Math Lab 5 MS 4

Reviewing Problems and Units That Deal with Transfer of Heat Between Objects at Different Temperatures

Solving Practical Problems That Deal with Heat Transfer

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach students how to use correct values and units in the thermal energy transfer equation, $H = mc\Delta T$.
2. Teach students how to use the equation, $H = mc\Delta T$, to solve practical problems that involve heat transfer between hot and cold objects.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that your students understand the correct answers.
2. Complete the Math Skills Lab activities.
   a. Go over Activity 1: "Reviewing Problems and Units That Deal With Transfer of Heat Between Objects at Different Temperatures." Go through the solved examples with your students. Review units.
   b. Have your students solve problems using the equation, $H = mc\Delta T$, in Activity 2: "Solving Practical Problems That Deal With Heat Transfer." Help your students arrive at the correct solution.
   c. Assign the **Student Challenge Problems** to those class members who complete the first two activities.
3. Before the class ends, ask your students to read Lab 5T1, "Equilibrium Temperature of a Mixture," for homework.

Math Skills Laboratory

Lab 5 MS 4

MATH ACTIVITIES

Activity 1:** **Reviewing Problems and Units That Deal with Transfer of Heat Between Objects at Different Temperatures

Activity 2:** **Solving Practical Problems That Deal with Heat Transfer

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Substitute appropriate numerical values and units in the heat-transfer equation, $H = mc\Delta T$, to determine a specified unknown quantity.***
2. ***Solve practical problems involving heat transfer between hot and cold objects.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Examples A, B and C.***
3. ***Work the problems in Activity 2.***

ACTIVITY 1

Reviewing Problems and Units That Deal with Transfer of Heat Between Objects at Different Temperatures

MATERIALS

For this activity, you'll need a calculator.

During classroom discussions, you learned that when a hot object of mass m and specific heat c decreases its temperature by ΔT while cooling, it loses an amount of heat energy H where $H = mc\Delta T$. If this heat energy is transferred to another object, the second object gains the same amount of heat energy.

In Activity 1, you'll study several sample problems to help you see how numbers and units are used in this equation. In Activity 2, you'll use this equation to solve problems similar to those that a technician might have to solve.

Part A: *The Thermal Energy Transfer Equation*

Let's look at the relationship of the various physical quantities in the thermal energy transfer equation.

$$\text{Thermal energy} = \text{Mass of the substance} \times \text{Specific heat of substance} \times \text{Temperature change of the substance}$$

$$H = mc\Delta T \qquad \textbf{\textit{Equation 1}}$$

where: H = thermal energy (in kcal or Btu)
m = mass of the substance (in kg or lb_m)
c = specific heat of the substance $\left[\text{in } \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ} \text{ or } \frac{\text{Btu}}{\text{lb}_m\cdot\text{F}^\circ}\right]$
ΔT = change in temperature of the substance (in C° or F°)

Part B: *Review of Units.*

Table 1 sums up typical units used with the physical quantities given in Equation 1. Study Table 1. Compare the units in SI and the English system.

TABLE 1. UNITS IN HEAT-TRANSFER EQUATION

	Typical Units			
	Equation 1: H = mcΔT			
System of Units	H	m	c	ΔT
English	Btu	lb_m (lb mass)	$\frac{\text{Btu}}{\text{lb}_m\cdot\text{F}^\circ}$	F°
SI	kcal	kg	$\frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$	C°

When mechanical energy (or fluid or electrical energy) is converted totally to heat energy, a certain relationship exists between units of heat energy (calories and Btu) and units of mechanical, fluid or electrical energy (foot·pounds, or joules). This relationship is called the "mechanical equivalent of heat." It's defined as follows:

$$\text{Mechanical Equivalent of Heat} = \frac{\text{Mechanical energy expended}}{\text{Heat energy produced when } \textbf{\textit{all}} \text{ mechanical energy goes into heat energy}}$$

Table 2 compares heat energy units to equivalent units of mechanical energy (or electrical or fluid energy).

ANSWERS FOR REVIEW OF UNITS

a. kcal or Btu

b. $\frac{\text{kcal}}{\text{kg}\bullet\text{C}^\circ}$

c. C°

d. kg; lbm

e. 4184 J

f. 778 ft•lb

TABLE 2: EQUIVALENT UNITS OF HEAT ENERGY
AND MECHANICAL ENERGY

Heat Energy	⟷	Mechanical Energy
1 Btu	=	1054 N·m or 1054 J
1 Btu	=	778 ft·lb
1 calorie	=	4.184 N·m or 4.184 J
1 calorie	=	3.086 ft·lb
1 kilocalorie	=	4184 N·m or 4184 J
1 kilocalorie	=	3086 ft·lb

Table 2 sets up equal relations between heat energy units and mechanical energy units. These equivalent relations are used to convert units of thermal energy to units of mechanical energy, or units of mechanical energy to units of thermal energy. Based on the equation given above and the data in Table 2, the mechanical equivalent of heat equals:

$$\text{Mechanical equivalent of heat} = \frac{4184\text{ J}}{1\text{ kcal}} \text{ or } \frac{778\text{ ft}\cdot\text{lb}}{\text{Btu}} \text{ or } \frac{1054\text{ J}}{\text{Btu}}$$

LET'S REVIEW UNITS!

Before you study the Practice Exercises in Activity 1, answer the following questions on thermal energy units. Fill in the blanks with the correct word or words. Use Table 1 and Table 2 to help you answer the questions.

a. Thermal energy is measured in units of ______ (kcal, Btu, lb, ohms).

b. In SI units, specific heat (c) has units of ______ $\left(\frac{\text{Btu}}{\text{lb}_m\cdot\text{F}^\circ}, \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ} \right)$.

c. In SI units, temperature difference (ΔT) has units of ______ (°C, C°, °F).

d. In SI units, mass (m) has units of ______ (kg, lb_m). In the English system of units, mass (m) has units of ______ (kg, lb_m).

e. One kilocalorie of heat energy equals ______ joules of mechanical energy.

f. One Btu of heat energy equals ______ ft·lb of mechanical energy.

PRACTICE EXERCISES FOR ACTIVITY 1

Example A: *Thermal Energy Stored in a Steel Mass*

The protective plastic coating (insulation) on electrical wire is often a material called PVC (polyvinyl chloride). An extruding machine is used to squeeze melted PVC through a die (a small round opening) and onto the wire. This process is similar to squeezing toothpaste from a tube.

To melt PVC, the extruder feed screw and cylinder are heated to 400°F from a room temperature of 80°F. This heat is stored in the steel screw and cylinder. Heat is transferred to the PVC as it moves through the cylinder. The added thermal energy melts the PVC. The feed screw pushes (extrudes) the melted PVC through the die and onto the wire.

Given: The screw and cylinder have a mass of 500 lb_m. The specific heat of steel is 0.35 Btu/($\text{lb}_m\cdot\text{F}^\circ$).

Find: The thermal energy stored in the 500 lb_m of screw and cylinder. (***Note:*** Ignore heat losses to the atmosphere.)

Solution: The thermal energy stored can be found from the equation:

$$H = m \times c \times \Delta T$$

where: $m = 500\ lb_m$

$c = 0.35\ Btu/(lb_m{\cdot}F^\circ)$

$\Delta T = 400°F - 80°F = 320\ F^\circ$

$$H = 500\ lb_m \times 0.35\ \frac{Btu}{lb_m{\cdot}F^\circ} \times 320\ F^\circ$$

$$H = 500 \times 0.35 \times 320\ \frac{\cancel{lb_m}{\cdot}Btu{\cdot}\cancel{F^\circ}}{\cancel{lb_m}{\cdot}\cancel{F^\circ}}$$ (Cancel like units.)

$$H = 56{,}000\ Btu.$$

Example B: *Thermal Energy Carried Off by Water in Cooling Coils*

Cooling coils that carry running water are wrapped around a laser rod to carry away excess heat while the laser operates. If excess heat isn't carried away, the laser rod overheats and stops operating.

Given: Ten liters of water pass through the cooling coils in one minute. The water enters the coils at a temperature of 20°C. Water leaves the coils at a temperature of 23°C.

Find: The number of kilocalories of excess laser heat energy carried away by the water each minute.

Solution: The heat energy carried away by the water is the heat energy transferred from the laser rod to the water. Use the heat energy equation, $H = mc\Delta T$, to find the heat energy transferred to the water.

$$H = mc\Delta T$$

where: $m = 10\ kg$ (That's because 1 liter equals 1000 cm^3. Each cubic centimeter of water has a mass of 1 gram. Thus, ten liters equal 10,000 cm^3, or 10,000 gm—10 kg—of water.)

$c = 1\ \frac{kcal}{kg{\cdot}C^\circ}$ (specific heat of water)

$\Delta T = 23°C - 20°C = 3\ C^\circ$

Substitute in values.

$$H = 10\ kg \times \left(1\frac{kcal}{kg{\cdot}C^\circ}\right) \times 3\ C^\circ$$

$$H = (10 \times 1 \times 3)\left[\cancel{kg}\frac{kcal}{\cancel{kg}{\cdot}\cancel{C^\circ}} \times \cancel{C^\circ}\right]$$ (Cancel units.)

$$H = 30\ kcal$$

Each minute, the cooling coils carry off 30 kcal of heat energy.

Example C: *Temperature Rise in Laser Rod*

Suppose the pump that circulates water through the cooling coils in Example B fails. That means that 30 kcal of heat energy each minute stay in the laser rod and increase its temperature.

Given: A laser rod of mass 2.2 kg with a specific heat of 0.18 kcal/(kg·C°). Rod absorbs 30 kcal of heat each minute.

Find: The temperature rise of the laser rod each minute (due to failure of cooling system).

Solution: The equation that relates heat energy (H) and temperature change (ΔT) in a material is $H = mc\Delta T$. Rearrange the equation, $H = mc\Delta T$, to isolate ΔT, as follows:

$$H = mc\Delta T$$

Divide each side by mc.

$$\frac{H}{mc} = \frac{\cancel{mc}\Delta T}{\cancel{mc}}$$ (Cancel mc.)

Reverse order of equation.

$$\Delta T = \frac{H}{mc}$$ (ΔT has been isolated.)

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2

Problem 1: $H = mc\Delta T$ where: $m = 8$ kg (since one liter of water has a mass of 1 kg)

$c = 1\ \dfrac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$

$\Delta T = 30^\circ\text{C} - 22^\circ\text{C} = 8\ \text{C}^\circ$

$$H = (8\ \text{kg})(1\ \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ})(8\ \text{C}^\circ)$$

$$H = (8 \times 1 \times 8)(\text{kg}\cdot\frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}\cdot\text{C}^\circ)$$

$$H = 64\ \text{kcal.}$$

Problem 2: a. $H = mc\Delta T$

Rearrange the equation to isolate "ΔT."

$\Delta T = \dfrac{H}{mc}$ where: $H = 64$ kcal

$m = 5$ kg

$c = 0.3\ \dfrac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$

$$\Delta T = \frac{64\ \text{kcal}}{(5\ \text{kg})(0.3\ \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ})} = (\frac{64}{5 \times 0.3})(\frac{\text{kcal}}{\text{kg}})(\frac{\text{kg}\cdot\text{C}^\circ}{\text{kcal}})$$

$$\Delta T = 42.7\ \text{C}^\circ.$$

b. The rod increases its temperature by 42.7 C° each minute. After five minutes, that's equal to 5 x 42.7 = 213.5°. So the rod temperature increases from 50°C to 263.5°C. The final rod temperature after five minutes is 263.5°C.

(See page T-121c for solution to Problem 3.)

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2, Continued

Problem 3: $H = mc\Delta T$ where:

$$m = 12.5 \text{ kg}$$

$$c_{water} = 1 \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$$

$$c_{mixture} = 0.53 \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$$

a. For water alone:

$$H = 12.5 \text{ kg} \times 1 \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ} \times 50 \text{ C}^\circ$$

$$H = (12.5 \times 1 \times 50)(\text{kg}\cdot\frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}\cdot\text{C}^\circ)$$

$$H = 625 \text{ kcal.}$$

b. For a 50/50 mixture of water and antifreeze:

$$H = 12.5 \text{ kg} \times 0.53 \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ} \times 50 \text{ C}^\circ$$

$$H = 12.5 \times 0.53 \times 50 \text{ kg} \cdot \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ} \cdot \text{C}^\circ$$

$$H = 331.25 \text{ kcal.}$$

c. The 50/50 mixture of water and antifreeze will absorb only about one-half as much thermal energy as the water alone. But the increased boiling point of the mixture compared to water alone--and the resulting increased heat-flow rate of the 50/50 coolant mixture--more than compensate for the mixture's inability to absorb as much thermal energy as an equal mass of water alone.

Now substitute in values and solve for ΔT.

$$\Delta T = \frac{H}{mc}$$

where: $c = 0.18\ \text{kcal/(kg·C°)}$
$m = 2.2\ \text{kg}$
$H = 30\ \text{kcal}$

$$\Delta T = \frac{30\ \text{kcal}}{(2.2\ \text{kg})\left(0.18\dfrac{\text{kcal}}{\text{kg·C°}}\right)}$$

$$\Delta T = \frac{30}{(2.2)\,(0.18)}\left(\frac{\cancel{\text{kcal}}}{\cancel{\text{kg}}} \times \frac{\cancel{\text{kg}}\text{·C°}}{\cancel{\text{kcal}}}\right)$$ (Cancel units.)

$$\Delta T = 75.8\ \text{C°}$$

Failure of water pump to operate properly causes laser rod to increase in temperature about 76 degrees each minute. This quickly develops into a serious problem.

ACTIVITY 2

Solving Practical Problems That Deal with Heat Transfer

The following problems are like those a technician must solve. Read the problems carefully. Use the equations and units learned in Activity 1 to solve each problem. Follow the "Hints."

Note: For the solution to each problem that follows, compare the units of the answer you get to the units given in Table 1. Do this. It will help you be sure the units to your answers are correct.

Problem 1: Given: Eight liters of water pass through a cooling-coil system each minute. The water that enters the coils has a temperature of 22°C. The water that leaves the coils has a temperature of 30°C.

Find: How much heat energy the flowing water in the cooling system absorbs each minute.

Solution: (***Hint:*** Use the equation, $H = mc\Delta T$. Convert 8 liters of water to kilograms of water. Remember that the specific heat of water is $c = 1\ \text{kcal/(kg·C°)}$.)

Problem 2: Given: The cooling system for a laser system fails. As a result, 64 kcal of heat are absorbed each minute by the laser rod in the laser system. The laser rod has a mass of 5 kg. It has a specific heat of 0.3 kcal/(kg·C°).

Find:
a. How much the temperature of the rod increases each minute. (***Hint:*** Use the equation, $H = mc\Delta T$. Solve for ΔT.)
b. If the rod's initial temperature was 50°C, find its temperature after 5 minutes.

Solution:

Problem 3: Automobiles use antifreeze in the winter to keep the water in the engine's cooling system from freezing when the engine isn't running. Water freezes at 32°F. A 50% water, 50% antifreeze mixture freezes at –37°F.

In the summer, antifreeze mixtures are used because the mixture boils at a higher temperature (approximately 230°F) than water (212°F). This greater temperature difference (a forcelike quantity) increases the ***heat-flow rate*** from the mixture in the radiator to the surrounding air.

Water alone can absorb more thermal energy than the antifreeze mixture. But the increased temperature difference of the mixture (due to the

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 2, Continued

Problem 4: Use the equation,

$H = mc\Delta T$. where: m = mass of Freon 12 = 2.2 kg

H = 31.57 kcal thermal energy absorbed by the Freon 12

$c = 0.41 \dfrac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$ for Freon 12

Rearrange the equation. Solve for "ΔT."

$$\Delta T = \frac{H}{mc}$$

$$\Delta T = \frac{31.57 \text{ kcal}}{2.2 \text{ kg} \times 0.41 \dfrac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}}$$

$$\Delta T = \frac{31.57}{2.2 \times 0.41} \times \frac{\text{kcal}\cdot\text{kg}\cdot\text{C}^\circ}{\text{kg}\cdot\text{kcal}} \qquad \left(\frac{1}{\dfrac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}} = \frac{\text{kg}\cdot\text{C}^\circ}{\text{kcal}}\right)$$

$$\Delta T = 35 \text{ C}^\circ.$$

Problem 5: Use the equation, $H = mc\Delta T$. where: H = 300 kcal

m = 6.8 kg

ΔT = 70 C°

Rearrange and solve the equation for "c."

$$c = \frac{H}{m\Delta T}$$

$$c = \frac{300 \text{ kcal}}{6.8 \text{ kg} \times 70 \text{ C}^\circ}$$

$$c = \frac{300}{6.8 \times 70} \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$$

$$c = 0.63 \frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}.$$

(See page T-122c for solution to Problem 6.)

SOLUTION TO STUDENT CHALLENGE

Problem 6: Work the problem in steps:

a. Heat 10 kg ice at -5°C to 10 kg ice at 0°C. (No "change of state.")

$H = mc\Delta T$ where: $m = 10$ kg
$c = 0.5$ kcal/kg•C°
$\Delta T = 5$ C°

$$H = (10\ \text{kg})(0.5\ \frac{\text{kcal}}{\text{kg}\cdot\text{C°}})(5\ \text{C°}) = (10 \times 0.5 \times 5)(\text{kg}\cdot\frac{\text{kcal}}{\text{kg}\cdot\text{C°}}\cdot\text{C°})$$

$H = 25$ kcal.

b. Change 10 kg ice at 0°C to 10 kg water at 0°C. (No temperature change.)

$$H = (10\ \text{kg})(80\ \frac{\text{kcal}}{\text{kg}})$$

(Latent heat needed to change ice to water at 0°C is 80 kcal per kg. See Table 5.3.)

$H = 800$ kcal.

c. Change 10 kg water at 0°C to 10 kg of water at 100°C. (No "change of state.")

$H = mc\Delta T$ where: $m = 10$ kg
$c = 1$ kcal/kg•C°
$\Delta T = 100$ C°

$$H = (10\ \text{kg})(1\ \frac{\text{kcal}}{\text{kg}\cdot\text{C°}})(100\ \text{C°}) = (10 \times 1 \times 100)(\text{kg}\cdot\frac{\text{kcal}}{\text{kg}\cdot\text{C°}}\cdot\text{C°})$$

$H = 1000$ kcal.

d. Change 10 kg of water at 100°C to 10 kg steam at 100°C. (No temperature change.)

$$H = (10\ \text{kg})(540\ \frac{\text{kcal}}{\text{kg}})$$

(Latent heat needed to change water to steam is 540 kcal per kg. See Table 5-3.)

$H = 5400$ kcal.

e. Adding up the four parts, the total energy required is

$H_{TOT} = 25$ kcal + 800 kcal + 1000 kcal + 5400 kcal

$H_{TOT} = 7225$ kcal.

increased boiling point) causes an increased heat-flow rate. This makes the antifreeze mixture the better coolant. By solving the following problem, you'll see that water can absorb more thermal energy compared to an equal volume of water/antifreeze mixture.

Given: The capacity of a cooling system is 12.5 liters of liquid for a full-size car. This is equal to 12.5 kg mass of water. The specific heat of water is 1 kcal/kg·C°. Assume that a $^{50}\!/_{50}$ water/antifreeze mixture has nearly the same mass as water alone—12.5 kg. The specific heat of the mixture is 0.53 kcal/kg·C°.

Find: The thermal energy absorbed when the temperature of the liquid in the cooling system rises 50 C°

a. if the liquid is water.

b. if the liquid is a $^{50}\!/_{50}$ mixture.

c. Compare "a" and "b." Which liquid absorbs more heat?

Solution: (***Hint:*** Use the equation, $H = mc\Delta T$, to solve for H for each liquid.)

Problem 4: Given: An automobile air conditioner is charged with 2.2 kg of a refrigerant gas called "Freon 12." In liquid form, Freon 12 has a specific heat of 0.41 kcal/kg·C°. The 2.2 kg of Freon absorbs 31.57 kcal of thermal energy from the air.

Find: How much the temperature of the Freon rises.

Solution: (***Hint:*** Use the equation, $H = mc\Delta T$. Rearrange the equation to isolate ΔT. Then solve for ΔT.)

Problem 5: Richland Machining Company has purchased a CNC milling machine. It's equipped with an oil pump that circulates cutting oil over the work piece and cutting tool. The oil bath lubricates and cools the tool and material being cut.

Given: The milling machine has a capacity of 6.8 kg (about 8 liters) of cutting oil. The cutting tool generates 300 kcal of heat, all of which is absorbed by the cutting oil. The temperature difference between the cutting tool/steel work piece and the cutting oil is 70 C°.

Find: The specific heat of the oil in units of kcal/kg·C°.

Solution: (***Hint:*** Use the equation, $H = mc\Delta T$. Rearrange the equation to isolate c. Then solve for c.)

Student Challenge

Problem 6: Given: Ten kilograms of ice at a temperature of –5°C are heated until the ice is changed to ten kilograms of steam at 100°C.

Find: How much heat energy (in kcal) is needed to change the ice to steam.

Solution: (***Hint:*** Study Figure 5-25 and Table 5-3 in the text. Then apply similar reasoning to the solution of this problem. ***Note:*** The specific heat of ice is 0.5 kcal/kg·C°; the specific heat of water is 1.0 kcal/kg·C°.)