

# Math Skills Laboratory

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Lab 7<sup>M</sup> S<sup>2</sup>

## **MATH ACTIVITY**

**Activity:** *Solving Rotational “Force” Transformer Problems*

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## **MATH SKILLS LAB OBJECTIVES**

*When you complete these activities, you should be able to do the following:*

- 1. Solve and interpret rotational force transformer problems for mechanical advantage and efficiency.*
  - 2. Distinguish between force, torque and speed mechanical advantages.*
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## **LEARNING PATH**

- 1. Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
  - 2. Work the problems.*
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## **ACTIVITY**

### **Solving Rotational ‘Force’ Transformer Problems**

Lathes, bandsaws, drill presses, mills and shapers are machine tools. These tools are more useful when their operating speed can be changed.

A common way to change the operating speed of a rotational mechanical device involves using a “stepped-cone” pulley. This pulley system is shown in Figure 1. The term, “stepped-cone pulley,” means that the pulley has a cone shape and has different-diameter pulleys (steps) machined into each complete pulley.

If the pulleys are arranged as shown in Figure 1, the same belt can be connected to corresponding pulley steps. Moving the belt from one pair of pulley steps to another changes the operating speed of the device.

The ratio of circumferences (distance around) of the corresponding pulleys is one thing that determines the speed of the driven shaft. The rpm of the drive pulley also determines the operating speed of the device.

During this lab, you’ll learn to find the mechanical advantage of different types of rotational mechanical devices. You’ll solve technical problems that involve wheel-and-axle force transformers, belt-drive systems and friction-drive systems.

You should understand ratio and proportion before working these problems. You must also know how to rearrange simple formulas. Also, you should make drawings for each problem. Label forces, torques, radii, angular speeds (and so on) for each problem. This will help you solve the problems correctly.

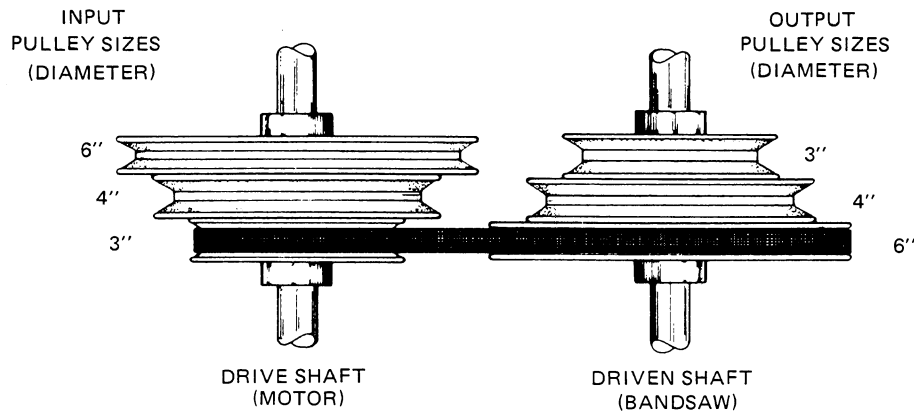
**Problem 1:** Given: A stepped-cone drive pulley is mounted on a motor shaft that turns at 1750 rpm (183 rad/sec). The driven stepped-cone pulley is mounted on the driven shaft of a bandsaw. Motor shaft torque is 4 lb·ft under full load. Figure 1 shows the diameter of each step on the pulley. The equations for ideal mechanical advantage are:

$$\text{IMA} = \frac{r_o}{r_i} = \frac{T_o}{T_i} = \frac{\omega_i}{\omega_o}$$

The conditions in Problem 1:

- (1) drive belt placed on the 4-inch drive pulley and 4-inch driven pulley.
- (2) drive belt placed on the 3-inch drive pulley and 6-inch driven pulley.
- (3) drive belt placed on the 6-inch drive pulley and 3-inch driven pulley.

**Note:** Assume ideal efficiency in the system.



**Fig. 1** Stepped-cone pulleys.

- Find:
- a. The **angular speed** of the bandsaw shaft with conditions (1), (2) and (3) above.
  - b. The bandsaw shaft torque with conditions (2) and (3) above.
  - c. The **ideal mechanical advantage** of the belt-drive system with conditions (2) and (3) above.

Solution:

Business and industrial buildings have doorways through which bulk materials and equipment can be carried in and out. This doorway is called a "loading dock."

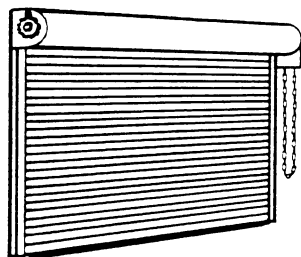
Frequently, the loading dock has a "roll-up" door similar to the one shown in Figure 2. When this type of door opens, it rolls up around a shaft that spans the doorway opening. The shaft around which the door wraps is actually an axle. This axle is driven by a wheel. The wheel is actually a sprocket. And the sprocket is driven by a chain.

Pulling on the chain causes the sprocket to rotate. This causes the axle to rotate. The direction in which the chain is pulled determines whether the door wraps around the axle and opens, or unwraps from the axle and closes.

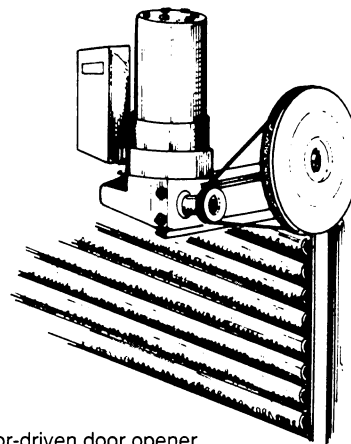
**Problem 2:** Given: A sheet-metal roll-up door weighs 400 lb. The chain-operated input wheel (sprocket) is 12 inches in diameter. The shaft (axle) the door cable wraps around is 1.5 inches in diameter. An input force of 50 pounds is required to open or close the door. The mechanical advantage of a wheel-and-axle force transformer can be determined with the following relationships.

$$\text{IMA} = \frac{r_i}{r_o} \text{ and } \text{AMA} = \frac{F_o}{F_i}$$

Assume ideal efficiency (no friction) for the door mechanism.



a. Hand-operated chain opener



b. Motor-driven door opener

**Fig. 2** Roll-up type door.

- Find:
- The ideal and actual mechanical advantages of the wheel and axle used with a sheet-metal roll-up door. Are the ideal and actual mechanical advantages equal?
  - The distance the chain must be pulled to raise a sheet-metal door 5 feet.

Solution:

**Problem 3:** Given: The sheet-metal door in Problem 2 is replaced with an aluminum roll-up door that weighs 250 pounds.

- Find:
- The mechanical advantage needed to raise the aluminum door if the input force remains 50 pounds. Assume ideal efficiency.
  - The distance the aluminum door will rise when the chain is pulled 5 feet.

Solution:

**Problem 4:** Given: The conditions in Problem 3.

- Find: The minimum wheel (sprocket) diameter that can be used with the aluminum door and provide a mechanical advantage of 5 if the axle diameter remains 1.5 inches.

**Note:** The ratio of the diameters is the same as the ratio of the radii.

Solution:

### Student Challenge

When you mix asphalt, you must dry the gravel used in the mix. Excess water must be removed from the gravel. This makes good adhesion between the asphalt and gravel possible.

Figure 3 shows one type of gravel-drying machine. It operates as a friction-drive rotational transformer. The equations that apply are the same as those in Table 7-2.

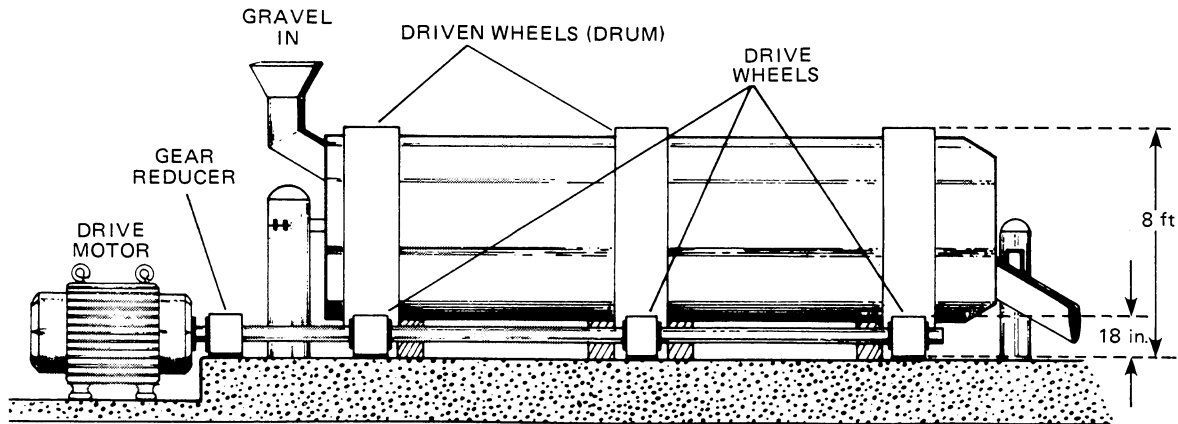


Fig. 3 Gravel dryer.

**Problem 5:** Given: The dryer drum is 8 feet in diameter. The drum rotates at 99 rpm. The rubber drive wheels are 18 inches in diameter. The drive motor rotates at 1500 rpm. The motor drive is coupled to a gear-type speed reducer. The speed reducer is coupled to the input shaft of the drive wheels.

- Find:
- The speed (rpm) of the drive wheels.
  - If the motor operates at 1050 rpm under load, what's the speed reduction ( $\omega_i$  to  $\omega_o$ ) of the gear reducer?

Solution:

In industry, most electric motors run at shaft speeds of 1050 rpm or 1725 rpm under full load. To use these motor outputs, most gear-train drive systems are made to reduce—rather than increase—speed. Mechanical devices that do this are called “speed reducers.” If a speed reducer links directly to the motor housing, it's called a “motorized speed reducer” or a “gear motor.”

**Problem 6:** Given: A gear motor develops 2000 lb·ft of motor torque. It operates with an output shaft speed of 350 rpm. Input motor speed is 1050 rpm.

- Find:
- The speed reduction ratio of the gear motor.
  - The torque mechanical advantage of the gear motor.
  - The output shaft torque.

Solution:

**Problem 7:** Given: A gear motor has a gear train. The gear train has a 20-tooth gear on the motor drive shaft. It also has a 40-tooth idler gear. There's an 80-tooth gear on the output shaft.

- Find:
- The speed reduction of the gear motor.
  - The speed of the motor if the output shaft turns 450 rpm.
  - The motor torque if output shaft torque is 2000 lb·ft.

Solution:

In industry, machines are more versatile when they can operate at various speeds. You know how stepped-cone pulleys can be used to change the operating speed of a machine. When gears are used to change the speed of a machine, the distance between the driving shaft and the driven shaft is fixed. (It stays the same.)

However, the speed of a gear-driven machine can be changed by installing matched sets of drive and driven gears on the shaft. This is because the gears of these “gear sets” have different diameters. Therefore, different gear sizes can change the shaft speed ratio.

For example, the drive and driven shafts on a machine are to be 4.5 inches apart (center to center). A 3-inch-diameter drive gear and a 6-inch-diameter driven gear would work as one matched set for the machine. A 4-inch-diameter drive gear and a 5-inch-diameter driven gear could be another matched set.

If you add the diameter of each gear in a matched set, you’ll notice that the total in this example always equals 9 inches. Why do gears in each matched set have this common total? Because the drive and driven shafts are 4.5 inches apart. And the gears must meet in the middle to mesh.

**Problem 8:** Given: A matched-gear set on a lathe produces an output shaft speed of 20 rpm when the input shaft speed is 80 rpm. The drive-gear diameter is 4 inches. The driven-gear diameter is 16 inches.

- Find:
- Ideal mechanical advantage (IMA) of the gear set.
  - Input torque when the output torque is 20 lb·ft.
  - Ideal mechanical advantage (IMA) if the input and output gears were exchanged.
  - Output shaft speed if the drive and driven gears were each 10 inches in diameter.

Solution: (**Hint:** Refer to Table 7-3 for formulas to use.)

**Problem 9:** Given: Matched-gear sets are used to speed up or slow down feed rollers on a printing press. Two gears are used to drive a roller. A 16-tooth gear is on the input shaft. A 24-tooth gear is on the driven-roller shaft.

- Find:
- Ideal mechanical advantage of the gear set.
  - To have the lower output torque on the feed rollers, which gear of a 16-tooth and 24-tooth matched-gear set would you use as the drive gear?
  - If  $IMA = 1.6$ , how many teeth would it take on an input gear if the output gear had 32 teeth?
  - The direction the roller turns if the input shaft rotates clockwise and there’s an idler gear between the drive and driven gears.

Solution: (**Hint:** Refer to Table 7-3 for formulas to use.)

**Problem 10:** Given: A worm-gear drive is made so that one full turn of the worm gear (input) turns the other gear by one tooth. The pitch of the worm gear is 1 inch.

- Find:
- Mechanical advantage of this gear system, if the worm gear is meshed with a 26-tooth output gear.
  - Revolutions made by the output shaft when the worm-gear shaft makes 10 revolutions.
  - Distance a point on the rim of the output gear moves during the 10 revolutions described in b above.
  - The output torque, if the input torque on the worm gear is 5 lb·ft.

Solution: