

# Math Skills Laboratory

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Lab 10 Ms 2

## **MATH ACTIVITY**

**Activity:** *Solving Energy-Conversion Problems  
for Fluid Energy Convertors*

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## **MATH SKILLS LAB OBJECTIVES**

*When you complete these activities, you should be able to do the following:*

- 1. Solve energy-conversion problems for fluid energy convertors.*
  - 2. Substitute correct numerical values and units in energy-conversion equations. Solve the equations for an unknown numerical value with the proper units.*
  - 3. Find the efficiency of fluid energy convertors.*
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## **LEARNING PATH**

- 1. Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
  - 2. Study the examples.*
  - 3. Work the problems.*
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## **ACTIVITY**

### **Solving Energy-Conversion Problems for Fluid Energy Convertors**

#### **MATERIALS**

For this activity, you'll need a calculator.

In this Math Skills Lab, you'll review the important formulas for work and energy. You'll review the units used to measure work, energy and power. You'll solve problems that involve fluid energy convertors.

### LET'S REVIEW FORMULAS AND UNITS!

Examine Tables 1, 2 and 3. (They are the same as those in Math Skills Lab 10MS1.)

Table 1 lists the important formulas for work and energy that you've already studied. Table 2 sums up common units that you'll use in energy-conversion problems. Table 3 lists some conversions between units.

You'll find these conversions helpful when you solve the problems in this activity.

TABLE 1. REVIEW OF BASIC FORMULAS FOR WORK AND ENERGY

Energy System	Formulas	Definition of Symbols
<b>MECHANICAL</b>		
Linear Work	$W = F \times D$	F = applied force D = distance moved
Rotational Work	$W = T \times \theta$	T = torque applied $\theta$ = angular distance rotated
Gravitational Potential Energy	$E_p = w \times h$	w = weight h = height raised above reference level
Elastic Potential Energy	$E_p = \frac{1}{2} kd^2$	k = force or spring constant d = distance spring is stretched or compressed
Linear Kinetic Energy	$E_k = \frac{1}{2} mv^2$	m = mass v = speed
Rotational Kinetic Energy	$E_k = \frac{1}{2} I\omega^2$	I = moment of inertia $\omega$ = angular speed
<b>FLUID</b>		
Fluid Work	$W = p \times (\Delta V);$ $W = (\Delta p) \times V$	p; $\Delta p$ = pressure or pressure difference $\Delta V$ ; V = volume of fluid moved
Fluid Kinetic Energy	$E_k = \frac{1}{2} (\rho V)v^2$	$\rho$ = mass density V = volume v = speed
<b>ELECTRICAL</b>		
Electrical Work	$W = q \times \Delta V$	q = electrical charge moved $\Delta V$ = voltage difference
Electrical Energy	$E_{elec} = P_{elec} \times t$	$P_{elec}$ = electrical power used t = time power is used
<b>THERMAL</b>		
Thermal Energy	$H = mc\Delta T$	m = mass c = specific heat $\Delta T$ = temperature difference

**TABLE 2. SUMMARY OF COMMON UNITS USED IN  
ENERGY CONVERSION CALCULATIONS**

<b>Quantity</b>	<b>Symbol</b>	<b>English Unit</b>	<b>SI Unit</b>
Mass	m	slug or (lb·sec <sup>2</sup> /ft)	kg
Weight	w	lb	N
Force	F	lb	N
Work	W	ft·lb	N·m or J
Kinetic Energy	E <sub>k</sub>	ft·lb	N·m or J
Potential Energy	E <sub>p</sub>	ft·lb	N·m or J
Power	P	(ft·lb)/sec; hp	(N·m)/sec or watt
Spring Constant	k	lb/ft; oz/in.	N/m; kgf/m *
Angular			
Displacement	θ	rad	rad
Angular Speed	ω	rad/sec	rad/sec
Linear Speed	v	ft/sec	m/sec
Mass Density	ρ	slug/ft <sup>3</sup> ; lbm/ft <sup>3</sup> **	kg/m <sup>3</sup>
Moment of Inertia	I	slug·ft <sup>2</sup>	kg/m <sup>2</sup>
Pressure or			
Pressure Difference	p; Δp	lb/ft <sup>2</sup> ; oz/in <sup>2</sup>	N/m <sup>2</sup>
Electrical Charge	q	—	coulomb (C)
Torque	T	lb·ft; lb·in.	N·m
Fluid Volume	V; ΔV	ft <sup>3</sup>	m <sup>3</sup>
Electrical Current	I	—	ampere (A)
Electrical Voltage	V; ΔV	—	volt (V)
Electrical Resistance	R	—	ohm (Ω)
Heat Energy	H	Btu	cal
Temperature	T	°F	°C
Temperature			
Difference	ΔT	F°	C°
Specific Heat	c	$\frac{\text{Btu}}{\text{lb} \cdot \text{F}^\circ}$	$\frac{\text{kcal}}{\text{kg} \cdot \text{C}^\circ}$

