Math Skills Laboratory

Lab 3 MS 1

MATH ACTIVITIES

Activity 1: Rearranging Symbols in Rate Equations to Isolate Certain Unknowns

Activity 2: Solving Mechanical Rate Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Rearrange the equation for constant linear speed,*** $v = \ell/t$. ***Isolate linear displacement*** (ℓ) ***or elapsed time*** (t).
2. ***Rearrange the equation for constant rotational speed,*** $w = \theta/t$. ***Isolate angular displacement*** (θ) ***or elapsed time*** (t).
3. ***Rearrange the equation for constant linear acceleration, given the equation,*** $a = \dfrac{v_f - v_i}{t}$. ***Isolate the final linear speed*** (v_f).
4. ***Rearrange the equation for constant angular acceleration, given the equation,*** $\alpha = \dfrac{\omega_f - \omega_i}{t}$. ***Isolate the final linear speed*** (ω_f).
5. ***Substitute correct numerical values and units in rate equations. Solve the equations for a numerical value with appropriate units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Examples A and B in Activity 1.***
3. ***Work the problems for Activities 1 and 2.***

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ACTIVITY 1

Rearranging Symbols in Rate Equations to Isolate Certain Unknowns

In previous math labs, you learned that equations and formulas are used to express a relationship between several physical quantities. Equation 1 relates the concept of linear speed, displacement and elapsed time, as follows:

$$\text{Linear Speed} = \frac{\text{Displacement}}{\text{Elapsed Time}} \qquad \textbf{\textit{Equation 1}}$$

This equation helps you determine the value of one physical quantity if you know the numerical value and units of the other two. Equations often are written with symbols rather than words. Thus, to express Equation 1 in symbol form, use the following symbols:

$$v = \frac{\ell}{t} \qquad \textbf{\textit{Equation 2}}$$

where: v = constant linear speed (miles/hour, feet/second, kilometers/hour, meters/second)
ℓ = displacement (miles, feet, kilometers, meters)
t = elapsed time (hours, seconds)

Other mechanical rate equations that express the concept of (1) constant linear acceleration, (2) angular speed, and (3) constant angular acceleration are expressed in Equations 3 through 8 below.

1. *Constant Linear Acceleration*

$$\text{Constant Linear Acceleration} = \frac{\text{Final Speed} - \text{Initial Speed}}{\text{Elapsed Time}} \qquad \textbf{\textit{Equation 3}}$$

To simplify Equation 3, use the following symbols:

$$a = \frac{v_f - v_i}{t} \qquad \textbf{\textit{Equation 4}}$$

where: a = constant linear acceleration (miles/hour^2, feet/second^2, kilometers/hour^2, meters/second^2)
v_f = final linear speed (miles/hour, feet/second, kilometers/hour, meters/second)
v_i = initial linear speed (same units as v_f)
t = elapsed time (hours, seconds)

2. *Rotational (Angular) Speed*

$$\text{Rotational (Angular) Speed} = \frac{\text{Angular Displacement}}{\text{Elapsed Time}} \qquad \textbf{\textit{Equation 5}}$$

To simplify Equation 5, use the following symbols:

$$\omega = \frac{\theta}{t} \qquad \textbf{\textit{Equation 6}}$$

where: ω = constant or average angular speed (revolutions/minute or radians/second)
θ = angular distance (revolutions or radians)
t = elapsed time (hours, minutes, seconds)

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3. *Constant Angular Acceleration*

$$\text{Constant Angular Acceleration} = \frac{\text{Final Angular Speed} - \text{Initial Angular Speed}}{\text{Elapsed Time}} \qquad \textbf{\textit{Equation 7}}$$

To simplify Equation 7, use the following symbols:

$$\alpha = \frac{\omega_f - \omega_i}{t} \qquad \textbf{\textit{Equation 8}}$$

where: α = constant angular acceleration (revolutions/minute^2 or radians/second^2)
ω_f = final angular speed (revolutions/minute or radians/second)
ω_i = initial angular speed (same units as ω_f)
t = elapsed time

You have already "solved" an equation by isolating symbols during Unit 2, "Work." Let's review the rules. To do this, you must always perform *identical mathematical operations* on *both sides of the equation*. ***Any operation done on one side of the equation is always done on the other side.*** *Therefore, take care to do the following:*

- ***Add or subtract the same quantity on both sides of an equation.***
- ***Multiply or divide both sides of an equation by the same quantity.***

Study Examples A and B. They show how to isolate a certain symbol by rearranging the given equation.

Example A: ***Rearranging the Linear Speed Equation to Isolate Displacement or Time***

Given: $v = \ell/t$ (Equation 2)

Find: a. ℓ by rearranging Equation 2.

b. t by rearranging Equation 2.

Solution: a. Isolate ℓ.

Step 1: In rearranging the equation, $v = \ell/t$, to isolate or "solve" for ℓ, start with the given equation.

$$v = \frac{\ell}{t}$$

Step 2: Multiply both sides by t to isolate ℓ.

$$v \times t = \frac{\ell}{\cancel{t}} \times \cancel{t}$$

(The variable t cancels out on the right side, leaving ℓ by itself.)

Step 3: Rewrite the equation with t on the right side removed.

$$v \times t = \ell$$

(The ℓ has been isolated.)

Step 4: Reverse the order of the equation so that ℓ is on the left.

$$\ell = v \times t$$

Equation 2, $v = \ell/t$, has been "solved" for ℓ. The symbol, "ℓ," has been isolated. The correct equation is $\ell = v \times t$.

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b. Isolate t.

Step 1: In rearranging the equation, $v = \ell/t$, to isolate or "solve" for t, start with the given equation:

$$v = \frac{\ell}{t}$$

Step 2: Multiply both sides by t.
(This step gets "t" out of the denominator.)

$$t \times v = \cancel{t} \times \frac{\ell}{\cancel{t}}$$

(The variable t cancels out on the right side.)

Step 3: Divide both sides by v.

$$\frac{t \times \cancel{v}}{\cancel{v}} = \frac{\ell}{v}$$

(The variable v cancels out on the left side, leaving t isolated.)

Step 4: This gives the equation for t.

$$t = \frac{\ell}{v}$$

Equation 2 has been "solved" for t. The symbol "t" has been isolated. Given numerical values and units for ℓ and v, we can now substitute in this equation and solve directly for a numerical value—and unit—for t.

Student Challenge

Example B: *Rearranging the Constant Linear Acceleration Equation to Isolate Speed*

Given: $a = \frac{v_f - v_i}{t}$ (Equation 4)

Find: v_f by rearranging Equation 4.

Solution: Step 1: In rearranging the equation, $a = (v_f - v_i)/t$, to isolate or "solve" for v_f, start with the given equation.

$$a = \frac{v_f - v_i}{t}$$

Step 2: Enclose the numerator with parentheses and multiply both sides by t to isolate the term "$v_f - v_i$."

$$a \times t = \frac{(v_f - v_i) \times \cancel{t}}{\cancel{t}}$$

(The variable t cancels out on the right side.)

Step 3: Rewrite the equation with the t's on the right side removed.

$$at = (v_f - v_i)$$

Step 4: Rewrite the equation with the parentheses removed.

$$at = v_f - v_i$$

Step 5: Add v_i to both sides of the equation.

$$at + v_i = v_f - \cancel{v_i} + \cancel{v_i}$$

(The v_i's "subtract out" on the right side.)

Step 6: Rewrite the equation with the v_i's on the right side removed.

$$at + v_i = v_f$$

(The v_f has been isolated.)

Step 7: Reverse the order of the equation so that v_f is on the left and "v_i" and "at" are reordered on the right.

$$v_f = v_i + at$$

Step 8: The equation, $v_f = v_i + at$, is the desired result. The term, v_f, has been isolated.

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PRACTICE EXERCISES FOR ACTIVITY 1

Use Examples A and B as a guide. Solve the following problems. Use the equations discussed above—Equations 2, 4, 6 and 8—in paragraphs A and B above. Solve each equation by isolating the symbol indicated.

Problem 1: Given: $\omega = \theta/t$ (Equation 6).

Find: θ.

Solution:

Problem 2: Given: $\alpha = \dfrac{\omega_f - \omega_i}{t}$ (Equation 8).

Find: ω_f.

Solution: (***Hint:*** Follow the solution outlined in Example B, step-by-step.)

Student Challenge

Problem 3: Given: Equations $\omega = \dfrac{\theta}{t}$ and $\alpha = \dfrac{\omega_f - \omega_i}{t}$

Find: t for each of these equations.

Solution:

ACTIVITY 2

Solving Mechanical Rate Problems

EQUIPMENT

For this activity, you'll need a hand calculator.

Mechanical rate formulas involved in this activity are given as Equations 2, 4, 6 and 8 in Activity 1. You'll need to choose the equation that is appropriate for the particular problem. If it's not in the correct form, first rearrange the formula and isolate the proper symbol. Then solve the rearranged equation for the unknown value. Your final answer should include a correct numerical answer AND the proper units.

PRACTICE EXERCISES FOR ACTIVITY 2

Problem 4: Given: A box sitting on a moving conveyor belt moves a distance of 88 feet in 8 seconds.

Find: The linear speed of the conveyor belt.

Solution: (***Hint:*** Use the equation, $v = \ell/t$.)

Problem 5: Given: The deposit cylinder at a teller drive-in window at a bank travels through the tube at a constant speed of 2 meters per second. It takes 21 seconds to travel from depositor to teller.

Find: The length of the tube the cylinder travels through: (Ignore acceleration; assume constant speed.)

Solution: (***Hint:*** Will the equation $v = \ell/t$ work here? What symbol do you need to isolate?)

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Problem 6: Given: The funny cars (dragsters) shown in the video portion of this subunit start from REST and reach a speed of 250 miles per hour (367 ft/sec) in 5.5 seconds.

Find: The acceleration of the dragster for this quarter-mile event.

Solution: (***Hint:*** The problem gives you information telling you that $v_i = 0$. Why? You are also given that $v_f = 367$ ft/sec and $t = 5.5$ seconds. You are asked to solve for a. Does the equation, $a = \frac{v_f - v_i}{t}$, look like the right one to choose?)

Problem 7: Given: The dragster in Problem 6 opens a drag parachute after the quarter-mile run to slow the car. The dragster decelerates from 367 ft/sec to a final speed of 67 ft/sec (about 45 mph) in 20 seconds.

Find: The constant linear deceleration during the 20-second time period.

Solution:

Problem 8: Given: The scanner dish of a weather radar antenna system completes 160 revolutions in one hour.

Find: The angular speed of the scanner dish in
- a. revolutions per minute (rpm).
- b. radians per second (rad/sec).

Solution: (***Hint:*** Recall that 1 hr = 60 min = 3600 sec, 1 rev = 6.28 radians, and 1 min = 60 sec.)

Problem 9: Given: An electric motor shaft, under load, starts from REST and reaches its design operating speed of 1040 revolutions per minute in 3.6 seconds after the motor is started.

Find: The angular acceleration of the electric motor in rad/sec^2. (See Problem 8 for unit conversions.)

Solution:

Problem 10: Given: A two-speed electric drill motor has a constant angular acceleration equal to 3 revolutions per second per second (3 rev/sec^2). The drill motor accelerates from a low speed of 1040 rpm to its high speed (ω_f) in 4 seconds.

Find: The angular speed of the drill motor at high speed (ω_f) in rpm.

Solution:

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