Math Lab 4 MS 3

Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

Solving Electrical Resistance Problems

For best results, print this document front-to-back and place it in a three-ring binder. Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There are Preparatory Math Skills Labs, located in a separate book entitled *PT Resource Manual*, that contain concepts your students should have mastered before they begin this Math Skills lab. These labs are coded and titled--

PMS8: "Substituting in Formulas"
PMS11: "Conducting Mathematical Operations with Numbers Expressed in Power-of-ten Notation"

Encourage students to refer to these preparatory labs if they need help.

RESOURCE MATERIALS

Student Text: Math Skills Lab

PT Resource Manual

CLASS GOALS

1. Teach students how to rearrange symbols in resistance equations to isolate certain numbers.
2. Teach students how to solve electrical resistance problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete the activities as time permits. Students should already have read the discussion material and looked at the examples for each activity before coming to this class. (How much is accomplished depends on the math skills that your students already have.)
   a. Summarize the explanatory material for Activity 1: "Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns." Then have students complete the Practice Exercises given at the end of Activity 1.
   b. Summarize the explanatory material for Activity 2: "Solving Electrical Resistance Problems." Then have students complete the Practice Exercises given at the end of Activity 2.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 4E1, "Ohm's Law and Series Circuits," as homework.

Math Skills Laboratory

Lab 4 MS 3

MATH ACTIVITIES

***Activity 1:** Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns*

***Activity 2:** Solving Electrical Resistance Problems*

MATH SKILLS LAB OBJECTIVES

When you've completed these activities, you should be able to do the following:

1. ***Isolate the change in voltage (ΔV), or the current (I), by rearranging Ohm's law given in the form, $R_E = \Delta V / I$.***
2. ***Isolate the resistivity (ρ), the length of the wire (ℓ), or the cross-sectional area (A), by rearranging the equation, $R_E = \rho \ell / A$.***
3. ***Find the resistance of a resistor using a table of resistor color codes.***
4. ***Substitute correct numerical values and units in resistance equations. Solve the equations for a numerical value with the proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the equations.***
3. ***Work the problems for Activities 1 and 2.***

ACTIVITY 1

Rearranging Symbols in Resistance Equations To Isolate Certain Unknowns

MATERIALS

For this activity, you'll need pencil and paper.

From previous math labs, you know that equations and formulas express a relationship between several physical quantities. Equation 1 shows the relationship between (1) electrical resistance, (2) voltage difference and (3) current. Equation 1 is a form of Ohm's law and is stated as follows:

$$\text{Electrical Resistance} = \frac{\text{Voltage Difference}}{\text{Current}}$$

This relationship can help you find the value of one physical quantity if you know the value and units of the other two physical quantities. The relationship is often written with symbols, as follows:

$$R_E = \frac{\Delta V}{I} \qquad \textbf{\textit{Equation 1}}$$

where: R_E = electrical resistance measured in ohms (Ω)

ΔV = voltage measured in volts (V)

I = current measured in amperes (A)

(Recall that 1 ohm = 1 volt/1 amp.)

Equation 1 describes resistance in terms of voltage difference (ΔV) and current flow (I). We can also describe ***resistance*** in terms of (1) length, (2) cross-sectional area, and (3) resistivity of the material, as follows:

$$\text{Resistance} = \frac{\text{Resistivity of Material} \times \text{Length of Material}}{\text{Cross-sectional Area of the Material}}$$

This relationship is written with symbols as follows:

$$R_E = \frac{\rho \ell}{A} \qquad \textbf{\textit{Equation 2}}$$

where: R_E = resistance in ohms (Ω)

ρ = resistivity in ohm·cm or microhm·cm (or ohm·in.)

ℓ = length in cm (or in.)

A = cross-sectional area in cm^2 (or in^2)

WHAT'S THE RESISTOR COLOR CODE?

Electronics industries have a specific color-coding system for resistors. Coding is based on the use of four bands of color. The position and color of each band have a specific meaning. Figure 1 shows a fixed, tubular-shaped resistor. The color bands are painted on it. The meaning of each band is explained.

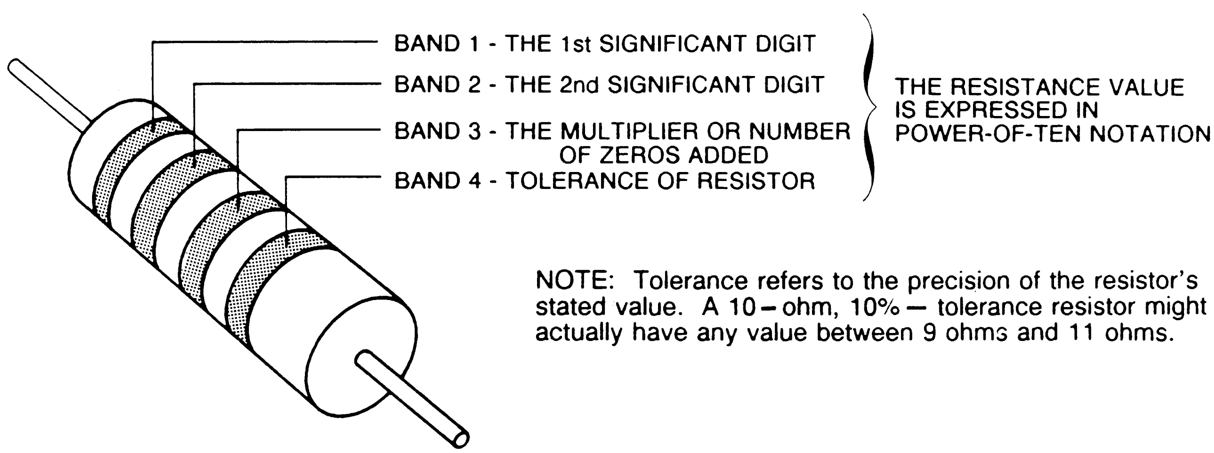


Fig. 1 Resistor color-coding.

Table 1 shows the colors used in resistor coding and explains what each color means with respect to the band.

TABLE 1. RESISTOR COLOR CODES

	Value in Band			
	Band 1	Band 2	Band 3	Band 4
Color	First Significant Digit	Second Significant Digit	Multiplier	Tolerance
Black	0	0	10^0	—
Brown	1	1	10^1	—
Red	2	2	10^2	—
Orange	3	3	10^3	—
Yellow	4	4	10^4	—
Green	5	5	10^5	—
Blue	6	6	10^6	—
Violet	7	7	10^7	—
Gray	8	8	10^8	—
White	9	9	10^9	—
Gold	—	—	10^{-1} or $\frac{1}{10}$	±5%
Silver	—	—	10^{-2} or $\frac{1}{100}$	±10%
No Color	—	—	—	±20%

For example, suppose the four color bands are *orange*, *green*, *red*, and *silver*:

Band 1:	orange	→	*First* digit is 3.
Band 2:	green	→	*Second* digit is 5.
Band 3:	red	→	*Multiplier* is 10^2.
Band 4:	silver	→	*Tolerance* is 10%.

The value of this resistor is 35×10^2 or 3500 Ω ±10% of 3500. (The symbol "±" means "plus or minus.") A tolerance of ±10% means the manufacturer "guarantees" the resistance value to be 3500 Ω ± 350 Ω. So the true value can be anywhere between 3150 Ω and 3850 Ω. Study Table 1. Refer to it as needed when you have to determine resistor values from the color code.

Table 2 summarizes the units used with the physical quantities given in Equations 1 and 2. The table is meant to help you learn the units. Study Table 2.

TABLE 2. ELECTRICAL RESISTANCE UNITS

		System of Units: English	SI
Equation 1: $R_E = \frac{\Delta V}{I}$	R_E	—	ohms (Ω)
	ΔV	—	volts (V)
	I	—	amperes (A)
Equation 2: $R_E = \frac{\rho \ell}{A}$	R_E	—	ohms (Ω)
	ρ	ohm·in.	ohm·cm
	ℓ	in.	cm
	A	in^2	cm^2

NOTE: The values of resistivity in Table 3 are values around room temperature. Of course, resistivity is a function of temperature, increasing with temperature for metals, and generally decreasing with temperature for semiconductors. The values given for the semiconductors in Table 3 are only order-of-magnitude values, since resistivity of semiconductors is extremely sensitive to temperature. An increase in temperature in a semiconductor causes increased electron (or hole) collisions with atoms and so, *increased* resistance--just as in metals. However, this effect is usually overcome in a semiconductor by an opposing effect that *reduces* resistance: the release of many more valence electrons (or holes) from the semiconductor and impurity atoms to provide enhanced conduction.

REVIEW OF UNITS

a. volts

b. amperes

c. ohms

d. volts/amps

e. may

f. may

REVIEW OF COLOR CODE

a. first

b. second

c. number of zeros

d. tolerance

e. 150 Ω

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1

Problem 1: The solution is given in the text of the lab.

Values of resistivity (ρ) are given for some common conductors in Table 3. Remember: 1 microhm·centimeter = 10^{-6} Ω·cm.

TABLE 3. RESISTIVITY OF COMMON CONDUCTORS

Material	Resistivity ($\mu\Omega$·cm)	Material	Resistivity ($\mu\Omega$·cm)
Glass	10^{20}	Lead	22
Silicon *	10^{6}	Aluminum	2.824
Germanium *	10^{6}	Gold	2.44
Carbon	10^{5}	Copper	1.724
Nichrome Wire	112	Silver	1.59

* The resistivity of semiconductors—like silicon and germanium—is very sensitive to temperature.

LET'S REVIEW UNITS!

Before starting your Practice Exercises, answer the following questions on units for electrical resistance. Complete the sentences with the correct word or words.

a. An electrical voltage in SI units is measured in ______ (volts, amperes, ohms).

b. An electrical current in SI units is measured in ______ (volts, amperes, ohms).

c. An electrical resistance in SI units is measured in ______ (volts, amperes, ohms).

d. Another expression of units for ohms is ______ (volts/amps, volts·amps).

e. Resistivity of electrical conductors ______ (may, may not) be measured in ohm·cm or ohm·inches.

f. The cross-sectional area of electric conductors ______ (may, may not) be measured in cm^2 or in^2.

LET'S REVIEW RESISTOR COLOR CODE!

a. The first color band tells the ______ (first, second) significant digit of the resistor value.

b. The second color band tells the ______ (first, second) significant digit of the resistor value.

c. The third color band tells the ______ (third significant digit, number of zeros) added to the resistor value.

d. The fourth color band is a ______ (multiplier, tolerance) band.

e. If the first three colors are brown-green-brown, the resistance (according to the color code) is ______ (15, 150, 1.5).

PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: Given: $R_E = {}^{\Delta V}\!/_{I}$ (Equation 1).

Find: ΔV. (Rearrange the equation and isolate ΔV.)

Solution: Step 1: First, write down the equation.

$$R_E = \frac{\Delta V}{I}$$

Step 2: Multiply both sides by I.

$$R_E \times I = \left(\frac{\Delta V}{\cancel{I}}\right) \times \cancel{I} \qquad \text{(Cancel I's on right side.)}$$

Problem 2: Solution given in text of lab.

Problem 3:

Step 1: Isolate "I" by rearranging Equation 1. First write down the equation.

$$R_E = \frac{(\Delta V)}{I}$$

Step 2: Use the form of the equation found in Problem 1.

$$(\Delta V) = R_E \times I$$

Step 3: Divide both sides by "R_E."

$$\frac{(\Delta V)}{R_E} = \frac{R_E \times I}{R_E} \quad \text{(Cancel "}R_E\text{" on right side.)}$$

Step 4: Rewrite equation without canceled "R_E's."

$$\frac{(\Delta V)}{R_E} = I$$

Step 5: Reverse the order of the equation.

$$I = \frac{(\Delta V)}{R_E}$$

The problem has been solved. Equation 1 has been rearranged, and "I" has been isolated.

Problem 4:

Step 1: Isolate "ℓ" by rearranging Equation 2. First write down the equation.

$$R_E = \frac{\rho \ell}{A}$$

Step 2: Multiply both sides by "A/ρ," which is the inverse of "ρ/A."

$$R_E \times \frac{A}{\rho} = \frac{\rho \ell}{A} \times \frac{A}{\rho} \quad \text{(Cancel "A's" and "}\rho\text{'s" on right side.)}$$

Step 3: Rewrite equation without "A's" and "ρ's" that canceled out.

$$R_E \times \frac{A}{\rho} = \ell \qquad \text{(The } \ell \text{ is isolated.)}$$

Step 4: Reverse the order of the equation with "ℓ" on the left.

$$\ell = R_E \times \frac{A}{\rho}$$

The problem has been solved. Equation 2 has been rearranged, and "ℓ" has been isolated.

(See page T-79c for Problem 5.)

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1, Continued

Problem 5:

Step 1: Isolate "A" by rearranging the equation. First write down the given equation.

$$R_E = \frac{\rho \ell}{A}$$

Step 2: Multiply both sides by "A" (to get "A" in the numerator) and cancel appropriately.

$$R_E \times A = \frac{\rho \ell}{A} \times A$$

Step 3: Divide both sides of the equation obtained in Step 2 by "R_E." Cancel the "R_E's" on the left.

$$\frac{R_E \times A}{R_E} = \frac{\rho \ell}{R_E}$$

Step 4: Write down the final result.

$$A = \frac{\rho \times \ell}{R_E}.$$ ("A" has been isolated.)

Step 3: Rewrite the equation without the I's that were canceled.

$R_E \times I = \Delta V$ (ΔV is isolated.)

Step 4: Reverse the order of the equation with (ΔV) on the left side.

$\Delta V = R_E \times I$

The problem has been solved. The equation, $R_E = {}^{\Delta V}\!/_{I}$, has been rearranged. ΔV has been isolated.

Problem 2: Given: $R_E = {}^{\rho\ell}\!/_{A}$ (Equation 2).

Find: ρ. (Rearrange the equation and isolate ρ.)

Solution: (***Hint:*** Examination of the right side of the equation indicates that multiplying both sides by the inverse of ${}^{\ell}\!/_{A}$, which is ${}^{A}\!/_{\ell}$, will isolate ρ on the right side when terms cancel out.)

Step 1: First, write down the equation.

$$R_E = \frac{\rho\ell}{A}$$

Step 2: Multiply both sides by ${}^{A}\!/_{\ell}$ (the inverse of ${}^{\ell}\!/_{A}$).

$$R_E \times \left(\frac{A}{\ell}\right) = \left(\frac{\rho\cancel{\ell}}{\cancel{A}}\right) \times \left(\frac{\cancel{A}}{\cancel{\ell}}\right)$$ (Cancel A's and ℓ's on the right side.)

Step 3: Rewrite the equation without the A's and ℓ's that you canceled.

$$R_E \times \left(\frac{A}{\ell}\right) = \rho$$ (The "ρ" is isolated.)

Step 4: Reverse the order of the equation with ρ on the left side.

$$\rho = R_E \times \left(\frac{A}{\ell}\right)$$

The problem has been solved. Equation 2 has been rearranged, and ρ has been isolated.

Problem 3: Given: $R_E = {}^{\Delta V}\!/_{I}$ (Equation 1).

Find: I.

Solution: (Rearrange the equation and isolate I.)

Problem 4: Given: $R_E = {}^{\rho\ell}\!/_{A}$ (Equation 2).

Find: ℓ.

Solution: (Rearrange the equation and isolate ℓ. ***Hint:*** Multiply both sides by ${}^{A}\!/_{\rho}$.)

Problem 5: Given: $R_E = {}^{\rho\ell}\!/_{A}$ (Equation 2).

Find: A.

Solution: (Rearrange the equation and isolate A. ***Hint:*** First multiply both sides by A. This gets A in the numerator. Then divide both sides by R_E.)

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2

Problem 6: Use Equation 1.

$R_E = \frac{\Delta V}{I}$ where: $(\Delta V) = 6\ V$

$R_E = \frac{6\ V}{3\ A}$

$R_E = 2\ \Omega.$

Problem 7: Equation 1, rearranged to isolate "ΔV," is $\Delta V = R_E \times I$.

$\Delta V = R_E \times I$ where: $R_E = 16.67\ \Omega$

$\Delta V = 16.67\ \Omega \times 6\ A$ $I = 6\ A$

$\Delta V = 100\ V.$

Problem 8: Equation 1, rearranged to isolate "I," is $I = \Delta V/R_E$.

$I = \frac{\Delta V}{R_E}$ where: $\Delta V = 50\ V$

$R_E = 20\ \Omega$

$I = \frac{50\ V}{20\ \Omega}$

$I = 2.5\ A.$

Problem 9: Use the equation, $R = \frac{\rho \times \ell}{A}$.

where: $\rho = 1.724 \times 10^{-6}\ \Omega\cdot cm$

$\ell = 120\ cm$

$A = 0.077\ cm^2$

$$R = \frac{1.724 \times 10^{-6}\ \Omega\cdot cm \times 120\ cm}{0.077\ cm^2}$$

$$R = \frac{1.724 \times 10^{-6} \times 120}{0.077}\left(\frac{\Omega\cdot cm \times cm}{cm^2}\right)$$

$$R = 2.687 \times 10^{-3}\ \Omega = \mathbf{0.00269\ \Omega}$$

(See page T-80c for Problem 10.)

SOLUTIONS TO STUDENT CHALLENGE

Problem 10: From Table 1--

Blue: 1st digit is 6.

Black: 2nd digit is 0.

Brown: Multiplier is 10^1 x 10.

Therefore, resistance is 60 x 10, or 600 Ω.

Use Equation 2 in the form obtained when Problem 4 in Activity 1 was solved.

$$\ell = \frac{R_E A}{\rho}$$

where: $R_E = 600\ \Omega$ or $6 \times 10^2\ \Omega$ (from resistor color code)

$A = 0.04\ \text{cm}^2 = 4 \times 10^{-2}\ \text{cm}^2$

$\rho = 112 \times 10^{-6}\ \Omega\cdot\text{cm}$ from Table 3

$$\ell = \frac{6 \times 10^2\ \Omega \times 4 \times 10^{-2}\ \text{cm}^2}{112 \times 10^{-6}\ \Omega\cdot\text{cm}}$$

$$\ell = \left(\frac{6 \times 4 \times 10^{2-2}}{112 \times 10^{-6}}\right)\left(\frac{\Omega\cdot\text{cm}^2}{\Omega\cdot\text{cm}}\right) = \frac{24}{112} \times 10^{+2-2+6}\ \text{cm}$$

$$\ell = \frac{24}{112} \times 10^6\ \text{cm}$$

$$\ell = 0.21429 \times 10^6\ \text{cm}$$

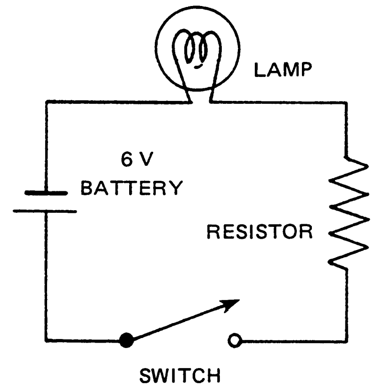
$$\ell = 214{,}290\ \text{cm}.$$

It would take 214,290 cm of nichrome wire to provide a resistance of 600 Ω!

(See page T-80e for Problem 11.)

SOLUTIONS TO STUDENT CHALLENGE, Continued

Problem 11: Schematically, this simple circuit should look like the one at the right.



First, we must find the resistance of the lamp (R_X) by Ohm's law ($R = \Delta V/I$), where $\Delta V = 1.5$ V and $I = 1$ A.
Thus: $R_X = 1.5 \text{ V}/1 \text{ A} = 1.5\ \Omega$.

If we connected the battery directly to the lamp, we would find the current flowing through the lamp to be:

$I = \Delta V/R$ (This equation comes from Ohm's law, $R = \Delta V/I$, by isolating the variable, I.)

$I = 6 \text{ V}/1.5\ \Omega = 4$ A. (This current would burn out the miniature lamp.)

To keep current flow (I) through the lamp limited to 1 A, we must increase the total resistance in the circuit. By using Ohm's law, in the form $R = \Delta V/I$, the total resistance needed is:

$R = 6 \text{ V}/1 \text{ A} = 6\ \Omega$ where: 6 V = battery voltage
1 A = total current in circuit

Therefore, total circuit resistance must be 6 ohms in order to limit the current to 1 ampere. Since we already have $1.5\ \Omega$ in the circuit due to the lamp, we need an additional resistance (R_X) of:

$R_X = 6\ \Omega - 1.5\ \Omega$

$R_X = 4.5\ \Omega$.

If we add a fixed resistance (R_X) equal to $4.5\ \Omega$ to the circuit, we can limit the current through the miniature lamp to 1 ampere.

ACTIVITY 2

Solving Electrical Resistance Problems

Electrical resistance problems in Activity 2 involve Equations 1 and 2 from Activity 1. If the equation isn't in the correct form, first rearrange the equation. Isolate the symbol that represents the physical quantity you want to find. Then solve this equation for the unknown value.

Your final answer should include a correct numerical answer with the proper units. Use Tables 1, 2 and 3 to help you solve the problems and check the units in your answer.

PRACTICE EXERCISES FOR ACTIVITY 2

Problem 6: Given: An electrical circuit with a 6-V battery operates a floodlight that has 3 A of current flowing through it.

Find: The resistance of the floodlight.

Solution:

Problem 7: Given: A current of 6 A flows through an electrical device. Resistance is 16.67 ohms.

Find: The voltage difference causing the current.

Solution:

Problem 8: Given: A voltage difference of 50 V causes current in an electrical circuit. Series resistance in the circuit is 20 ohms.

Find: The current in the circuit.

Solution:

Problem 9: Given: The resistivity of copper wire is $\rho = 1.724 \times 10^{-6}\ \Omega\cdot\text{cm}$. A copper wire has a cross-sectional area of 0.077 cm^2 and is 120 cm long.

Find: The resistance of this piece of wire.

Solution:

Student Challenge

Problem 10: Given: A resistor has the following color code: 1st band = blue, 2nd band = black, 3rd band = brown, 4th band = gold.

Find: The *length* in cm of a piece of nichrome wire that has the same resistance value as the resistor. The wire has a cross-sectional area of 0.04 cm^2.

Solution: (***Hint:*** First use Figure 1 and Table 1 to find the correct resistance for the given resistor.)

Problem 11: Given: A circuit must be made from a 6-V battery, conducting wire, a switch, a miniature lamp base, and a single 1.5-V miniature lamp. When the voltage across the lamp is 1.5 volts, the current through it is 1 ampere. If more than one ampere flows, the lamp's life expectancy is severly reduced.

Find: The amount of resistance needed to limit the voltage across the miniature lamp to 1.5 V and limit the current through it to 1 A.

Solution: (***Hint:*** Draw a schematic of the circuit. Find the lamp resistance. Subtract it from the total resistance in the circuit. The difference is the amount of fixed resistance you must add in series with the lamp to avoid rapid burnout.)