Math Skills Laboratory

Lab 9 MS 1

MATH ACTIVITIES

Activity 1: Describing the Trigonometric Ratios—Sine, Cosine, Tangent
Activity 2: Plotting a Sine Curve

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Identify the parts of a right triangle.***
2. ***Describe the trigonometric ratios (sine, cosine, tangent) by using a right triangle.***
3. ***Solve problems using trigonometric tables (or calculators) where sine, cosine, and tangent of an angle are involved.***
4. ***Plot a sine curve.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

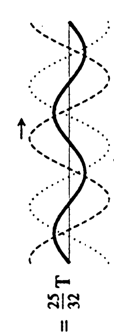
ACTIVITY 1:

Describing the Trigonometric Ratios (Sine, Cosine, Tangent)

MATERIALS

For this activity, you'll need a scientific calculator, graph paper, a protractor and a scaled ruler or straightedge.

Certain ratios of lengths of the sides of a right triangle are important in technical work. Before explaining the ratios, let's study the parts of a right triangle.



Part A: *Right triangles*

A ***right*** triangle (see Figure 1) contains:

- One 90° angle. (A 90° angle is called a "right" angle.)
- Three sides: A, B and C.
  - — The side directly across the triangle from the 90° angle (side C) is called the "hypotenuse." It's the longest side of the triangle.
  - — The other two sides (sides A and B) are shorter. Sometimes, they're called "legs."
- Three angles that always total 180°.

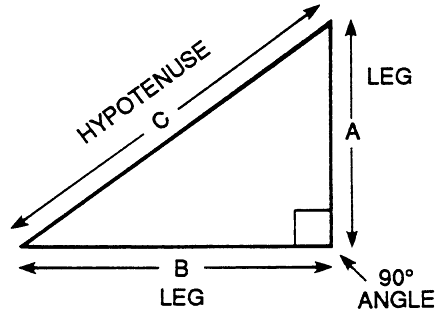


Fig. 1 Right triangle.

The relationship between the length of the hypotenuse and the lengths of the other two sides of a right triangle is expressed in the ***"Pythagorean theorem."*** This theorem states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse (side C). In equation form, this theorem is written as follows.

$$A^2 + B^2 = C^2$$

This is the basis for the familiar *three-four-five triangle method* used by carpenters and masons to check for squareness (90° angles) on everything from concrete slabs to door and window frames. The following example shows us that if the legs of a ***right triangle*** are 3 feet and 4 feet long, the hypotenuse has to be 5 feet long.

Example A: *The "3-4-5" Right Triangle*

Given: The two legs of a ***right triangle*** have the following lengths:

Leg A = 3 ft

Leg B = 4 ft

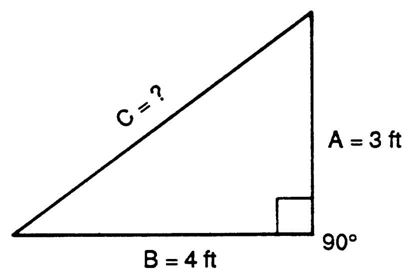
Find: The length of the hypotenuse (side C).

Solution: The Pythagorean theorem states that the sides are related as follows:

$C^2 = A^2 + B^2$

Or

$(\text{hypotenuse})^2 = (\text{leg A})^2 + (\text{leg B})^2$



Substitute in the equation for the lengths of leg A and leg B. Then solve for C.

$C^2 = (3\text{ ft})^2 + (4\text{ ft})^2$

$C^2 = 9\text{ ft}^2 + 16\text{ ft}^2$

$C^2 = 25\text{ ft}^2$

So

$C = \sqrt{25\text{ ft}^2} = \sqrt{25} \times \sqrt{\text{ft}^2} = (5)\ (\text{ft})$

$C = 5\text{ ft}$

The hypotenuse must be 5 ft if the legs are 3 ft and 4 ft. But remember, this is true only for a ***right triangle.***

Take out a sheet of paper. Are the corners "square"? They probably are. But check to see by using the "3-4-5" right-triangle method.

Look at Figure 2. It's an outline of the page, with the long edge horizontal. If the corners are square, the sides must meet at 90°. Check the squareness of the lower left corner. Do this as follows.

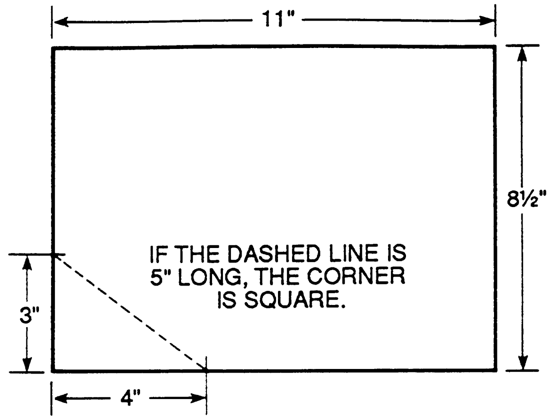


Fig. 2 Checking a corner for squareness.

Start from the corner and draw a 4-inch line along the bottom side. Now draw a 3-inch line up along the left side. Connect the ends of these lines by drawing a dotted line as shown.

Measure the length of the dotted line. If the length is 5 inches, the corner angle is 90°, and the corner is square. If it's not 5 inches (longer or shorter), the corner isn't square.

Part B: *Trigonometric ratios—sine, cosine, tangent*

A right triangle is shown in Figure 3. The angle "θ" (pronounced "THAY-tuh"), is shown. Legs A and B are now named in terms of their location relative to angle θ.

- Leg A is opposite angle θ.
- Leg B is adjacent to angle θ.
- The hypotenuse is still the side across from the 90° angle. It's also adjacent to angle θ, but it's not a "leg." So don't confuse it with the "adjacent side," leg B.

Several useful ratios of interest are:

$$\frac{\text{Length of side opposite angle } \theta}{\text{Length of hypotenuse}} = \frac{A}{C}$$

$$\frac{\text{Length of side adjacent to angle } \theta}{\text{Length of hypotenuse}} = \frac{B}{C}$$

$$\frac{\text{Length of side opposite angle } \theta}{\text{Length of side adjacent to angle } \theta} = \frac{A}{B}$$

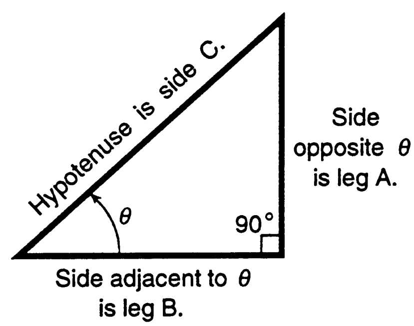


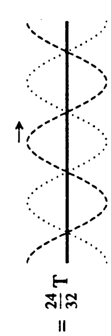
Fig. 3 Right triangle.

The above ratios are used often in drafting, construction, civil engineering, etc. These ratios have specific names.

$$\text{Sine } \theta = \frac{A}{C} = \frac{\text{Length of side opposite } \theta}{\text{Length of hypotenuse}}$$

$$\text{Cosine } \theta = \frac{B}{C} = \frac{\text{Length of side adjacent to } \theta}{\text{Length of hypotenuse}}$$

$$\text{Tangent } \theta = \frac{A}{B} = \frac{\text{Length of side opposite } \theta}{\text{Length of side adjacent to } \theta}$$



These expressions are read *sine of theta, cosine of theta,* and *tangent of theta.* The words sine, cosine and tangent are abbreviated usually as ***sin, cos,*** and ***tan.*** Letters A and B, used to represent the sides of the right triangle, are changed to x and y when the right triangle is drawn on graph paper with x- and y-axes. (See Figure 4.) The examples that follow will use this notation. Therefore, in graph drawings like the one shown in Figure 4:

$$\sin\theta = \frac{y}{h} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{x}{h} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{y}{x} = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

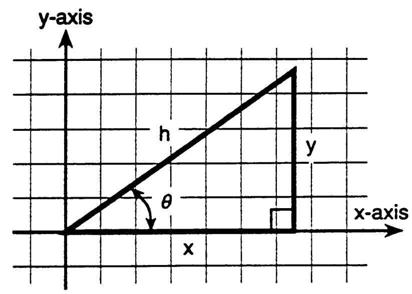


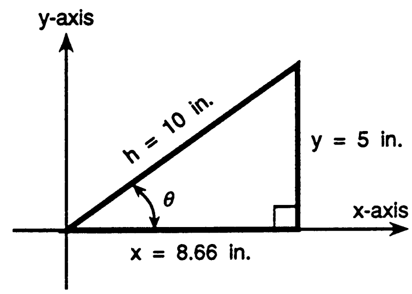
Fig. 4 Right triangle on "x-y" graph paper.

The following example shows how these ratios are computed in a specific triangle.

Example B: *Sine, Cosine, and Tangent Ratios for an Angle of a Right Triangle*

Given: A right triangle has angles and sides with the values shown in the sketch below.

Find: Sin θ, cos θ, tan θ.



Solution:

a. $\sin\theta = \frac{y}{h}$

$\sin\theta = \frac{5\text{ in.}}{10\text{ in.}}$

$\sin\theta = 0.500$

b. $\cos\theta = \frac{x}{h}$

$\cos\theta = \frac{8.66\text{ in.}}{10\text{ in.}}$

$\cos\theta = 0.866$

c. $\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{5\text{ in.}}{8.66\text{ in.}}$

$\tan\theta = 0.577$

In Example B, we computed numbers that represent the sine, cosine and tangent of an angle between 0° and 90°—in this case, $\theta = 30°$. In this range, values of sin θ and cos θ are always between 0.000 and 1.000. Values for tan θ are greater than 1.000 for values above 45°—and these values become very large as θ approaches 90°.

Part C: *Finding θ when you know the sine, cosine or tangent value of θ*

In the previous example, angle θ is contained in a right triangle with sides 5 inches, 8.66 inches, and 10 inches in length. But we still don't know the angle θ. However, we do know that any **similar** right triangle—regardless of size—that contains the same angle θ as in Example B would have the same values for the sine, cosine and tangent of angle θ.

If the drawing shown in Example B is to scale, we can find θ by using a protractor and reading it directly, as we've already learned. We could also use a table that lists values of sin θ, cos θ, and tan θ for all the angles between 0° and 180°. This table is called a "Table of Trigonometric Functions."

Many good texts discuss how to use this kind of table. Learning how to use the tables involves knowing how to "interpolate," or calculate values for angles not listed in the table. This requires more time than we have for this laboratory exercise.

So instead of reading tables, we'll use a *scientific calculator*. We can get the answer faster—and with more accuracy. Besides, it's more fun!

So let's use our scientific calculators to solve problems that involve sine, cosine, and tangent of a certain angle. Here's how.

To find sine, cosine, or tangent of an angle when the angle is known:

Step 1. Turn calculator ON. Make sure it's set to "DEG" (that is, degrees).

Step 2. Enter the angle value into the display, such as "30" for a 30° angle.

Step 3. Push the key for the function whose value you seek.
- To find the sine of the angle, press **SIN**.
- To find the cosine of the angle, press **COS**.
- To find the tangent of the angle, press **TAN**.

Step 4. The display will show the value of the ratio. For example, for an angle of 30°, the calculator should yield the following values (rounded to three decimal places):
- *0.500*, if you pressed **SIN**. Thus, sin 30° = 0.500.
- *0.866*, if you pressed **COS**. Thus, cos 30° = 0.866.
- *0.577*, if you pressed **TAN**. Thus, tan 30° = 0.577.

To find the angle θ when the value of sin θ, cos θ, or tan θ is known:

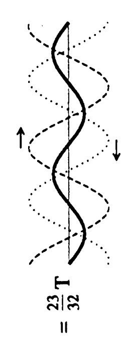
Step 1. Turn calculator ON. Make sure it's set to "DEG" (that is, degrees).

Step 2. Enter the value of the ratio sin θ, cos θ, or tan θ into the display. For example, you could enter the values from Example B.
- The ratio for sin θ: 0.500.
- The ratio for cos θ: 0.866.
- The ratio for tan θ: 0.577.

Step 3. With the ratio in the display, press **INV** and the function key that describes the ratio you entered: **SIN**, **COS**, or **TAN**.
- If you entered the ratio for sin θ (that is, 0.500), press **INV SIN**.
- If you entered the ratio for cos θ (that is, 0.866), press **INV COS**.
- If you entered the ratio for tan θ (that is, 0.577), press **INV TAN**.

Step 4. The display will show the angle (in degrees) that produced the ratio you entered.
- *30*, if you entered 0.500 and pressed **INV SIN**. Thus 30° is the angle whose sine is equal to 0.500.
- *30.002911*, if you entered 0.866 and pressed **INV COS**. Thus 30° is the angle whose cosine is nearly equal to 0.866.
- *29.984946*, if you entered 0.577 and pressed **INV TAN**. Thus 30° is the angle whose tangent is nearly equal to 0.577.

In each case above from Example B, we've shown the angle is θ = 30°.



Part D: ***How can we use trigonometric ratios?***

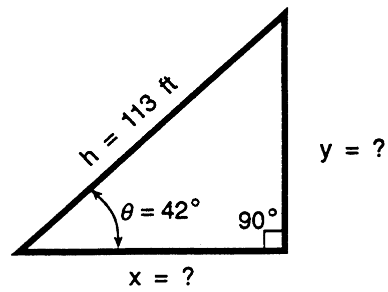
Frequently, sine, cosine and tangent ratios are used with right triangles to:

- find unknown sides.
- find unknown angles.

Examples D and E show how to find an unknown side and an unknown angle in a right triangle.

Example D: *Finding Unknown Sides of a Right Triangle*

Given: A right triangle has the values shown in the adjoining sketch.



Find:
a. The length of side y.
b. The length of side x.

Solution:
a. The sine definition contains y and h (that is, sin $\theta = y/h$). The hypotenuse h is given. The angle θ is given, so we can calculate sin θ. That leaves y as the only unknown.

$$\sin\theta = \frac{y}{h}$$

where $\theta = 42°$ and h = 113 ft.

Rearrange the equation to isolate y. This gives:

$y = (h)(\sin\theta)$

$y = (113\text{ ft})(\sin 42°)$ (Using a calculator and rounding to three decimal places, $\sin 42° \approx 0.669$.)

$y = (113\text{ ft})(0.669)$

$y = 75.6\text{ ft}$ (rounded to three digits)

The height y has been found to be 75.6 ft.

b. Now let's find side x. The cosine definition contains x and h.

$$\cos\theta = \frac{x}{h}$$

where again $\theta = 42°$ and h = 113 ft.

Rearranging to isolate x gives:

$x = (h)(\cos\theta)$

$x = (113\text{ ft})(\cos 42°)$ (Using a calculator and rounding to three decimal places, $\cos 42° \approx 0.743$.)

$x = (113\text{ ft})(0.743)$

$x = 84.0\text{ ft}$ (rounded to three digits)

The base x has been found to be 84.0 ft.

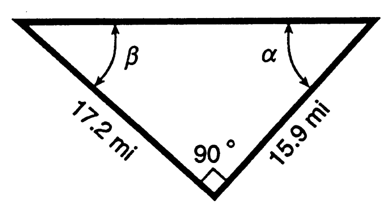
Note: You can check the values by using the Pythagorean theorem, as follows:

$$x^2 + y^2 = h^2$$
$$(84.0\text{ ft})^2 + (75.6\text{ ft})^2 = (113\text{ ft})^2$$
$$7056\text{ ft}^2 + 5715.36\text{ ft}^2 = 12769\text{ ft}^2$$
$$12771.36\text{ ft}^2 = 12769\text{ ft}^2$$

The results agree, with only a small error due to rounding of the x and y values above.

Example E: *Finding an Unknown Angle α in a Certain Right Triangle*

Given: A right triangle has values shown in the adjoining diagram.



Find: The angles α (alpha) and β (beta) in degrees.

Solution: The triangle isn't oriented in an x-y plane. It doesn't have to be! We simply go back to our basic definition of the different trig functions. Examine the diagram.

- We know the length of the side *opposite* angle α is 17.2 miles.
- We know the length of the side *adjacent* to angle α is 15.9 miles.
- We know that *side opposite* and *side adjacent* are involved in the tangent ratio:

$$\tan\alpha = \frac{\text{length of side opposite angle } \alpha}{\text{length of side adjacent to angle } \alpha}$$

Substituting in numbers:

$$\tan\alpha = \frac{17.2\text{ miles}}{15.9\text{ miles}}$$

$$\tan\alpha = 1.08176$$

To find the angle α, use your calculator. Turn it ON and enter 1.08176 into the display. Press the **INV** key and then the **TAN** key. Display will read approximately *47.2*. Therefore, $\alpha = 47.2°$.

The remaining angle β (beta, pronounced "BAY-tuh") may be found in the same way.

$$\tan\beta = \frac{\text{length of side opposite angle }\beta}{\text{length of side adjacent to angle }\beta} = \frac{15.9\text{ miles}}{17.2\text{ miles}}$$

$$\tan\beta = 0.924418$$

To find angle β, use your calculator. Turn it ON, and enter 0.924418 into the display. Press the **INV** key and then the **TAN** key. Display will read approximately *42.8*. Therefore, $\beta = 42.8°$.

The angle β can also be found by using the fact that all the angles added together must equal 180°. Thus,

$$90° + \alpha + \beta = 180°$$

$$90° + 47.2° + \beta = 180° \qquad \text{(Use } \alpha = 47.2° \text{ as found above.)}$$

$$137.2° + \beta = 180°$$

$$\beta = 180° - 137.2°$$

$$\beta = 42.8°$$

This is the same value as that found from $\tan\beta = 0.924418$.

ACTIVITY 2

Plotting a Sine Curve

MATERIALS

For this activity, you'll need a calculator, graph paper and Table 1.

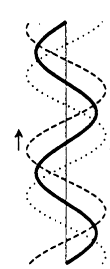
We mentioned earlier that all numerical values of $\sin\theta$ and $\cos\theta$, for angles θ between 0° and 90°, lie between 0.000 and 1.000. Table 1 gives values for $\sin\theta$ where θ varies from 0° to 360°. Looking at Table 1, we can see that:

- $\sin\theta$ increases from 0.000 to 1.000 for angles between $\theta = 0°$ and $\theta = 90°$.
- $\sin\theta$ decreases from 1.000 to 0.000 for angles between $\theta = 90°$ and $\theta = 180°$.
- $\sin\theta$ goes from 0.000 to –1.000 for angles between $\theta = 180°$ and $\theta = 270°$.
- $\sin\theta$ goes from –1.000 to 0.000 for angles between $\theta = 270°$ and $\theta = 360°$.

TABLE 1. VALUES OF θ AND SIN θ, 0° TO 360°

θ	$\sin\theta$*	θ	$\sin\theta$*	θ	$\sin\theta$*	θ	$\sin\theta$*
0°	0.000	100°	0.985	200°	–0.342	300°	–____
10°	0.174	110°	0.940	210°	–____	310°	–0.766
20°	____	120°	____	220°	–0.643	320°	–0.643
30°	0.500	130°	0.766	230°	–0.766	330°	–____
40°	0.643	140°	0.643	240°	–____	340°	–0.342
50°	____	150°	____	250°	–0.940	350°	–____
60°	0.866	160°	0.342	260°	–0.985	360°	0.000
70°	____	170°	0.174	270°	–1.000		
80°	0.985	180°	0.000	280°	–0.985		
90°	1.000	190°	–0.174	290°	–0.940		

* The values of *sin θ* are rounded to three decimal places.



$t = \frac{22}{32}T$

Problem 1: Complete Table 1. Use your calculator to find sin θ for those angles for which no value of sin θ is given. Refer to Activity 1c if you need help.

Problem 2: a. Using a piece of graph paper, construct an x-y system of axes—called a *rectangular coordinate system.* See Figure 5. Lay out the x-axis (horizontal axis) by dividing it into 10° intervals for angle θ between 0° and 360°. Lay out the y-axis (vertical axis) for sin θ by dividing it into 0.1 intervals between −1.0 and +1.0. Graph paper with 0.5-cm square works well for this activity. The graph, when completed, will look like that shown in Figure 5.

b. Plot the data from Table 1 on the graph. For each angle θ given, plot the value of sin θ. (Round off the numerical values of sin θ listed in Table 1 to the nearest hundredth.) The result should be the graph of a sine wave.

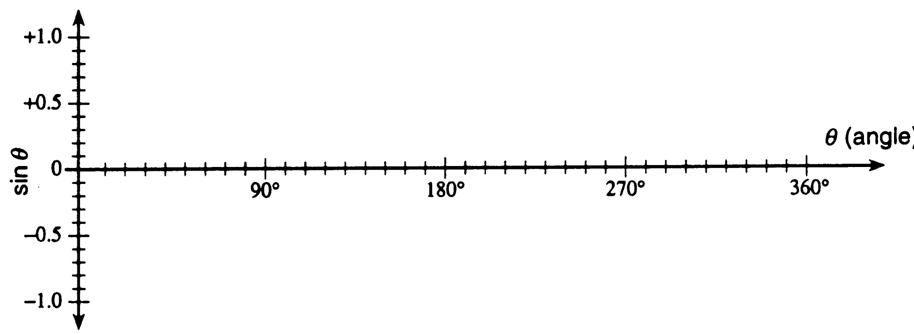


Fig. 5 A rectangular coordinate system for θ and sin θ.

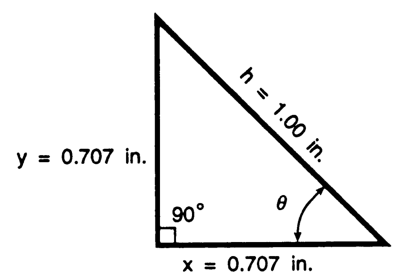
Student Challenge

Problem 3: In Problem 2 you plotted sin θ versus θ for angles $\theta = 0°$ to $\theta = 360°$. Now plot cos θ versus θ for the same range of angle. Use your calculator to find cos θ for angle θ—for θ from 0° to 360°, every 10 degrees. Plot the data on the same graph you used to plot the sine function in Problem 2b.

Problem 4: Given: The right triangle shown in the sketch here. Values for the sides are given.

Find: The angle θ using the definition for either sin θ or cos θ.

Solution:



Problem 5: Given: A right triangle with values as shown in the sketch here.

Find: a. Length of side A. (Use the ratio for sin θ.)

b. Length of side B. (Use the ratio for cos θ.)

Solution: