Math Lab 12 MS 2

Solving Problems That Involve Nuclear Radiation

For best results, print this document front-to-back and place it in a three-ring binder. Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach your students to solve problems that involve nuclear radiation.
2. Teach your students to use the Periodic Table to find elements, atomic number and mass number of isotopes.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure your students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to this class. (How much you accomplish will depend on the math skills your students already have.) Summarize the explanatory material for the activity: "Solving Problems That Involve Nuclear Radiation." Then have students complete the Practice Exercises given at the end of the activity.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 12*3, "Nuclear Radiation." This lab requires only one set of apparatus. **You will conduct a "demonstration" lab for your students, using a small demonstration set such as that offered by CENCO--"Radioactivity Demonstrator Set" (cat. no. 71201-003). Be sure you have the apparatus. Read the Teacher's Guide provided by CENCO before you attempt the demonstration lab.**

Math Skills Laboratory

Lab 12 MS 2

MATH ACTIVITY

Activity: Solving Problems That Involve Nuclear Radiation

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. *Identify isotopes of an element by their atomic number and mass number.*
2. *Use a periodic table to find the average mass number of an element.*
3. *Find the number of neutrons in an isotope from the atomic number and mass number.*
4. *Express the mass of atoms in kilograms or atomic mass units (u).*
5. *Convert atomic mass to an energy equivalent using Einstein's equation, $E = mc^2$.*

LEARNING PATH

1. *Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
2. *Work through Examples 1 and 2.*
3. *Work the problems.*

ACTIVITY

Solving Problems That Involve Nuclear Radiation

MATERIALS

For this activity, you'll need pencil, paper and a calculator.

LET'S REVIEW!

From your text and classroom activity, you learned that each chemical element in nature contains isotopes represented by symbols. These symbols are presented in a form that looks like:

$$_{Z}(\text{element})^{A}$$

where: element = the ***chemical symbol*** for each element
A = the ***mass number*** for each isotope (nuclide)
Z = the ***atomic number*** for each isotope (nuclide)

You also learned the following definitions:

- The ***atomic number*** (Z) for a given isotope equals the number of protons in the nucleus of the isotope.
- The ***mass number*** (A) for a given isotope equals the total number of protons (Z) and neutrons (N) in the nucleus of the isotope. In equation form, $A = Z + N$.
- Each of the different elements listed in an organized table of the elements—called the "Periodic Table"—has a different atomic number. See Table 1. For lighter elements, there's a ratio of about one neutron to one proton in the nucleus. For heavier elements, this ratio may be as many as one and one-half neutrons to every proton.
- In all electrically neutral atoms, there's one electron outside the nucleus for each proton inside the nucleus.
- The term ***isotope*** refers to atoms of the ***same element*** that have different numbers of neutrons in their nuclei but the same number of protons. Thus, their neutron numbers—and their mass numbers—are different. For instance, naturally occurring carbon is a mixture of two isotopes—$_6C^{12}$ and $_6C^{13}$. Carbon-12 ($_6C^{12}$) has six protons and six neutrons. Carbon-13 ($_6C^{13}$) has six protons and seven neutrons.
- Some elements have a large number of isotopes. The percentage of each type of isotope in a sample determines the ***average*** atomic mass for each element.

In the partial Periodic Table shown in Table 1, note that carbon (C) has an average atomic mass of 12.01. This is an average mass for natural carbon which contains a mixture of isotopes $_6C^{12}$ and $_6C^{13}$.

If you look closely, you'll notice that these average mass numbers (non-integers like 12.01) mean that many of the elements have isotopes. Note also that the atomic number (Z) increases by one as you move from left to right through the elements in a row.

NOTE: Table 2 has all the values needed to work the problems in this Math Lab.

TABLE 1
Periodic Table (Cutaway)

AVERAGE ATOMIC MASS → 10.81
SYMBOL FOR ELEMENT → B
ATOMIC NUMBER → 5

		V	VI	VII	
					4.00 He 2
10.81 B 5	12.01 C 6	14.01 N 7	16.00 O 8	19.00 F 9	20.18 Ne 10
26.98 Al 13	28.09 Si 14	30.98 P 15	32.07 S 16	35.46 Cl 17	39.94 Ar 18
72	72.59 Ge 32	74.92 As 33	78.96 Se 34	79.91 Br 35	83.80 Kr 36
			127.61		

The following definitions, along with Table 2, will help you work the problems in this Math Skills Lab.

- A ***nuclide*** is a type of atom characterized by its atomic number (Z) and its mass number (A).
- ***Isotopes*** are nuclides that have the same atomic number (Z) but different neutron numbers (N) and mass numbers (A)—and hence, different atomic masses.
- One ***atomic mass unit*** (u) is exactly equal to one-twelfth the mass of the most abundant form of the carbon isotope $_6C^{12}$. Its value in units of kilograms is 1.6605×10^{-27} kg.
- To change mass into energy, use the equation, $\boldsymbol{E = mc^2}$.

Table 2 lists the rest mass and energy equivalent of the more common nuclear particles.

TABLE 2

Rest Mass of Particles			
Particle	*Mass in kg*	*Mass in amu*	*Energy equivalent*
Electron	9.108×10^{-31}	5.485×10^{-4} u	8.197×10^{-14} J
Proton	1.673×10^{-27}	1.0075 u	1.5057×10^{-10} J
Neutron	1.675×10^{-27}	1.0087 u	1.5075×10^{-10} J
α-particle	6.646×10^{-27}	4.0026 u	5.9814×10^{-10} J
Useful Constants			
Speed of Light: $c = 3 \times 10^8 \frac{\text{m}}{\text{sec}}$ $1 \text{ amu} = 1 \text{ u} = 1.6605 \times 10^{-27}$ kg			

Note: You should recognize that "amu" and "u" are the same. Some text materials use the older symbol (amu); newer texts use the symbol, "u."

Use the data in Table 2 to work typical problems that will help you understand nuclear radiation.

Begin by reviewing the notation used to identify nuclides (the nuclei of isotopes). Be sure you understand how this notation gives you information about the number of neutrons and protons in the nucleus of an atom. Example 1 reminds you how this is done.

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Example 1: *Atomic Numbers and Mass Numbers of Different Isotopes*

Given: Four examples of isotopes with their atomic numbers and mass numbers expressed in nuclide symbol notation.

(1) Hydrogen $_{1}H^{1}$
(2) Carbon $_{6}C^{12}$
(3) Thorium $_{90}Th^{234}$
(4) Uranium $_{92}U^{235}$

Find: a. The atomic number (Z) for each isotope from the general form—

$$_{Z}(\text{element})^{A}$$

b. The mass number (A) for each isotope from the general form—

$$_{Z}(\text{element})^{A}$$

c. The number of neutrons in the nucleus of each isotope from the equation, $A = Z + N$.

Solution: From the general form—

$$_{Z}(\text{element})^{A}$$

—identify the "Z" subscript values as:

a. (1) Hydrogen ($_{1}H^{1}$): $Z = 1$
(2) Carbon ($_{6}C^{12}$) : $Z = 6$
(3) Thorium ($_{90}Th^{234}$): $Z = 90$
(4) Uranium ($_{92}U^{235}$): $Z = 92$

b. From the general form—

$$_{Z}(\text{element})^{A}$$

—identify the" A" superscript values as:

(1) Hydrogen ($_{1}H^{1}$): $A = 1$
(2) Carbon ($_{6}C^{12}$): $A = 12$
(3) Thorium ($_{90}Th^{234}$): $A = 234$
(4) Uranium ($_{92}U^{235}$): $A = 235$

c. From the equation, $A = Z + N$, solve for the neutron number (N), using appropriate values from Parts "a" and "b" above:

(1) Hydrogen: $N = A - Z = 1 - 1 = 0$
(2) Carbon: $N = 12 - 6 = 6$
(3) Thorium: $N = 234 - 90 = 144$
(4) Uranium: $N = 235 - 92 = 143$

Note: It's obvious that light elements, such as hydrogen, have a small number of protons and neutrons. Heavy elements, such as uranium, have a large number. For example, hydrogen has one proton and no neutrons. Uranium-235 has 92 protons and 143 neutrons.

SOLUTIONS TO PRACTICE EXERCISES

Problem 1: From the Periodic Table--

a. ${}_{2}He^{4}$ d. ${}_{13}Al^{27}$

b. ${}_{6}C^{12}$ e. ${}_{14}Si^{28}$

c. ${}_{8}O^{16}$ f. ${}_{34}Se^{79}$

For example, the table shows that the **average** atomic mass for carbon (C) is 12.01. The nearest integer is 12. So the nuclide symbol is ${}_{6}C^{12}$. Consider another example. For selenium (Se), the Table gives 78.96 for the average atomic mass. The nearest integer is 79. So the nuclide symbol is ${}_{34}Se^{79}$.

Problem 2: Since A = Z + N, solve for "N" to get the formula, N = A - Z.

a. ${}_{2}He^{4}$	N = A - Z = 4 - 2 = 2
b. ${}_{6}C^{12}$	N = 12 - 6 = 6
c. ${}_{8}O^{16}$	N = 16 - 8 = 8
d. ${}_{13}Al^{27}$	N = 27 - 13 = 14
e. ${}_{14}Si^{28}$	N = 28 - 14 = 14
f. ${}_{34}Se^{79}$	N = 79 - 34 = 45

Now let's review how to convert mass to energy. Example 2 shows how to do this.

Example 2: ***Converting Mass to Energy***

Given: Einstein's energy equation and the relationships in Table 2.

Find: The energy corresponding to one atomic mass unit (u).

Solution: $E = mc^2$ where: $m = 1\ u = 1.6605 \times 10^{-27}$ kg
$c = 3 \times 10^8$ m/sec

$$E = (1.6605 \times 10^{-27}\ \text{kg})(3 \times 10^8\ \text{m/sec})^2$$

$$E = (1.6605 \times 10^{-27}\ \text{kg}) \times (9 \times 10^{16} \frac{\text{m}^2}{\text{sec}^2})$$

$$E = (1.6605 \times 9 \times 10^{-27+16}) \frac{\text{kg}\cdot\text{m}^2}{\text{sec}^2}$$

$$E = 1.494 \times 10^{-10}\ \text{kg} \frac{\text{m}^2}{\text{sec}^2}$$

(Recall from Unit 5, "Energy," that 1 $\text{kg}\cdot\text{m}^2/\text{sec}^2 = 1$ J.)
$E = 1.494 \times 10^{-10}$ J.

Therefore, 1 atomic mass unit—only 1.6605×10^{-27} kg—is equal to 1.494×10^{-10} J of energy.

Now solve the following problems.

Problem 1: Given: Table 1 (portion of the Periodic Table) and the atomic numbers of the following elements:

Element	*Atomic Number (Z)*	*Element*	*Atomic Number (Z)*
a. Helium (He)	2	d. Aluminum (Al)	13
b. Carbon (C)	6	e. Silicon (Si)	14
c. Oxygen (O)	8	f. Selenium (Se)	34

Find: The correct symbolic form (nuclide symbol) for each element where "A" is the ***integer nearest the average atomic mass number*** given in Table 1.

Solution: Write answers in the form $_Z(\text{element})^A$ for "a" through "f."

Problem 2: Given: The results of Problem 1.

Find: The correct number of neutrons in the nucleus of the isotope for the elements in "a" through "f."

Solution: (***Hint:*** $A = Z + N$.)

SOLUTIONS TO PRACTICE EXERCISES, Continued

Problem 3: $_{88}RA^{226} \longrightarrow {}_{Z}Rn^{A} + {}_{2}He^{4}$ + kinetic energy

To balance the equation, find A and Z such that 226 = A + 4 and 88 = Z + 2.

88 = Z + 2 or Z = 88 - 2 = 86

226 = A + 4 or A = 226 - 4 = 222

Therefore, the equation is $_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4}$.

STUDENT CHALLENGE PROBLEM

Problem 4: From the masses given calculate "mass before."

$_{92}U^{235}$	=	235.0439 u	
$_{0}n^{1}$	=	1.008665 u	
		236.0526 u	(mass before)

Now calculate "mass after."

$_{56}Ba^{138}$	=	137.9050 u	
$_{36}Kn^{95}$	=	94.9000 u	
$3(_{0}n^{1})$	=	3.0261 u	
		235.8311 u	(mass after)

The missing mass = mass before - mass after

missing mass = 236.0526 u - 235.8311 u

missing mass = 0.2215 u.

SOLUTIONS TO PRACTICE EXERCISES, Continued

STUDENT CHALLENGE PROBLEM

Problem 5:

a. Missing Mass = 0.2215 u (from Problem 4)
From Example 2, 1 atomic mass unit (u) = 1.494×10^{-10} J.
So the energy released = the missing mass that is converted to energy.
To convert "u" to joules, use the relationship,
$1u = 1.494 \times 10^{-10}$ J.

$$\text{Energy Released} = 0.2215\ u \times \frac{1.494 \times 10^{-10} J}{1u}$$

$$\text{Energy Released} = \frac{0.2215 \times 1.494 \times 10^{-10}}{1}\ \frac{\cancel{u} \cdot J}{\cancel{u}}$$

Energy Released = 33.09×10^{-12} J

Energy Released = 33.09 picojoules;
(1 picojoule = 1×10^{-12} J).

b. To find the energy released by the fission of 1 kg of U-235, multiply the energy released per atom by the number of atoms per kg.

$$\text{Number of } U^{235} \text{ atoms per kg} = \frac{6.022 \times 10^{26} \text{ atoms}/\cancel{\text{kgmole}}}{235 \text{ kg}/\cancel{\text{kgmole}}}$$

Number of U^{235} atoms per kg = 2.563×10^{24} atoms/kg

So,

Energy per kg U^{235} = (Energy per U^{235} atom) x (Number of U^{235} atoms per kg)

Energy per kg U^{235} = $(33.09 \times 10^{-12}$ J/atom) x $(2.563 \times 10^{24}$ atoms/kg)

$$\text{Energy per kg } U^{235} = (33.09 \times 2.563) \times (10^{-12+24}) \times \left(\frac{J \cdot \cancel{\text{atoms}}}{\cancel{\text{atoms}} \cdot kg}\right)$$

Energy per kg U^{235} = 84.81×10^{12} J/kg

Energy per kg U^{235} = 8.481×10^{13} J/kg

STUDENT CHALLENGE PROBLEM

Problem 6: Use Einstein's energy equation, $E = mc^2$, and then the work equation $W = F \times D$. The potential energy (E_p) converted to work to lift the load is:

$E_p = W = E = mc^2$ where: m = mass of the ink = 3×10^{-9} kg

$c = 3 \times 10^8$ m/sec

$$W = mc^2 = (3 \times 10^{-9}\ \text{kg})\left(3 \times 10^8\ \frac{\text{m}}{\text{sec}}\right)^2$$

$$W = (3 \times 10^{-9}\ \text{kg})\left(9 \times 10^{16}\ \frac{\text{m}^2}{\text{sec}^2}\right)$$

$$W = 3 \times 9 \times 10^{-9+16}\ \text{kg}\cdot\frac{\text{m}^2}{\text{sec}^2};\quad 1\ \text{kg}\ \frac{\text{m}^2}{\text{sec}^2} = 1\ \text{N}\cdot\text{m or } 1\ \text{J}$$

$W = 27 \times 10^7$ N•m

$W = F \times D$ (Solve for "D")

$D = W/F$ where: $W = 27 \times 10^7$ N•m

F = load (bus and students) = $m \times g$

F = [mass of bus + students] x gravitational attraction "g"

$F = [5000\ \text{kg} + (30 \times 60\ \text{kg})] \times 9.8\ \text{m/sec}^2$

$F = [5000\ \text{kg} + 1800\ \text{kg}] \times 9.8\ \text{m/sec}^2$

$$F = 6800\ \text{kg} \times 9.8\ \text{m/sec}^2 = 6.664 \times 10^4\ \frac{\text{kg}\cdot\text{m}}{\text{sec}^2};\quad \frac{1\ \text{kg}\cdot\text{m}}{\text{sec}^2} = 1\ \text{N}$$

$$D = \frac{W}{F} = \frac{27 \times 10^7\ \text{N}\cdot\text{m}}{6.664 \times 10^4\ \text{N}} = 4.052 \times 10^3\text{m} = 4.052\ \text{km}$$

D = 4.052 Km (about 2.5 miles) is the distance straight up that the energy from 3 micrograms of ink will raise the busload of students.

Problem 3: Given: An alpha particle is really a helium nucleus. It has the symbol, "$_2He^4$." When radium (Ra) decays by alpha emission, it produces radon (Rn) plus an alpha particle.

The mass number of radium is 226 and the atomic number is 88.

Find: The mass number (A) and atomic number (Z) of radon (Rn) if the decay of radium (Ra) follows the form—

$$_{88}Ra^{226} \rightarrow {}_{Z}Rn^{A} + {}_{2}He^{4} + \text{kinetic energy of moving parts}$$

Solution: ***Note:*** The mass number on the left side must be equal to the sum of the mass numbers on the right side. The atomic number on the left side must equal the sum of the atomic numbers on the right side.

Student Challenge

Problem 4: Given: During a fission process, neutrons are emitted and a large amount of kinetic energy of the moving parts is released. An equation describing this reaction is given for the fission of uranium-235.

$$_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{56}Ba^{138} + {}_{36}Kr^{95} + 3{}_{0}n^{1} + \text{kinetic energy}$$

The masses involved are as follows:

$_{92}U^{235}$ = 235.0439 u $_{36}Kr^{95}$ = 94.9000 u
$_{56}Ba^{138}$ = 137.9050 u $_{0}n^{1}$ = 1.0087 u

Find: The "missing mass" after the reaction in atomic mass units.

Solution: (***Hint:*** See Example 12-F in the text.)

Problem 5: Given: Problem 4 and the missing mass from Problem 4.

Find: a. The energy released in joules during one fission reaction.

b. The energy released if one kg of $_{92}U^{235}$ fissions.

Solution: (***Hint:*** See Example 12-G in the text.)

Problem 6: Given: The ink used to print the words "Principles of Technology" has a mass of about 3 micrograms (3×10^{-6} gm = 3×10^{-9} kg). If the mass of this ink could be entirely converted into energy, there would be enough energy to raise a 5000-kg bus with 30 students—each of mass 60 kg—high into the air.

Find: The distance the load (bus and students) would be raised if the energy from the ink is converted into potential energy to do work in lifting the load.

Solution: Use Einstein's equation, $E = mc^2$, and the linear work equation, $W = F \times D$.

Note: $c = 3 \times 10^8$ m/sec, and $g = 9.8$ m/sec^2.