutions, degrees and radians, we have the following relationships:
1 revolution = 360° (The symbol "°" stands for degree.)
1 revolution = 6.28 radians (abbreviated "rad")
1 radian = 57.3°

C. When ***changing degrees to radians***, multiply angle in degrees by the fraction $\frac{1 \text{ radian}}{57.3 \text{ degrees}}$. This fraction is equal to *one*!

Example: Change 90° to radians.

$$(90 \cancel{\text{degrees}}) \times \left[\frac{1 \text{ radian}}{57.3 \cancel{\text{degrees}}}\right] = \underline{\underline{\boldsymbol{1.57 \text{ rad}}}} \quad \text{(Cancel degrees.)}$$

Example: Change 287° to radians.

$$(287 \cancel{\text{degrees}}) \times \left[\frac{1 \text{ radian}}{57.3 \cancel{\text{degrees}}}\right] = \underline{\underline{\boldsymbol{5.01 \text{ rad}}}} \quad \text{(Cancel degrees.)}$$

D. When ***changing revolutions (rev) to radians***, multiply angle in revolutions by the fraction $\frac{6.28 \text{ radians}}{\text{revolution}}$. This fraction is equal to *one*!

Example: Change 2.5 revolutions to radians.

$$(2.5 \cancel{\text{rev}}) \times \left[\frac{6.28 \text{ rad}}{\cancel{\text{rev}}}\right] = \boldsymbol{15.7 \text{ rad}} \quad \text{(Cancel rev.)}$$

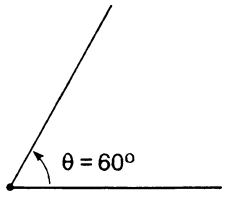
Example: Change 10 revolutions to radians.

$$(10 \cancel{\text{rev}}) \times \left[\frac{6.28 \text{ rad}}{\cancel{\text{rev}}}\right] = \boldsymbol{62.8 \text{ rad}} \quad \text{(Cancel rev.)}$$

E. Measure the following angles with a protractor. Change angle in degrees to angle in radians.

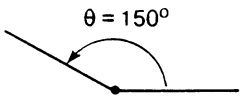
Example:
θ measured = 60°
θ in radians is equal to:

$$60° \times \left[\frac{1 \text{ rad}}{57.3°}\right] = \boldsymbol{1.05 \text{ rad}}$$



Example:
θ measured = 150°
θ in radians is equal to:

$$150° \times \left[\frac{1 \text{ rad}}{57.3°}\right] = \boldsymbol{2.62 \text{ rad}}$$



© 2012, CORD

© 2012, CORD

NOTE: Encourage students to use their hand calculators to solve for numerical answers in Problems 1, 2, 3 of the Practice Exercises for Activity 1.

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1

Problem 1:

a. $30° = 30\cancel{°} \times \left(\frac{1 \text{ rad}}{57.3\cancel{°}}\right) = 0.52 \text{ rad.}$

b. $57.3° = 57.3\cancel{°} \times \left(\frac{1 \text{ rad}}{57.3\cancel{°}}\right) = 1.0 \text{ rad.}$

c. $360° = 360\cancel{°} \times \left(\frac{1 \text{ rad}}{57.3\cancel{°}}\right) = 6.28 \text{ rad } (2\pi \text{ rad}).$

Problem 2:

a. $0.5 \text{ rev} = 0.5 \cancel{\text{rev}} \times \left(\frac{6.28 \text{ rad}}{1 \cancel{\text{rev}}}\right) = 3.14 \text{ rad.}$

b. $5 \text{ rev} = 5 \cancel{\text{rev}} \times \left(\frac{6.28 \text{ rad}}{1 \cancel{\text{rev}}}\right) = 31.4 \text{ rad.}$

c. $3.2 \text{ rev} = 3.2 \cancel{\text{rev}} \times \left(\frac{6.28 \text{ rad}}{1 \cancel{\text{rev}}}\right) = 20.1 \text{ rad.}$

Problem 3:

(Student should draw an angle of 85° with a protractor.)

$85° = 85\cancel{°} \times \left(\frac{1 \text{ rad}}{57.3\cancel{°}}\right) = 1.48 \text{ rad.}$

© 2012, CORD

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2

Problem 1: a. $W = F \times D$ $\quad F = 50\ N$, $D = 40m$

$W = 50\ N \times 40\ m$

$W = 2000\ N \cdot m.$

b. Linear mechanical work.

Problem 2: $W = F \times D$ $\quad W = 150\ ft \cdot lb$, $D = 3\ ft$

Solve for F.

$$F = \frac{W}{D} = \frac{150\ \cancel{ft} \cdot lb}{3\ \cancel{ft}}$$

$F = 50\ lb.$

A greater lifting force would accelerate the object.

© 2012, CORD

PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: Change each angle in degrees to an angle in radians. (Remember, 1 radian = 57.3°.)

a. 30° = _____.
b. 57.3° = _____.
c. 360° = _____.

Problem 2: Change revolutions to angle in radians. (Remember, 1 revolution = 6.28 radians.)

a. 0.5 revolution = _____.
b. 5 revolutions = _____.
c. 3.2 revolutions = _____.

Problem 3: Draw an angle of 85° with a protractor. What's the angle equal to in radians?

ACTIVITY 2

Solving Mechanical Work Problems

MATERIALS

For this activity, you'll need a hand calculator.

Mechanical work formulas needed for this activity are given below. The work formulas include formulas for linear mechanical work, torque, rotational mechanical work and efficiency. You'll need to choose the proper formula to solve each problem for the unknown value. Your final answer should include a *correct numerical answer* AND the *proper units*. Do a careful "unit-analysis" in each problem. Some unit relations are also given below.

$$\text{Work} = \begin{matrix}\text{Force Applied}\\ \text{to Object}\end{matrix} \times \begin{matrix}\text{Distance Object Moves}\\ \text{While Force Acts}\end{matrix} \qquad W = F \times D;\ F = \frac{W}{D};\ D = \frac{W}{F}$$

$$\text{Work} = \text{Torque Applied} \times \text{Angle Moved Through} \qquad W = T \times \theta;\ T = \frac{W}{\theta};\ \theta = \frac{W}{T}$$

$$\text{Torque} = \text{Force Applied} \times \text{Lever Arm} \qquad T = F \times L;\ F = \frac{T}{L};\ L = \frac{T}{F}$$

$$\text{Eff (\%)} = \frac{\text{Output Work}}{\text{Input Work}} \times 100\ \%$$

1 N·m = 1 joule; 1 in. = 2.54 cm

PRACTICE EXERCISES FOR ACTIVITY 2

Problem 1:
Given: A horizontal force of 50 newtons is applied to push a desk 40 meters across a warehouse floor.
Find: a. How much work is done to move the desk?
b. What type of work is done?
Solution:

Problem 2:
Given: A test reveals that 150 ft·lb of work is required to lift an object 3 ft at a ***constant speed.***
Find: The weight of the object.
Solution:

© 2012, CORD

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2, Continued

Problem 3: $W = T \times \theta$ and $T = F \times L$

a. $T = F \times L$ $\quad F = 50$ lb, $L = 18$ in. $= 1.5$ ft

$T = 50 \text{ lb} \times 1.5 \text{ ft}$

$T = 75 \text{ lb}\cdot\text{ft}$

b. $W = T \times \theta$ $\quad T = 75$ lb·ft (from "a"), $\theta = 2\pi$ radians $= 6.28$ radians

$W = 75 \text{ lb}\cdot\text{ft} \times 6.28 \text{ rad}$

$W = 471 \text{ ft}\cdot\text{lb}$ (Drop rad and change lb·ft [torque units] to ft·lb [work units].)

Problem 4: $W = T \times \theta$ $\quad T = 300$ lb·ft, $W = 400$ ft·lb

Solve for θ

$$\theta = \frac{W}{T} = \frac{400 \cancel{\text{ft}\cdot\text{lb}}}{300 \cancel{\text{lb}\cdot\text{ft}}}$$

$$\theta = \frac{4}{3} = 1.33 \text{ radians.}$$

Problem 5: $$\text{Efficiency (\%)} = \frac{\text{Output Work}}{\text{Input work}} \times 100\%$$

$$\text{Eff (\%)} = \frac{1200 \cancel{\text{ft}\cdot\text{lb}}}{1400 \cancel{\text{ft}\cdot\text{lb}}} \times 100\%$$

$$\text{Eff (\%)} = 85.7\%.$$

Here's an example of a problem with extra information (600–lb load) not needed to solve the problem. This is closer to real-life situations, where you have to select the useful data from all the information available.

Problem 6: $W = F \times D$ $\quad W = 180$ J $= 180$ N·m, $D = 130$ cm $= 1.3$ m

Solve for F.

$$F = \frac{W}{D} = \frac{180 \text{ N}\cdot\cancel{\text{m}}}{1.3 \cancel{\text{m}}}$$

$F = 138.46$ N

$F = 138.5$ N.

© 2012, CORD

SOULUTIONS TO PRACTICE EXERCISES, ACTIVITY 2, Continued

Problem 7: $W = F \times D$ $\quad F = 20 \frac{lb}{\cancel{box}} \times 5 \frac{\cancel{boxes}}{person} = 100 \frac{lb}{person}$

$D = 10$ ft

a. $W = 100 \frac{lb}{person} \times 10$ ft

$W = 1000 \frac{ft \cdot lb}{person}$.

b. Each man did 1000 ft·lb of work. The time it took to do the work is not involved in the definition of work. This example sets the stage for the concept of power, treated later, which does involve the time.

An alternate method

a. $W = F \times D$ $\quad F = 20$ lb

$D = 10 \frac{ft}{\cancel{trip}} \times 5 \frac{\cancel{trips}}{person} = 50 \frac{ft}{person}$

$W = 20 \text{ lb} \times 50 \frac{ft}{person}$

$W = 1000 \frac{ft \cdot lb}{person}$.

Problem 8: $Eff\ (\%) \quad \frac{\text{Output Work}}{\text{Input Work}} \times 100\ \%$

$Eff\ (\%) = \frac{3000\ \cancel{ft \cdot lb}}{3000\ \cancel{ft \cdot lb}} \times 100\ \%$

$Eff = 100\%$.

No jack is 100% efficient. The measured output work must be less than the input work. An actual auto jack cannot be completely frictionless.

© 2012, CORD

ANSWERS TO STUDENT CHALLENGE PROBLEMS

Problem 9: 1 in. = 2.54 cm, so $18 \cancel{\text{in.}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{in.}}} = 45.72 \text{ cm} = 0.4572 \text{ m}$

$T = F \times L$
$T = 196 \text{ N} \times 0.4572 \text{ m}$
$T = 89.6 \text{ N·m.}$

$W = T \times \theta$
$W = 89.6 \text{ N·m} \times 0.7854 \text{ rad}$
$W = 70.37 \text{ N·m}$
$W = 70.37 \text{ J.}$

$\theta = \pi/4 \text{ rad} = 0.7854 \text{ rad.}$
(Drop rad.)

Problem 10: a. $W = T \times \theta$

$W = 600 \text{ J} = 600 \text{ N·m}$
$\theta = 1 \text{ rev} = 6.28 \text{ rad}$

Solve for T.

$$T = \frac{W}{\theta}$$

$$T = \frac{600 \text{ N·m}}{6.28 \text{ rad}}$$

$T = 95.54 \text{ N·m.}$

b. $T = F \times L$

$T = 95.54 \text{ N·m}$
$L = 1 \text{ m}$

Solve for F.

$$F = \frac{T}{L} = \frac{95.54 \text{ N·} \cancel{\text{m}}}{1 \cancel{\text{m}}}$$

$F = 95.54 \text{ N.}$

© 2012, CORD

Problem 3: Given: To tighten a nut, a force of 50 lb is applied at the end of an 18-inch torque wrench. The wrench moves the nut through one revolution (2π radians) on the bolt.

Find: a. The torque generated or applied.
b. The work done on the nut.

Solution:

Problem 4: Given: A torque of 300 lb·ft is applied to a lever. The lever does 400 ft·lb of work in lifting a stone.

Find: The angle (in radians) the lever moves through.

Solution:

Problem 5: Given: A block and tackle requires 1400 ft·lb of input work to accomplish 1200 ft·lb of output work while lifting a 600-lb load.

Find: The % efficiency of the block and tackle.

Solution:

Problem 6: Given: To lift a transformer straight up from the floor to a truck bed located 130 cm above the floor requires 180 joules of work.

Find: The weight of the transformer.

Solution:

Problem 7: Given: Bill Johnston and Tim Brown have to push 5 boxes each (with a force of 20 lb on each box) to move the ten boxes from a packing table to a conveyor, a horizontal distance of 10 ft. Bill moves two boxes, goes to the water fountain, returns and moves the other three. Tim Brown moves his 5 boxes in 5 minutes without stopping for a drink.

Find: a. The work done by each person.
b. Who did more work in moving the boxes?

Solution:

Problem 8: Given: An auto jack that's considered frictionless raises an auto. The input work on the jack is 3000 ft·lb. The output work on the auto is 3000 ft·lb.

Find: The % efficiency of the jack. (Is this answer reasonable?)

Solution:

Student Challenge

Problem 9: Given: A 196-N force is applied to the end of a torque wrench. The arm of the torque wrench is 18 inches long. It causes the wrench to move through an angle of $\pi/4$ radians.

Find: The work done by the applied force in ***joules.***

Solution:

Problem 10: Given: A heavy spinning flywheel has a radius of 1 meter. It requires 600 joules of energy (work) to bring it to a stop in 1 revolution.

Find: a. The braking ***torque*** applied to the flywheel (in newton-meters).
b. The braking ***force*** required to stop the flywheel in one revolution (in newtons).

Solution:

© 2012, CORD