Math Lab 13 MS 3

Solving Problems That Involve Laser Radiant Power

Solving Problems That Involve Laser Power Density

For best results, print this document front-to-back and place it in a three-ring binder. Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach students how to determine the radiant power and power density of laser light.
2. Teach students the proper units for solutions to problems involving radiant power and power density.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete the activities. Students should have read the discussion material and looked at the examples for each activity before coming to this class. You should summarize the main points in each activity, work an example or two, and have the students complete the Practice Exercises for each activity on their own.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell students to read Lab 13*6, "Prism Experiments."

Math Skills Laboratory

Lab 13 MS3

MATH ACTIVITIES

Activity 1: Solving Problems That Involve Laser Radiant Power

Activity 2: Solving Problems That Involve Laser Power Density

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve for the radiant power and power density output of typical lasers.***
2. ***Give solutions to radiant power and power density problems in proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

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ACTIVITY 1

Solving Problems That Involve Laser Radiant Power

WHAT'S RADIANT POWER?

Radiant power is the amount of radiant energy delivered in a given unit of time. It's a measure of the power of laser light delivered, in units of joules/seconds or watts. The equation for radiant power is expressed as:

$$\text{Power (watts)} = \frac{\text{Energy (joules)}}{\text{Time (seconds)}}$$

From this equation it's evident that the shorter the time the laser acts to deliver the same energy, the greater the power. Example A compares the power produced when the same amount of energy is delivered over two different time intervals.

Example A: ***Comparing Radiant Power Produced by Two Lasers***

Given: Laser A and laser B each deliver 20 joules of energy.
Laser A delivers this energy in 50 microseconds.
Laser B delivers the same energy in 50 nanoseconds.

Find: The radiant power of laser A and laser B.

Solution: Use the basic equation for power.

$$\text{Radiant Power} = \frac{\text{Energy}}{\text{Time}}$$

In symbols,

$$P = \frac{E}{t}$$

where: $E = 20$ joules
$t_A = 50\ \mu\text{sec} = 50 \times 10^{-6}$ sec
$t_B = 50$ nsec $= 50 \times 10^{-9}$ sec

a. $$P_A = \frac{20 \text{ joules}}{50 \times 10^{-6} \text{ sec}} = \left(\frac{20}{50} \times 10^6\right)\frac{\text{J}}{\text{sec}} = 0.4 \times 10^6 \text{ J/sec}$$

$P_A = 4 \times 10^5$ J/sec $= 400$ kilowatts.

b. $$P_B = \frac{20 \text{ joules}}{50 \times 10^{-9} \text{ sec}} = 0.4 \times 10^9 \text{ J/sec}$$

$P_B = 400 \times 10^6$ J/sec $= 400{,}000$ kilowatts.

Note: P_B is 1000 times greater than P_A because laser B delivers the same energy in one one-thousandth of the time that laser A does.

ANSWERS TO PRACTICE EXERCISES

Activity 1:

Problem 1:

$$P = \frac{E}{t} \qquad \text{where:} \quad E = 50\ J, \quad t = 1 \times 10^{-3}\ sec$$

$$P = \frac{50\ J}{1 \times 10^{-3}\ sec} = \left(\frac{50 \times 10^{3}}{1}\right) \frac{J}{sec} = 50 \times 10^{3}\ W$$

$$P = 5.0 \times 10^{4}\ watts$$

Problem 2:

$$P = \frac{E}{t} \qquad \text{where:} \quad P = 1 \times 10^{5}\ W = 1 \times 10^{5}\ J/sec, \quad E = 15\ J$$

Rearrange equation to isolate t.

$$t = \frac{E}{P}$$

$$t = \frac{15\ J}{1 \times 10^{5}\ J/sec} = \left(\frac{15 \times 10^{-5}}{1}\right)\left(\frac{\cancel{J}}{1} \cdot \frac{sec}{\cancel{J}}\right)$$

$$t = 1.5 \times 10^{-4}\ sec = 0.15\ millisec$$

Problem 3:

a. <u>Doubling the delivery time</u>:

$$P = \frac{E}{t} \qquad \text{where:} \quad E = 15\ J, \quad t = 2 \times 1.5 \times 10^{-4}\ sec = 3 \times 10^{-4}\ sec$$

$$P = \frac{15\ J}{3 \times 10^{-4}\ sec} = \left(\frac{15 \times 10^{4}}{3}\right) J/sec$$

$$P = 5 \times 10^{4}\ W$$

b. <u>Doubling the delivery time and the radiant energy in the pulse</u>:

$$P = \frac{E}{t} \qquad \text{where:} \quad E = 2 \times 15\ J = 30\ J, \quad t = 2 \times 1.5 \times 10^{-4}\ sec = 3 \times 10^{-4}\ sec$$

$$P = \frac{30\ J}{3 \times 10^{-4}\ sec} = \left(\frac{30 \times 10^{4}}{3}\right) J/sec$$

$$P = 10 \times 10^{4}\ J/sec = 1 \times 10^{5}\ W$$

Doubling the energy delivered and the time of delivery leaves the power unchanged at 1×10^{5} W, same as that given in Problem 2.

NOTE: **In the text that follows ACTIVITY 2, it is mentioned that a power density of 100 W/cm² is 10,000 times larger than 100 W/m². Have your students show this as follows:**

$$1\,\frac{W}{cm^2} = \left(1\,\frac{W}{\cancel{cm^2}}\right) \times \left(\frac{10{,}000\ \cancel{cm^2}}{1\ m^2}\right) = 10{,}000\,\frac{W}{m^2}$$

At this point, work Problems 1 through 3 in the Practice Exercises.

PRACTICE EXERCISES

Problem 1:	Given:	A laser delivers 50 joules of energy in one millisecond.
	Find:	The radiant power of the laser in watts.
	Solution:	

Problem 2:	Given:	A laser is capable of delivering 10^5 watts of power when 15 joules of radiant energy are generated.
	Find:	The time in seconds for the laser to deliver the energy.
	Solution:	

Problem 3:	Given:	Same conditions as in Problem 2.
	Find:	a. The effect of doubling the delivery time on the power delivered. b. The effect of doubling both the delivery time and the radiant energy on the power delivered.
	Solution:	

ACTIVITY 2

Solving Problems That Involve Laser Power Density

Power density is an expression for the amount of power that's concentrated on a unit area of the target. It's often measured in watts per square centimeter (watts/cm^2).

Knowing the total output of a laser is important. But knowing the area onto which the power is focused—or concentrated—is also important. A power density of 100 watts per square centimeter is much larger than one of 100 watts per square meter. In fact, it's 10,000 times larger!

In optical systems, laser beams often are focused down to very small areas on the target. When they are, the laser beams are so powerful they can cut through steel.

The amount of radiant power per unit area is called "irradiance" (or power density) and is given by the equation:

$$\text{Irradiance (Power Density)} = \frac{\text{Power (watts)}}{\text{Area (cm}^2\text{)}}$$

Let's compare a laser beam's irradiance when the beam is focused (concentrated) on different areas of the target.

Problem 4:

a. First find the power delivered.

$$P = \frac{E}{t} \qquad \text{where: } E = 75\ \text{J},\quad t = 1 \times 10^{-3}\ \text{sec}$$

$$P = \frac{75\ \text{J}}{(1 \times 10^{-3}\ \text{sec})} = \left(\frac{75 \times 10^{3}}{1}\right)\ \text{J/sec} = 7.5 \times 10^{4}\ \text{J/sec}$$

$$P = 7.5 \times 10^{4}\ \text{W}$$

b. Next find the irradiance or power density.

$$\text{Irradiance} = \frac{P}{A} \qquad \text{where: } A = 0.05\ \text{cm}^2,\quad P = 7.5 \times 10^{4}\ \text{W}$$

$$\text{Irradiance} = \frac{7.5 \times 10^{4}\ \text{W}}{0.05\ \text{cm}^2} = \left(\frac{7.5 \times 10^{4}}{0.05}\right) \frac{\text{W}}{\text{cm}^2}$$

$$\text{Irradiance} = 1.5 \times 10^{6}\ \text{W/cm}^2\ (1.5\ \text{megawatts/cm}^2)$$

Problem 5:

$$\text{Irradiance} = \frac{P}{A} \qquad \text{Irradiance} = 250\ \text{W/cm}^2,\quad A = 0.2\ \text{cm}^2$$

Rearrange equation to isolate power (P), then solve for P.

$$P = \text{Irradiance} \times A$$

$$P = (250\ \text{W/cm}^2)(0.2\ \text{cm}^2)$$

$$P = (250 \times 0.2)\ \left(\frac{\text{W}}{\cancel{\text{cm}^2}} \cdot \cancel{\text{cm}^2}\right)$$

$$P = 50\ \text{watts}$$

Therefore 50 watts of beam power strike the 0.2 cm² target area.

Example B: *Laser-beam Concentration and Irradiance*

Given: A laser beam delivers a radiant power equal to one watt. The laser beam is focused onto:

a. a surface area of 1 cm^2
b. a surface area of 0.01 cm^2

Find: The irradiance (power density) of the laser on target for conditions a and b.

Solution:

a. $\text{Irradiance} = \dfrac{\text{Power}}{\text{Area}}$

where: Power = 1 watt
Area = 1 cm^2

$$\text{Irradiance} = \frac{1 \text{ watt}}{1 \text{ cm}^2} = 1 \text{ watt/cm}^2.$$

b. $\text{Irradiance} = \dfrac{\text{Power}}{\text{Area}}$

where: Power = 1 watt
Area = 0.01 cm^2

$$\text{Irradiance} = \frac{1 \text{ watt}}{0.01 \text{ cm}^2} = \left(\frac{1}{0.01}\right)\frac{\text{watt}}{\text{cm}^2} = 100 \text{ watt/cm}^2.$$

Note: The power density in Part a, 1 watt/cm^2, will burn a hole through a sheet of paper. The power density in Part b, 100 watts/cm^2, will burn a hole through aluminum foil. That's because the irradiance on the 0.01-cm^2 target area is 100 times greater than on the 1-cm^2 target area. The same power, 1 watt, has been concentrated from a 1-cm^2 area onto an area of 0.01 cm^2, 1/100 as large.

At this point, work Problems 4 and 5 in the ***Practice Exercises.***

PRACTICE EXERCISES

Problem 4: Given: A laser delivers 75 joules of energy in one millisecond.

Find: The irradiance (power density) when the power is focused by a lens onto a target area of 0.05 cm^2.

Solution:

Problem 5: Given: A laser-beam irradiance of 250 watts/cm^2 is concentrated on a 0.2-cm^2 area of the target material.

Find: The beam power striking the 0.2-cm^2 area.

Solution: