Math Lab 3 MS 4
Using SI Prefixes
Solving Thermal-Rate Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There is a Preparatory Math Skills Lab in a separate book entitled *PT Resource Manual* that contains concepts your students should have mastered before they begin this Math Skills Lab. The preparatory lab is coded and titled--PMS12: "Identifying Names of Symbols to Indicate Power-of-Ten Prefixes." Encourage students who need these skills to review the material in PMS12.

RESOURCE MATERIALS

Student Text: Math Skills Lab
PT Resource Manual

GLASS GOALS

1. Teach students how to convert between "power-of-ten" notation and scientific notation.
2. Teach students how to solve thermal rate problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through Student Exercises. Make sure that your students understand the correct answers.
2. Complete as many activities as time permits. Students should already have read the discussion material and looked at the examples for each activity before coming to class. You should summarize the main points in each activity, work an example or two, and have the students complete the Practice Exercises for each activity on their own.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, assign students the homework reading materials for Lab 3T1, "Measuring Heat-flow Rate."

Math Skills Laboratory

Lab 3 MS 4

MATH ACTIVITIES

Activity 1: Using SI Prefixes

Activity 2: Solving Thermal-rate Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. *Express a numerical value and unit, such 3.4×10^{-3} meters, in prefix notation (3.4 millimeters).*
2. *Rearrange the equation for heat-flow rate, $Q_H = H/t$. Solve for heat energy (H) or elapsed time (t).*
3. *Rearrange the equation for heat-flow rate caused by a temperature difference across a substance, $Q_H = \frac{kA\Delta T}{\ell}$. Solve for thermal conductivity (k), area of the substance (A), temperature difference across the material (ΔT) or the material thickness (ℓ).*
4. *Substitute appropriate numerical values and units in rate equations. Solve the equations.*

LEARNING PATH

1. *Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
2. *Study examples given in Table 1.*
3. *Work the problems for Activities 1 and 2.*

ACTIVITY 1

Using SI Prefixes

Table 1 gives ***examples*** of certain physical quantities expressed as numbers with units. These are listed in Column 1. In the next column, these numbers are restated in equal *power-of-ten* units. In the final column, these same numbers are written with correct prefixes.

TABLE 1. USE OF SI PREFIXES

Physical Quantity	Quantity Restated in Power-of-ten Notation	Quantity Expressed with Prefix Notation
12,000 newtons (N)	12×10^3 N	12 kN
0.000325 meters (m)	0.325×10^{-3} m or 325×10^{-6} m	0.325 mm 325 μm
1401 volts (V)	1.401×10^3 V	1.401 kV
34,200 watts (W)	34.2×10^3 W	34.2 kW
1.87×10^{-7} seconds (sec)	0.187×10^{-6} sec or 187×10^{-9} sec	0.187 μsec 187 nsec
3.28×10^7 joules (J)	32.8×10^6 J	32.8 MJ
9,500 grams	9.5×10^3 gm	9.5 kg

Let's take a look at the first entry in Table 1. You can see that 12,000 N can be written in different ways.

$$12{,}000 \text{ N} = 12 \times 10^3 \text{ N} = 12 \text{ kilonewtons (or 12 kN)}$$

The prefix "kilo" equals 10^3. The other examples in Table 1 are done the same way. Note carefully that there's more than one choice for writing a number in *power-of-ten* notation. For example, 0.000325 m can be written ***either*** as 0.325×10^{-3} m or 325×10^{-6} m. The first choice leads to 0.325 millimeter, the second to 325 micrometers.

Note to the Student: If you have difficulty changing a number to power-of-ten notation or prefix notation, as in Table 1, you may wish to read ***Preparatory Math Skills Activities 9, 10, 11, and 12***. These activities will show you how to write decimal numbers as power-of-ten numbers and ***vice-versa.*** You'll also learn about scientific notation and names/symbols used to indicate power-of-ten prefixes. After you have studied these activities, you should understand what's going on in Table 1.

SOLUTIONS TO PRACTICE EXERCISE FOR ACTIVITY 1

18.6×10^7 joules	=	$186{,}000 \times 10^3$ J	186,000 kJ
or	=	186×10^6 J	186 MJ
6050 hertz	=	6.05×10^3 Hz	6.05 kHz
1000 volts	=	1×10^3 V	1 kV
1.91×10^{-7} seconds	=	0.191×10^{-6} sec	0.191 μsec
or	=	191×10^{-9} sec	191 nsec
3200 grams	=	3.2×10^3 g	3.2 kg

PRACTICE EXERCISES FOR ACTIVITY 1

Complete the following exercises by expressing the physical quantity (number and unit) given in power-of-ten units and prefix units. Follow the examples in Table 1.

Physical Quantity		*Power-of-Ten Units*	*Prefix Units*
18.6×10^7 joules		________ $\times\ 10^3$ J	________________
	or	________ $\times\ 10^6$ J	________________
6050 hertz		________ $\times\ 10^3$ Hz	________________
1000 volts		________ $\times\ 10^3$ V	________________
1.91×10^{-7} seconds		________ $\times\ 10^{-6}$ sec	________________
	or	________ $\times\ 10^{-9}$ sec	________________
3200 grams		________ $\times\ 10^3$ gm	________________

ACTIVITY 2

Solving Thermal-rate Problems

EQUIPMENT

For this activity, you'll need a hand calculator.

This activity will give you practice in combining the skills learned in rearranging equations to isolate an unknown. This activity also will help you solve for an unknown quantity by substituting appropriate numerical values and units in thermal rate equations. Before you begin to solve problems, review the following equations and what the symbols mean.

The equation for heat-flow rate is stated as follows:

$$\text{Heat-flow Rate} = \frac{\text{Amount of Heat Transferred}}{\text{Elapsed Time}} \qquad \textbf{\textit{Equation 1}}$$

To simplify Equation 1, use the following symbols:

$$Q_H = \frac{H}{t} \qquad \textbf{\textit{Equation 2}}$$

where: Q_H = heat-flow rate (Btu/hr, kcal/sec, or cal/sec)
H = amount of heat energy transferred (Btu or cal)
t = elapsed time (hour, min, sec)

The equation for amount of heat transferred is as follows:

$$\begin{matrix}\text{Amount of} \\ \text{Heat Transferred}\end{matrix} = \text{Mass} \times \begin{matrix}\text{Specific} \\ \text{Heat}\end{matrix} \times \begin{matrix}\text{Temperature} \\ \text{Difference}\end{matrix} \qquad \textbf{\textit{Equation 3}}$$

To simplify Equation 3, use the following symbols:

$$H = mc\Delta T \qquad \textbf{\textit{Equation 4}}$$

where: H = amount of heat energy added or transferred to a body to cause a temperature rise (Btu, kcal, or cal)
m = mass of the body that undergoes a temperature rise (kg, gm or lb)
c = specific heat $\frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$ or $\frac{\text{Btu}}{\text{lb}\cdot\text{F}^\circ}$
ΔT = temperature change of the body (C° or F°)

SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 2

Problem 1:

$$Q_H = \frac{H}{t}$$ where: $H = 250{,}000$ Btu, $t = 30$ min $= 0.5$ hr

$$Q_H = \frac{250{,}000 \text{ Btu}}{0.5 \text{ hr}}$$

$$Q_H = 500{,}000 \text{ Btu/hr.}$$

Problem 2:

$$Q_H = \frac{H}{t}$$ Isolate and solve for "H."

$$H = Q_H \times t$$ where: $Q_H = 36{,}000$ Btu/hr, $t = 3.5$ hr

$$H = 36{,}000 \frac{\text{Btu}}{\text{hr}} \times 3.5 \text{ hr}$$

$$H = 126{,}000 \text{ Btu.}$$

Problem 3:

$$Q_H = \frac{H}{t}$$ Isolate and solve for "t."

$$t = \frac{H}{Q_H}$$ where: $H = 1.8 \times 10^5$ cal

$$Q_H = 850 \frac{\text{cal}}{\text{min}} = 8.5 \times 10^2 \frac{\text{cal}}{\text{min}}$$

$$t = \frac{1.8 \times 10^5 \text{ cal}}{8.5 \times 10^2 \frac{\text{cal}}{\text{min}}}$$

$$t = 0.212 \times 10^{5-2} \frac{\text{cal}}{\text{cal/min}}$$

$$t = 0.212 \times 10^3 \text{ min}$$

$$t = 212 \text{ min} = 3 \text{ hr, } 32 \text{ min.}$$

Problem 4:

$$Q_H = kA \frac{\Delta T}{\ell}$$ where: $k = 0.30 \frac{\text{Btu·in}}{\text{hr·ft}^2\text{·F°}}$

$A = 72 \text{ ft}^2$

$\ell = 3$ in

$\Delta T = 80°\text{F} - 30°\text{F} = 50$ F°

$$Q_H = \frac{\frac{0.30 \text{ Btu·in}}{\text{hr·ft}^2\text{·F°}} \times 72 \text{ ft}^2 \times 50 \text{ F°}}{3 \text{ in}}$$

$$Q_H = \frac{0.30 \times 72 \times 50}{3} \; \frac{\text{Btu·in·ft}^2\text{·F°}}{\text{hr·ft}^2\text{·F°·in}}$$

$$Q_H = 360 \text{ Btu/hr.}$$

The heat-flow rate through a substance of uniform thickness with a temperature difference across opposite faces is given by the following equation:

$$Q_H = \frac{kA\,\Delta T}{\ell} \qquad \textbf{\textit{Equation 5}}$$

where: Q_H = heat-flow rate (Btu/hr, kcal/sec, cal/sec) through the material

k = thermal conductivity constant $\left(\frac{\text{Btu·in.}}{\text{hr·ft}^2\text{·F°}} \text{ or } \frac{\text{cal·cm}}{\text{sec·cm}^2\text{·C°}}\right)$

A = area of material (ft^2, in^2, m^2, cm^2)

ΔT = temperature difference (F°, C°)

ℓ = uniform thickness of material (in., cm)

PRACTICE EXERCISES FOR ACTIVITY 2

Use one of the preceding equations to solve the thermal rate problems in the exercises that follow.

Problem 1: Given: A plywood-drying oven produces 250,000 Btu of heat during a 30-minute drying cycle.

Find: The heat-flow rate (Q_H) for the oven in Btu/hr.

Solution: (***Hint:*** Use the equation, $Q_H = {}^H\!/_t$. Change t = 30 min to t = 0.5 hr before substituting into the equation.)

Problem 2: Given: The refrigeration unit on a meat-delivery truck is rated at 3 ton/hr (that is, 36,000 Btu/hr).

Find: The amount of heat the refrigeration unit will remove if it operates for 3.5 hours during an 8-hr on/off cycle.

Solution: (***Hint:*** Use the equation, $Q_H = {}^H\!/_t$. Rearrange to isolate H. Then solve for H, knowing t and Q_H.)

Problem 3: Given: A sterilizer heating element used in a hospital is rated at 850 cal/min. It requires 1.8×10^5 calories of heat to sterilize instruments.

Find: The amount of time the heating element must operate to meet the sterilization standard.

Solution: (***Hint:*** Use the equation, $Q_H = {}^H\!/_t$. Rearrange to isolate t. Then solve for t, knowing Q_H and H.)

Problem 4: Given: The refrigerated truck in Problem 2 is insulated with 3 inches of corkboard. Insulation made of corkboard has a thermal conductivity of $k = 0.30 \frac{\text{Btu·in.}}{\text{hr·ft}^2\text{·F°}}$. The surface area on ***one*** side of the truck is 72 ft^2. The outside temperature is 80°F, while the refrigerator temperature inside is 30°F.

Find: The heat-flow rate into the truck through one side. (Disregard the aluminum truck siding that usually covers the corkboard.)

Solution: (***Hint:*** Use the equation, $Q_H = \frac{kA\,\Delta T}{\ell}$. Substitute values for k, A, ΔT and ℓ. Handle the units carefully.)

STUDENT CHALLENGE PROBLEMS

Problem 5: From Problem 4, $Q_H = 360$ Btu/hr.

So, $H = 360$ Btu

$t = 1$ hr

From Problem 2, $Q_H = 36{,}000$ Btu/hr.

$Q_H = H/t$ Isolate and solve for t.

$t = \frac{H}{Q_H}$ where: $H = 360$ Btu

$$Q_H = 36{,}000 \frac{\text{Btu}}{\text{hr}} = 3.6 \times 10^4 \frac{\text{Btu}}{\text{hr}}$$

$$t = \frac{360 \text{ Btu}}{3.6 \times 10^4 \frac{\text{Btu}}{\text{hr}}}$$

$$t = 100 \times 10^{-4} \frac{\text{Btu}}{\text{Btu/hr}}$$

$$t = 0.01 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}}$$

$$t = 0.6 \text{ min}$$

Problem 6: $Q_H = kA \frac{\Delta T}{\ell}$ Isolate and solve for "A."

$A = \frac{Q_H \ell}{k \Delta T}$ where: $Q_H = 2320$ Btu/hr

$\ell = 3$ in

$k = 0.30 \frac{\text{Btu} \cdot \text{in}}{\text{hr} \cdot \text{ft}^2 \cdot \text{F}^\circ}$

$\Delta T = 50$ F°

$$A = \frac{2320 \frac{\text{Btu}}{\text{hr}} \times 3 \text{ in}}{0.30 \frac{\text{Btu} \cdot \text{in}}{\text{hr} \cdot \text{ft}^2 \cdot \text{F}^\circ} \times 50 \text{ F}^\circ}$$

$$A = \frac{6960}{15} \frac{\frac{\cancel{\text{Btu} \cdot \text{in}}}{\cancel{\text{hr}}}}{\frac{\cancel{\text{Btu} \cdot \text{in} \cdot \text{F}^\circ}}{\cancel{\text{hr}} \cdot \text{ft}^2 \cdot \cancel{\text{F}^\circ}}}$$ (all units cancel except ft^2)

$A = 464 \text{ ft}^2$.

Problem 7: a. $Q_H = H/t$ Isolate and solve for "H."

$H = Q_H \times t$ where: $Q_H = 15.16$ cal/sec

$t = 20$ min $= 1200$ sec

$$H = 15.16\ \frac{\text{cal}}{\text{sec}} \times 1200\ \text{sec}$$

$H = 18{,}192$ cal.

b. $H = mc\Delta T$ Isolate and solve for "m."

$m = \dfrac{H}{c\Delta T}$ where: $H = 18{,}192$ cal (from part "a")

$c = 0.091$ cal/g·C°

$\Delta T = 80°C - 40°C = 40\ C°$

$$m = \frac{18{,}192\ \text{cal}}{0.091\ \frac{\text{cal}}{\text{g}\cdot\text{C}°} \times 40\ \text{C}°}$$

$$m = \frac{18{,}192}{3.64}\ \frac{\cancel{\text{cal}}}{\frac{\cancel{\text{cal}}}{\text{g}\cdot\cancel{\text{C}°}}\cdot\cancel{\text{C}°}}$$

$m = 4997.8\ \text{g} = 4.9978 \times 10^3 \text{g}$

$m = 5.00$ kg.

Problem 8: a. For Water: where: $m = 1000$ kg

$c = 1$ kcal/kg·C°

$\Delta T = 40$ C°

$H = mc\Delta T$

$$H = 1000\ \text{kg} \times \frac{1\ \text{kcal}}{\text{kg}\cdot\text{C}°} \times 40\ \text{C}°$$

$H_W = 40{,}000$ kcal.

For Stone: where: $m = 2400$ kg

$c = 0.192$ kcal/kg·C°

$\Delta T = 40$ C°

$$H = 2400\ \text{kg} \times 0.192\ \frac{\text{kcal}}{\text{kg}\cdot\text{C}°} \times 40\ \text{C}°$$

$H_S = 18{,}432$ kcal.

b. "H" for water is much greater than "H" for stone. So the technician should use water, since more heat is stored in water than in stone at 40 C° temperature rise.

Problem 5: Given: Conditions stated in Problems 2 and 4.

Find: The time, in minutes, that the 36,000-Btu/hr refrigeration unit in Problem 2 should operate ***each hour*** to remove the heat that comes in through the 72-ft^2 section of wall on the truck each hour (from Problem 4).

Solution: (***Hint:*** Use the answers to Problem 2 and Problem 4 to help you solve this problem. You can do it!)

Problem 6: Given: Conditions stated in Problem 4, where thermal conductivity of corkboard is $k = 0.30 \dfrac{\text{Btu}\cdot\text{in.}}{\text{hr}\cdot\text{ft}^2\cdot\text{F}^\circ}$, and the corkboard is 3 inches thick. The ***temperature difference*** between the inside and outside of the truck refrigerator is 50 F°.

Find: The total surface area (A) of the refrigerator truck in Problem 4 if the heat-flow rate into the truck is 2320 Btu/hr.

Solution: (***Hint:*** Use the equation, $Q_H = \dfrac{kA\,\Delta T}{\ell}$ and rearrange it to isolate A. Then substitute values for Q_H, ℓ, k and ΔT.)

Problem 7: Given: A piece of brass, with specific heat, c = 0.091 cal/gm·C°, is cooled from 80°C to 40°C. The heat-flow rate out of the material is 15.16 cal/sec for 20 minutes. (Assume the heat-flow rate is constant.)

Find:

a. The heat given up by the brass.

b. The mass of the piece of brass.

Solution: (***Hint:*** In Part a use the equation, $Q_H = H/t$. Rearrange to solve for H. Then substitute in known values of Q_H and t. In Part b, rearrange the equation, $H = mc\,\Delta T$, to solve for m. Then substitute known values of H [from Part a], c and ΔT.)

Problem 8: Given: While helping build a new home, the heating and air-conditioning technician decided to install a solar-powered heating system that uses a heat-storage tank. Space available for the tank is limited to one cubic meter (1 m^3). The tank will hold 1000 kg of water (which has a specific heat c = 1.0 kcal/kg·C°) or 2400 kg of crushed stone (which has a specific heat c = 0.192 kcal/kg·C°). The temperature of either substance is to be raised 40 C°.

Find:

a. The amount of heat that can be stored in each substance with a 40-C° temperature rise.

b. Which substance the technician should use if all other conditions are equal.

Solution: (***Hint:*** For Part a, use the heat-transfer equation, $H = mc\,\Delta T$, for each substance—water and crushed stone.)