\* kgf ⇒ kilogram force

\*\* lbm ⇒ pound mass

**TABLE 3. USEFUL CONVERSIONS FOR UNITS**

1 slug	=	32 pound mass (32 lbm)
1 kilogram force (kgf)	=	9.8 newtons
1 newton-meter	=	1 joule
1 volt-coulomb	=	1 joule
1 ampere-second	=	1 coulomb
1 radian	=	57.3°
1 horsepower	=	550 ft·lb/sec
1 horsepower	=	746 watts
1 watt	=	1 newton-meter per second
1 watt	=	1 joule per second
1 watt	=	1 volt-ampere
1 watt-sec	=	1 joule = 0.7376 ft·lb
1 kilowatt-hour (kWh)	=	3.6 × 10 <sup>6</sup> joules
1 calorie	=	3.086 foot-pounds
1 calorie	=	4.184 joules
1 Btu	=	778 foot-pounds
1 Btu	=	1055 joules

Use the contents of Tables 1, 2 and 3 to help you answer the following questions.

1. The formula for fluid work is \_\_\_\_.
2. The formula for fluid kinetic energy is \_\_\_\_.
3. The formula for rotational mechanical kinetic energy is \_\_\_\_.
4. The formula for electrical work is \_\_\_\_.
5. The formula for electrical energy used by a device, given the electrical power and time of operation, is \_\_\_\_.
6. The kinetic energy of a moving fluid may be found if you know the mass and speed of the fluid. It may also be found if you know the mass density of the fluid, the \_\_\_\_ of the fluid, and the speed of the fluid.
7. In a fluid energy convertor that is 100% efficient, the fluid kinetic energy of the input equals the \_\_\_\_.
8. A fluid energy convertor changes 100 joules of input fluid energy into 83.7 joules of output rotational mechanical energy. At the same time, 3.896 calories of energy are wasted as heat. One calorie of energy equals \_\_\_\_ joules of energy.
9. The fluid energy convertor in Question 8 has an efficiency of \_\_\_\_ %.
10. A fluid energy convertor changes 778 ft·lb of input fluid energy into 1054 joules of output mechanical energy. A fluid energy of 778 ft·lb equals \_\_\_\_ joules of energy.
11. The fluid energy convertor in Question 10 has an efficiency of \_\_\_\_ %.
12. The quantity  $\rho V$  can be used for mass in fluid kinetic-energy problems. The SI units of  $\rho$  are \_\_\_\_\_. The SI units of  $V$  are \_\_\_\_\_. Therefore, SI units of  $\rho V$  are \_\_\_\_\_.
13. A fluid energy convertor changes 1000 N·m of fluid energy each second into 890 watts of electrical power. One N·m per second equals \_\_\_\_ watt(s).
14. The fluid energy convertor in Question 13 has an efficiency of \_\_\_\_ %.

### ENERGY-CONVERSION PROBLEMS THAT INVOLVE FLUID ENERGY CONVERTORS

Study the two examples of energy-conversion problems given here. Then solve the **Practice Exercises** that follow.

#### Example A: Efficiency of a Windmill Rotor

##### (Fluid-to-Mechanical Energy Convertor)

**Given:** A windmill rotor consists of aluminum blades, gears and an output shaft. The windmill extracts 60,000 N·m of energy from the air each second while the wind blows past the blades at a speed of 12.5 m/sec. The windmill output shaft delivers 48,000 J of energy to the input shaft of a water pump.

**Find:** The efficiency of the windmill rotor as an energy convertor.

**Solution:** The efficiency of the windmill rotor is given as:

$$\text{Eff}(\%) = \frac{E_{\text{OUT}}}{E_{\text{IN}}} \times 100\% \quad \text{where: } E_{\text{OUT}} = 48,000 \text{ J} \\ E_{\text{IN}} = 60,000 \text{ N}\cdot\text{m or } 60,000 \text{ J}$$

$$\text{Eff}(\%) = \frac{48,000 \cancel{\text{J}}}{60,000 \cancel{\text{J}}} \times 100\% \quad (\text{Cancel units.})$$

$$\text{Eff}(\%) = 0.8 \times 100\% = 80\%$$

Thus, this windmill is 80% efficient.

**Example B: Efficiency of a Hydraulic Motor**  
**(Fluid-to-Mechanical Energy Converter)**

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**Given:** One-sixth of a gallon ( $631 \text{ cm}^3$ ) of hydraulic fluid moves through a hydraulic motor each second at a pressure of 1000 psi ( $690 \text{ N/cm}^2$ ). The hydraulic motor converts 3610 J of fluid energy each second to mechanical energy of the motor output shaft.

**Find:** a. The energy contained in the fluid that enters and drives the hydraulic motor.  
b. The efficiency of the hydraulic motor as an energy converter. (Neglect any internal friction in the system.)

**Solution:** a. Energy in the fluid that enters and drives the motor equals the work done on the fluid. Therefore, use the following formula.

$$E_{\text{IN}} = W_{\text{IN}} = p \times \Delta V \quad \text{where: } p = 690 \text{ N/cm}^2 \\ \Delta V = 631 \text{ cm}^3$$

So,

$$E_{\text{IN}} = 690 \text{ N/cm}^2 \times 631 \text{ cm}^3$$

$$E_{\text{IN}} = (690 \times 631) \left[ \frac{\text{N}}{\text{cm}^2} \times \text{cm}^3 \right] \quad (\text{Cancel units.})$$

$$E_{\text{IN}} = 435,390 \text{ N}\cdot\text{cm}$$

Change to N·m by using  $1 \text{ cm} = 10^{-2} \text{ m}$ .

$$E_{\text{IN}} = 435,390 \text{ N}\cdot\text{cm} \times \left[ \frac{10^{-2} \text{ m}}{1 \text{ cm}} \right] \quad (\text{Cancel units.})$$

$$E_{\text{IN}} = 4353.9 \text{ N}\cdot\text{m}$$

$$E_{\text{IN}} = 4354 \text{ N}\cdot\text{m} \text{ or } 4354 \text{ J (rounded)} \quad (\text{Remember } 1 \text{ N}\cdot\text{m} = 1 \text{ J.})$$

b. Efficiency of the hydraulic motor equals the mechanical output energy  $E_{\text{OUT}}$  divided by the total input energy of the fluid  $E_{\text{IN}}$ .

$$\text{Eff}(\%) = \frac{E_{\text{OUT}}}{E_{\text{IN}}} \times 100\% \quad \text{where: } E_{\text{OUT}} = 3610 \text{ J, the fluid energy converted to mechanical energy}$$

$$E_{\text{IN}} = 4354 \text{ J, the total energy of the fluid that enters the hydraulic motor}$$

$$\text{Eff}(\%) = \frac{3610 \text{ J}}{4354 \text{ J}} \times 100\% \quad (\text{Cancel units.})$$

$$\text{Eff}(\%) = 0.829 \times 100\% = 82.9\%$$

Thus, this hydraulic motor is 82.9% efficient.

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Examples A and B deal with two energy converters—each of which converts fluid energy to rotational mechanical energy. The windmill's mechanical energy output is an intermediate stage in energy conversion. That's because the rotational mechanical energy becomes input energy for conversion into electrical energy in a generator—or fluid energy in a pump.

The mechanical energy output from the hydraulic motor described in Example B most often is used to do rotational mechanical work. A hydraulic-powered gearbox or winch is a good example of uses for hydraulic motor output energy.

## PRACTICE EXERCISES

**Problem 1:** Given: Over a certain period of time, a dam releases 2,341,248 ft·lb of fluid kinetic energy onto the wheel of a water turbine. The turbine output shaft is coupled to an electric power generator input shaft. During the same time period, the water turbine changes the fluid kinetic energy into 2800 Btu of output mechanical energy to the input shaft of the generator.

(**Note:** The output mechanical energy normally **wouldn't be rated in Btu**. We're using that unit in this problem to give you practice with conversion of energy units.)

Find: The water turbine efficiency.

Solution: (**Hint:** Change all energy values into the same units.)

**Problem 2:** Given: The electric generator in Problem 1 produces  $2.80 \times 10^6$  J of electrical energy from the input mechanical energy it receives.

Find: The electric generator efficiency.

Solution:

**Problem 3:** Given: A vane-type air motor is chosen to operate a paint stirrer. The motor is rated at a shaft power of 50 ft·lb/sec. The efficiency of the pneumatic-to-rotational mechanical energy conversion is 78%.

Find: The fluid energy converted into rotational mechanical energy.

Solution: (**Hint:** Power = Energy/Time; so Energy = Power  $\times$  Time, or  $E = P \times t$ .)

**Problem 4:** Given: The fluid volume displacement (movement) through a hydraulic gear motor is 1575 cm<sup>3</sup> each second at a constant pressure of  $2 \times 10^3$  N/cm<sup>2</sup>. The gear motor converts  $2.8 \times 10^4$  J of fluid energy each second to output shaft mechanical energy.

Find: a. The kinetic energy contained in the fluid that enters the gear motor.

b. The efficiency of the gear motor.

Solution: (**Hint:** Model solution after Example B.)

**Problem 5:** Given: Aircraft carriers use steam catapults to launch planes. One of the catapults extracts 400 Btu of energy from superheated steam each second during a launch. The catapult delivers  $30 \times 10^4$  ft·lb of thrust energy to the plane during each second of the launch.

Find: The efficiency of the steam catapult.

Solution:

### **Student Challenge**

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**Problem 6:** Given: A mass of steam equal to 62.4 lbm at a temperature of 212°F is superheated to steam at 970°F and 4500 psi. All of the energy gained by the steam is extracted by a steam turbine. The steam returns to the boiler condenser line, cooled back down to 212°F. The steam turbine develops  $46.363 \times 10^6$  J of output mechanical energy in the turbine shaft.

**(Note:** The specific heat of steam is nearly 1 Btu/lbm·F° at pressures as high as 4500 psi.)

Find: The efficiency of the turbine.

Solution: