

Supplemental Experiments Instructors Guide

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Measuring Vector Forces

Procedure

Lab Setup

- ❑ 1. Lab setup assembly is straightforward and should pose no difficulties.
- ❑ 4. Be sure to zero the scales carefully for best results.

NOTE: Make certain that the scales are not twisted or snagged. This could produce a fictitious resultant force. When done properly, this experiment should produce a very small resultant force.

Observations and Data Collection

1. Convert the mass of the hanging weights from grams to kilograms.

$$1 \text{ Gram} = \frac{1}{1000} \text{ Kilogram}$$

$$\begin{aligned} \text{Weight} &= 250 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} \\ &= \underline{0.25} \text{ kg} \end{aligned}$$

2. Use the following conversion equation to convert the mass of the hanging weights to units of force (weight) in Newtons.

$$\mathbf{F \text{ (Newtons)} = \text{Mass (kilograms)} \times \frac{9.8 \text{ Newtons}}{\text{kilograms}}}$$

$$\text{Force (N)} = \text{Mass (kg)} \times \frac{9.8 \text{ N}}{\text{kg}}$$

$$\text{Force (N)} = 0.25 \text{ (kg)} \times \frac{9.8 \text{ N}}{\text{kg}}$$

$$\text{Force (N)} = \underline{2.45 \text{ N}}$$

Data Table 1

| Component | Angle L (°) | Force Magnitude F (N) |
|---------------|-------------------------|-----------------------------|
| Scale A | $L_A = \underline{-30}$ | $F_A = \underline{2}$ |
| Scale B | $L_B = \underline{60}$ | $F_B = \underline{1.27}$ |
| Weight Hanger | $L_W = 180^\circ$ | $F_W = \underline{2.45}$ |

Measuring Vector Forces

- When using the protractor, be sure to carefully align the center index hole with the center of the O-ring.

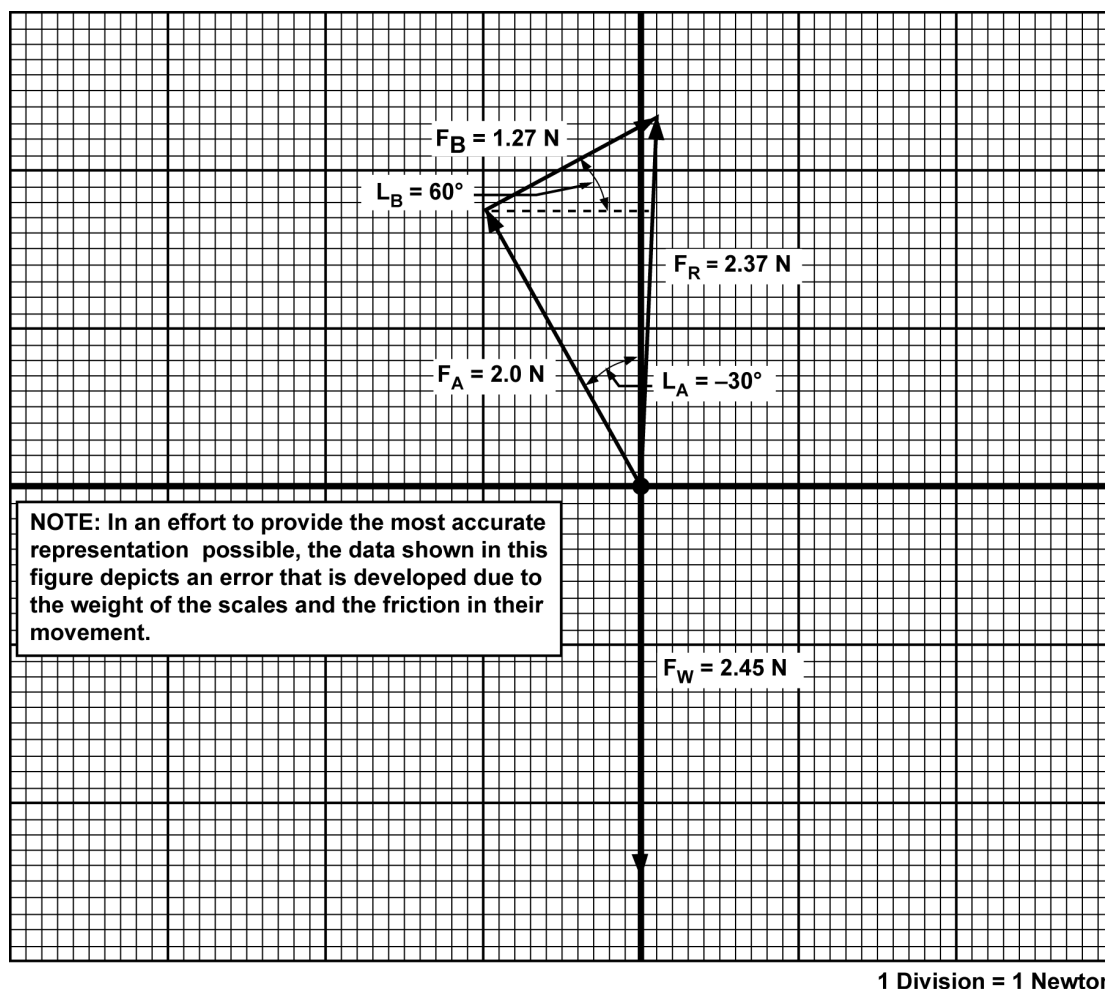
NOTE: The metal “O” ring provides a rather large origin. You may find it difficult to align the protractor so it is properly positioned. Misalignment of the protractor can account for a fictitious resultant force. You may want to practice measuring the forces and filling in the graph before your students attempt the experiment so you can help them to do it properly.

- Optional — If you wish to use a 360° vector notation, measure and record all angles as positive values measured clockwise from the vertical 0° axis.

Graphing the Data

- It is best to demonstrate to the students how to do the graphical solution before they attempt it themselves.
- Complete the following Data Graph.

Data Graph 1



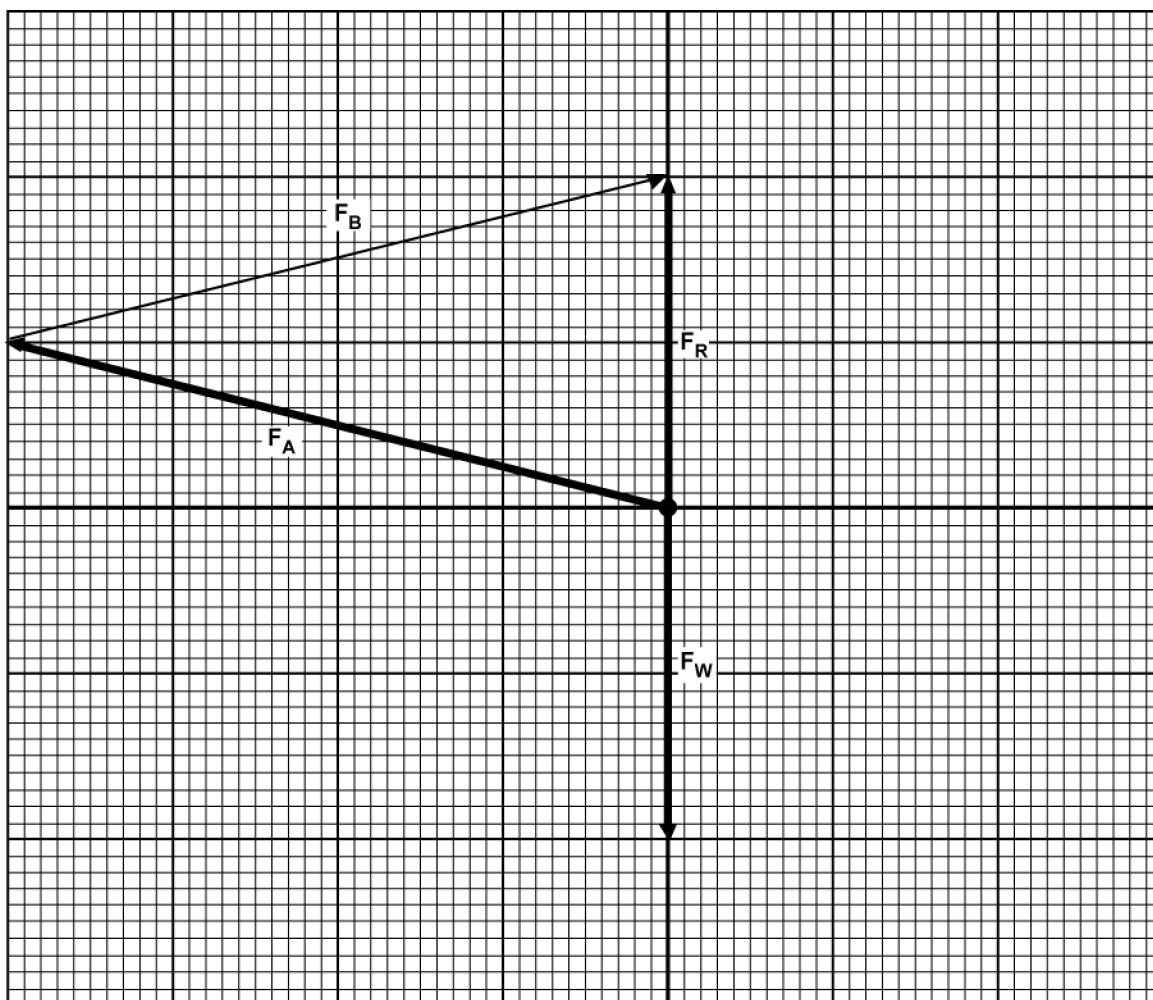
4. $F_R = \underline{2.37 \text{ N}}$ Angle of $F_R = \underline{+2.4^\circ}$

Questions and Interpretations

1. What should your resultant vector have been? It should be the same length as F_W at an angle of 0° . Was it as expected? It was very close to what was expected.

NOTE: A non-zero resultant force in question 2 refers to the non-zero angle in the resultant force vector.

2. Give as many reasons as you can for a non-zero resultant force. Improper zeroing of the scales, scale friction, improper measurement, etc. Anything that would introduce error into the collection of data is acceptable as an answer.
3. Figure 11 shows vectors acting at a point, similar to the experiments you did. Vector F_R is the resultant of vectors F_A and F_B . Draw the missing vector, F_B , on the graph in the Student Journal.



1 Major Division = 1 Newton

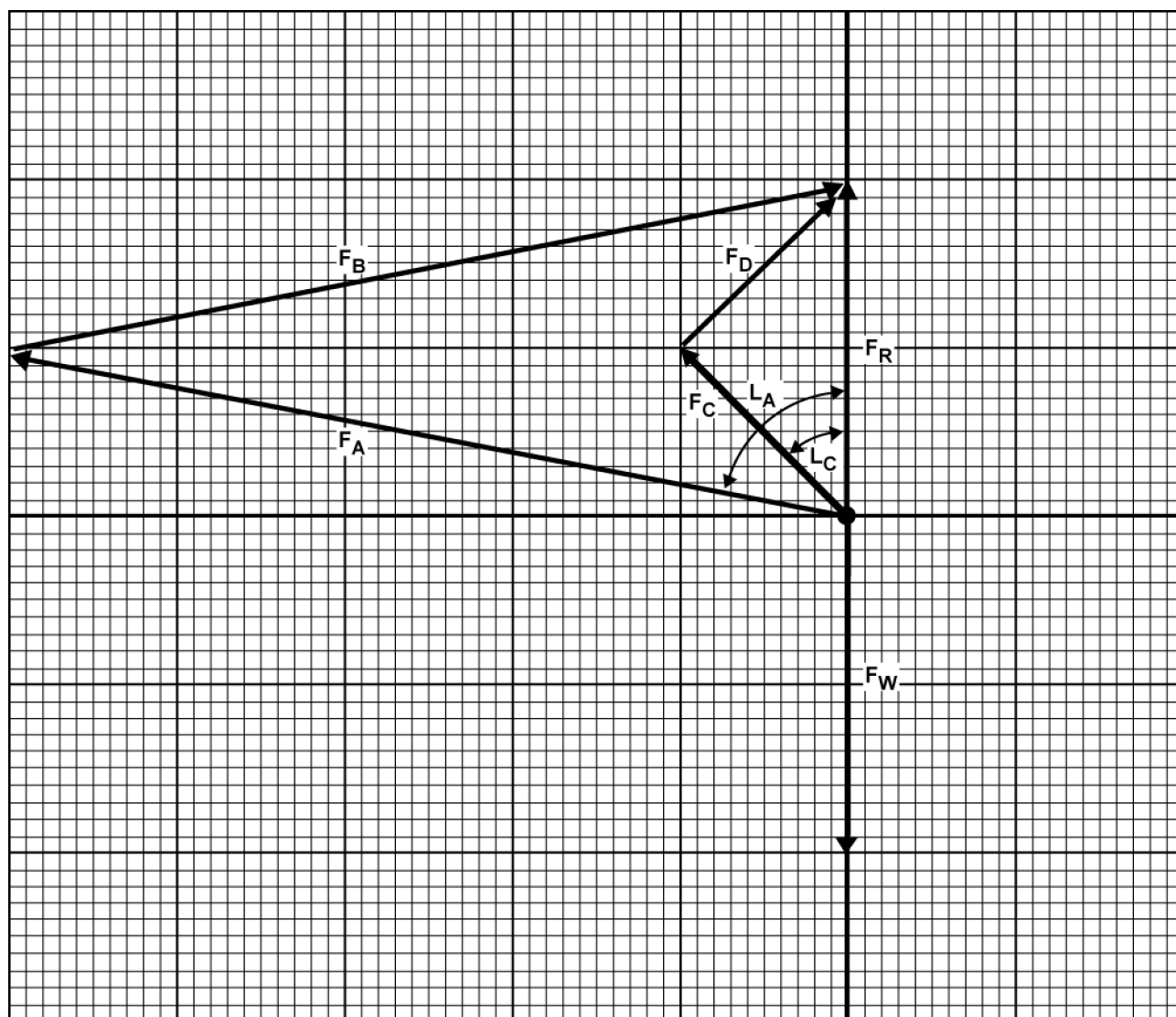
Measuring Vector Forces

4. If vector F_W in Figure 11 represents a force of 2 N, what force does vector F_A represent? Record the value in the Student Journal. *Since the forces are in equilibrium, the resultant vector F_R is equal to vector F_W . The vertical component of vector F_A is 1/2 of vector F_R or 1 N. The horizontal component of vector F_A spans 4 major divisions. With each major division equal to 1 Newton, the horizontal component of vector F_A is 4 N.*

$$F_A = \sqrt{1^2 + 4^2}$$

$$F_A = 4.12 \text{ N}$$

5. Looking at the vectors in Figure 12 below, the resultant vector can consist of the sum of vectors F_A and F_B or the sum of vectors F_C and F_D . Comparing the angles L_A and L_C , can we say that the vector (or the force that it represents) increases or decreases as the angle increases? *Increases.*



1 Major Division = 2 Newtons

Solving Vector Forces Using Trigonometric Functions

Observations and Data Collection

NOTE: Instructors should expect θ “theta” to be somewhere in between 70° – 80° .
The best estimate for θ is 75° .

Instructors should expect ϕ “phi” to be somewhere in between 30° – 40° .
The best estimate for ϕ is 35° .

Instructors should expect F_2 to be somewhere in between 0.8–1.4 N.

Instructors should expect F_1 to be somewhere in between 3.2–3.8 N.

1. Complete the following data tables.

Data Table 1

| | |
|-------------|--------------------|
| Angle $A =$ | <u>15</u> $^\circ$ |
| Angle $B =$ | <u>55</u> $^\circ$ |
| $\theta =$ | <u>75</u> $^\circ$ |
| $\phi =$ | <u>35</u> $^\circ$ |
| $F_1 =$ | <u>3.42</u> N |
| $F_2 =$ | <u>1.08</u> N |

Data Table 2

| | |
|------------|---------------|
| $F_{X1} =$ | <u>0.88</u> N |
| $F_{X2} =$ | <u>0.88</u> N |

Horizontal Balance of Forces

2. Although in the experiment the horizontal forces cannot be measured directly, they can be calculated mathematically. Enter the values in Data Table 2.

$$\begin{aligned} \mathbf{F_{X1}} &= \mathbf{F_1 \cos \theta} \\ &= 3.42 \text{ N} \times 0.259 \\ &= \underline{0.88} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F_{X2}} &= \mathbf{F_2 \cos \theta} \\ &= 1.08 \text{ N} \times 0.819 \\ &= \underline{0.88} \text{ N} \end{aligned}$$

-
2. Figure 9 shows the vertical component of F_2 . The vertical component of F_2 can be written as:

$$\begin{aligned} F_{Y2} &= F_2 \sin\phi \\ &= 1.08 \text{ N} \times 0.573 \\ &= \underline{0.619} \text{ N} \end{aligned}$$

Enter F_{Y2} in Data Table 3.

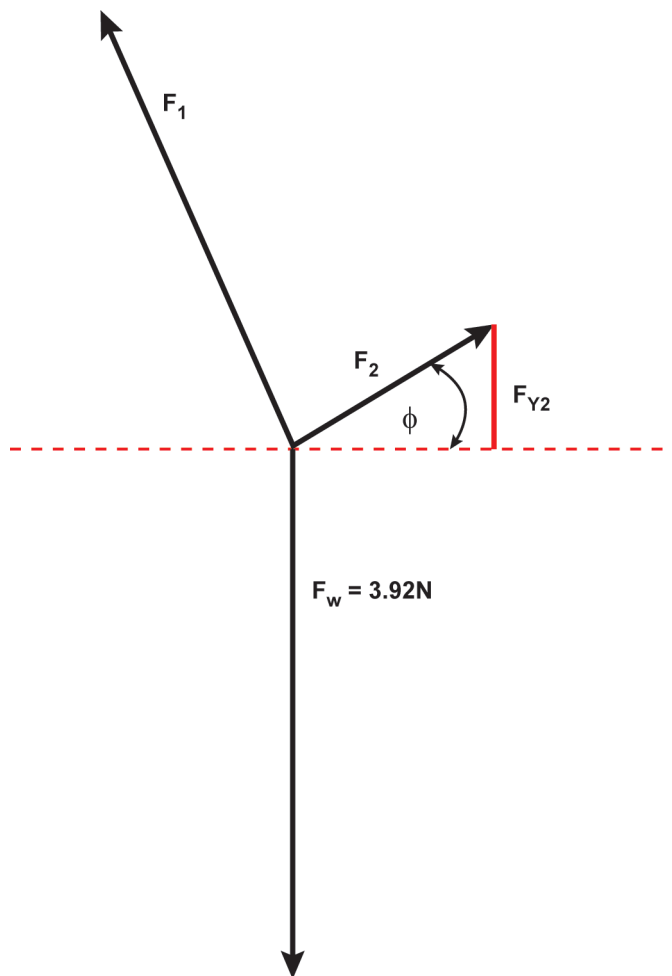


Figure 9
The vertical component of F_2 .

Data Table 3

| | |
|----------|----------------|
| F_{Y1} | <u>3.30</u> N |
| F_{Y2} | <u>0.619</u> N |

Solving Vector Forces Using Trigonometric Functions

3. Since the vertical components are acting in the same direction, they can be added as shown in Figure 10. The summation can be written as:

$$\begin{aligned}\mathbf{F_R} &= \mathbf{F_{Y1}} + \mathbf{F_{Y2}} \\ &= 3.30 \text{ N} + 0.619 \text{ N} \\ &= \underline{3.919 \text{ N}}\end{aligned}$$

Enter the value in Data Table 4.

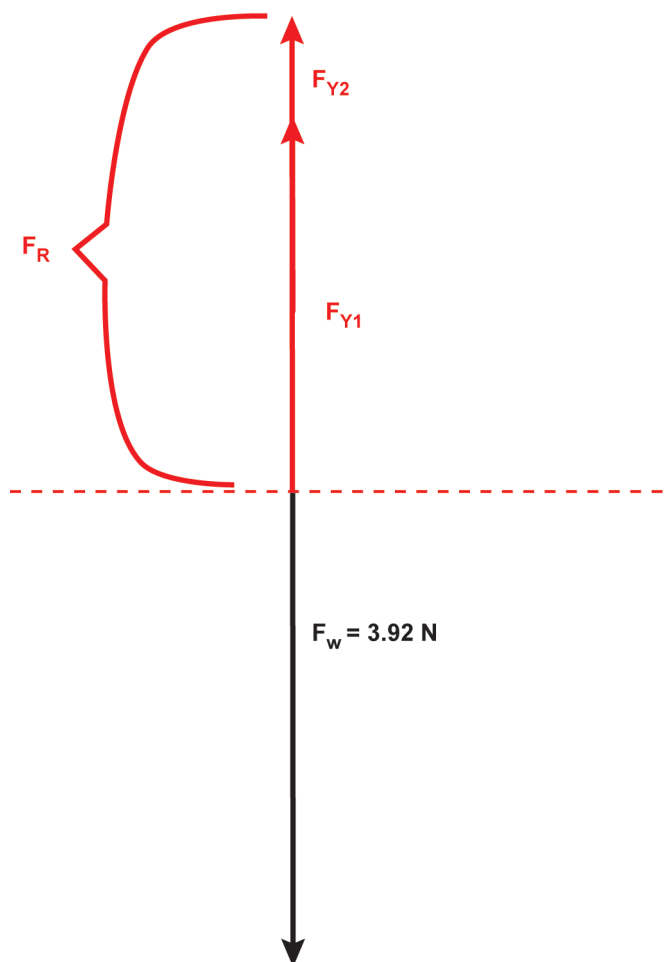


Figure 10
The addition of vectors

- ☐ 4. Convert the mass of the hanging mass to kilograms. Enter the value in Data Table 4.

$$\begin{aligned}\text{Mass of weight} &= 0.40 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} \\ &= \underline{0.40} \text{ kg}\end{aligned}$$

- 5. Use the following equation to convert the mass of the hanging weight to units of force in Newtons.

$$\begin{aligned} F_w &= \text{Mass (kg)} \times \frac{9.81 \text{ N}}{\text{kg}} \\ &= 0.40 \times \frac{9.81 \text{ N}}{\text{kg}} \\ &= \underline{3.92} \text{ N} \end{aligned}$$

Enter your answer in Data Table 4.

Data Table 4

| | |
|-------|----------------|
| F_R | <u>3.919</u> N |
| Mass | <u>0.40</u> kg |
| F_W | <u>3.92</u> N |

- 6. Is F_w equal to F_R ? Enter your answer in the Student Journal. F_w should be equal to or very close to F_R .

Questions and Interpretations

1. Show mathematically how F_{Y1} is the same as $F_1 \sin \theta$, and F_{Y2} is the same as $F_2 \sin \phi$. Refer to the equations in step 6.

$$\sin \theta = \frac{F_{Y1}}{F_1}$$

$$F_1 \sin \theta = \cancel{(F_1)} \frac{F_{Y1}}{\cancel{F_1}} = \underline{F_{Y1}}$$

$$\sin \theta = \frac{F_{Y2}}{F_2}$$

$$F_2 \sin \theta = \cancel{(F_2)} \frac{F_{Y2}}{\cancel{F_2}} = \underline{F_{Y2}}$$

Solving Vector Forces Using Trigonometric Functions

2. If θ “theta” was to equal 47° and ϕ “phi” was to equal 12° , what would the new F_1 and F_2 be? Assume $F_R = 3.92$ N.

$$F_2 = \frac{3.92}{\frac{\sin\theta \cos\phi}{\cos\theta} + \sin\phi}$$

$$F_2 = \frac{3.92}{\frac{\sin 47 \cos 12}{\cos 47} + \sin 12}$$

$$F_2 = \frac{3.92}{\frac{0.7153}{0.6819} + 0.2079}$$

$$F_2 = \frac{3.92}{1.0489 + 0.2079} = \frac{3.92}{1.2568} = \underline{3.119 \text{ N}}$$

$$F_1 = \frac{F_2 \cos\phi}{\cos\theta}$$

$$F_1 = \frac{3.119 \cos 12}{\cos 47}$$

$$F_1 = \frac{(3.119)(0.9781)}{0.6819} = \underline{4.473 \text{ N}}$$

3. If we changed the height of 2.5 Newton scale from 9 cm below the crossbar to 3 cm below the crossbar, how would that affect the vector force of the 5 newton scale (F_1)? Will it increase, decrease, or stay the same? Refer to Figure 5. decrease
4. Where should you position the 2.5 Newton scale in order for the 5 Newton scale to support the whole weight (vertically in line with the weight)? Perpendicular to 5 Newton scale.

Projectile Motion

Procedure Part 1

To get the best time for the ball toss, the ball should be tossed as high as possible.

- ☐ 4. Complete the following Data Table.

Data Table 1

| | Slow | Medium | Fast |
|------------|-------------|-------------|-------------|
| Time (sec) | <u>0.53</u> | <u>0.52</u> | <u>0.57</u> |

- ☐ 5. Did the speed of the rolling ball matter? If so, how? No. The speed of the rolling ball does not matter.

Procedure Part 2

- ☐ 4. The following formula uses initial velocity and acceleration to find the distance.

$$d = v_i t + 0.5 at^2$$

To use this equation the value of t must be one half of the measured t.

$$t = \frac{0.75 \text{ s}}{2}$$

$$= \underline{0.375 \text{ s}}$$

$$d = 0.0 \text{ m/s} + 0.5 \times 9.81 \text{ m/s} \times (0.375)^2$$

$$d = \underline{0.7 \text{ m}}$$

- ☐ 5. Find the final velocity.

$$v_f = v_i + at$$

$$v_f = 0.0 \text{ m/s} + 9.81 \text{ m/s} \times 0.375 \text{ s}$$

$$= \underline{3.7 \text{ m/s}}$$

OR

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 0.0 \text{ m/s} + 2 \times 9.81 \text{ m/s} \times 0.7 \text{ m}$$

$$v_f^2 = 13.73 \text{ m}^2/\text{s}$$

$$v_f = \underline{3.7 \text{ m/s}}$$

If time permits balls of different mass can be tried.

Data Table 2

| | |
|----------------------------|-----------------|
| t = | <u>0.75</u> sec |
| d = | <u>0.7</u> m |
| v _{ball thrown} = | <u>3.71</u> m/s |
| v _{ball caught} = | <u>3.71</u> m/s |

**Procedure
Part 3**

- 7. Complete the following Data Table.

Data Table 3

| | |
|-----------------------------------|---------------|
| Mass of the ball | <u>14.2</u> g |
| Height of the table | <u>91</u> cm |
| Initial piston position, ℓ_i | <u>50</u> mm |
| Final piston position, ℓ_f | <u>115</u> mm |

- 14. Complete the following Data Table.

Data Table 4

| | Trial 1 | Trial 2 | Trial 3 | Average |
|----------------------------------|----------------|----------------|----------------|----------------|
| Flight time | <u>0.56</u> s | <u>0.60</u> s | <u>0.72</u> s | <u>0.63</u> s |
| Horizontal displacement | <u>105</u> cm | <u>110</u> cm | <u>120</u> cm | <u>112</u> cm |
| Estimated height above the table | <u>32</u> cm | <u>35</u> cm | <u>40</u> cm | <u>36</u> cm |

Observations and Calculations

Finding the Ejection Velocity

1. Find the average flight time, the average horizontal displacement, and the estimated height above the table.

$$\begin{aligned}\text{Average flight time} &= \frac{0.56 \text{ s} + 0.60 \text{ s} + 0.72 \text{ s}}{3} \\ &= \underline{0.63 \text{ s}}\end{aligned}$$

$$\begin{aligned}\text{Average horizontal distance} &= \frac{105 \text{ cm} + 110 \text{ cm} + 120 \text{ cm}}{3} \\ &= \underline{112 \text{ cm}}\end{aligned}$$

$$\begin{aligned}\text{Estimated height above the table} &= \frac{32 \text{ cm} + 35 \text{ cm} + 40 \text{ cm}}{3} \\ &= \underline{36 \text{ cm}}\end{aligned}$$

4. Complete the following data table.

Data Table 5

| | |
|-----------------------------------------|------------------|
| Total moving mass, m_{total} | <u>71.1</u> g |
| | <u>0.0711</u> kg |
| Distance the piston moves, $\Delta\ell$ | <u>65</u> mm |
| | <u>0.065</u> m |
| Ejection velocity | <u>10.1</u> m/s |

Calculate the total moving mass.

$$\begin{aligned} m_{\text{total}} &= m_{\text{ball}} + 56.9 \text{ g} \\ &= 14.2 \text{ g} + 56.9 \text{ g} \\ &= \underline{71.1 \text{ g}} \end{aligned}$$

5. Convert the moving mass to kilograms.

$$\begin{aligned} m_{\text{total}} &= 71.1 \text{ g} \frac{1 \text{ kg}}{1000 \text{ g}} \\ &= \underline{0.0711 \text{ kg}} \end{aligned}$$

6. Find the distance.

$$\begin{aligned} \Delta\ell &= \ell_f - \ell_i \\ &= 115 \text{ mm} - 50 \text{ mm} \\ &= \underline{65 \text{ mm}} \end{aligned}$$

7. Convert $\Delta\ell$ from millimeters to meters.

$$\begin{aligned} \Delta\ell &= 65 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} \\ &= \underline{0.065 \text{ m}} \end{aligned}$$

9. Find the ejection velocity.

$$\begin{aligned}V &= \sqrt{\frac{k(\Delta\ell)^2}{m_{\text{total}}}} \\&= \sqrt{\frac{170 \text{ N/m} \times (0.065 \text{ m})^2}{0.0711 \text{ kg}}} \\&= \sqrt{\frac{0.718 \text{ m}^2}{0.0711 \text{ s}^2}} \\&= \underline{10.1 \text{ m/s}}\end{aligned}$$

Determining the Final Velocity

5. Calculate the vertical component of the ejection velocity.

$$\begin{aligned}\frac{v_y}{v_i} &= \sin(70^\circ) \\v_y &= v_i \sin(70^\circ) \\&= 10.1 \text{ m/s} \times 0.940 \\&= \underline{9.5 \text{ m/s}}\end{aligned}$$

6. Convert the table height to meters.

$$\begin{aligned}\text{Table height} &= 91 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} \\&= \underline{0.91 \text{ m}}\end{aligned}$$

7. Convert the height of the projectile apparatus above the table to meters.

$$\begin{aligned}\text{Height of the projectile apparatus above the table} &= 30 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} \\&= \underline{0.30 \text{ m}}\end{aligned}$$

8. Find the height between points C and D.

$$\begin{aligned}\text{Height between points C and D} &= 0.91 \text{ m} + 0.30 \text{ m} \\&= \underline{1.21 \text{ m}}\end{aligned}$$

9. Find the vertical component of the final velocity at point D.

$$\begin{aligned}
 (v_{y\text{final}})^2 &= v_y^2 + 2ay \\
 &= (9.5 \text{ m/s})^2 + (2 \times 9.81 \text{ m/s}^2 \times 1.21 \text{ m}) \\
 &= 90.05 \text{ m}^2/\text{s}^2 + 23.74 \text{ m}^2/\text{s}^2 \\
 &= 113.8 \text{ m}^2/\text{s}^2 \\
 v_{y\text{final}} &= \sqrt{113.8 \text{ m}^2/\text{s}^2} \\
 &= \underline{10.7 \text{ m/s}}
 \end{aligned}$$

10. Find the final projectile velocity at point D.

$$\begin{aligned}
 \frac{v_x}{v_i} &= \cos(70^\circ) \\
 v_x &= v_i \cos(70^\circ) \\
 &= 10.1 \text{ m/s} \times 0.342 \\
 &= \underline{3.45 \text{ m/s}}
 \end{aligned}$$

11. Use the Pythagorean theorem to the final velocity at point D.

$$\begin{aligned}
 v_f^2 &= v_x^2 + v_{y\text{final}}^2 \\
 &= (3.45 \text{ m/s})^2 + (10.7 \text{ m/s})^2 \\
 &= 11.90 \text{ m}^2/\text{s}^2 + 115.5 \text{ m}^2/\text{s}^2 \\
 &= 127.4 \text{ m}^2/\text{s}^2 \\
 v_f &= \sqrt{127.4 \text{ m}^2/\text{s}^2} \\
 &= \underline{11.3 \text{ m/s}}
 \end{aligned}$$

Data Table 6

| | |
|----------------------------------------------------------|-----------------|
| Initial projectile velocity, v_i | <u>10.1</u> m/s |
| Initial projectile velocity, v_y | <u>9.5</u> m/s |
| Height of the table | <u>0.91</u> m |
| Height between points C to D | <u>0.30</u> m |
| Final velocity (vertical component), $v_{y\text{final}}$ | <u>1.21</u> m |
| v_x | <u>3.45</u> m/s |
| $v_{f(\text{point D})}$ | <u>11.3</u> m/s |

Determine the Maximum Height the Ball Reaches

1. Determine the flight time from point C to point D.

$$\begin{aligned}t_{(C \text{ to } D)} &= \frac{v_{y\text{final}} - v_y}{a} \\&= \frac{10.9 \text{ m/s} - 9.5 \text{ m/s}}{9.81 \text{ m/s}^2} \\&= \underline{0.14 \text{ s}}\end{aligned}$$

2. Find the flight time from point A to point C.

$$\begin{aligned}t_{(A \text{ to } C)} &= t_{\text{stopwatch}} - t_{(C \text{ to } D)} \\&= 0.63 \text{ s} - 0.14 \text{ s} \\&= \underline{0.49 \text{ s}}\end{aligned}$$

3. Find the flight time from point A to point B.

$$\begin{aligned}t_{(A \text{ to } B)} &= \frac{t_{(A \text{ to } C)}}{2} \\&= \frac{0.49 \text{ s}}{2} \\&= \underline{0.25 \text{ s}}\end{aligned}$$

5. Find the maximum height of the ball above the projectile apparatus.

$$\begin{aligned}y_{\text{max}} &= v_y t + 0.5 a t_{(A \text{ to } B)}^2 \\&= \{9.5 \text{ m/s} \times 0.25 \text{ s}\} + \{0.5 \times (-9.81 \text{ m/s}^2) \times (0.25 \text{ s})^2\} \\&= 2.37 \text{ m} - 0.31 \text{ m} \\&= \underline{2.0 \text{ m}}\end{aligned}$$

6. Find the total height above the table.

$$\begin{aligned}\text{Total height (above table)} &= y_{\text{max}} + 0.30 \text{ m} \\&= 2.0 \text{ m} + 0.30 \text{ m} \\&= \underline{2.5 \text{ m}}\end{aligned}$$

Data Table 7

| | |
|-----------------------------------|----------------|
| $T_{(C \text{ to } D)}$ | <u>0.14</u> s |
| $T_{(A \text{ to } C)}$ | <u>0.49</u> s |
| $T_{(A \text{ to } B)}$ | <u>0.25</u> s |
| V_y | <u>9.5</u> m/s |
| $Y_{(\text{max})}$ | <u>2.0</u> m |
| Calculated height above the table | <u>2.3</u> m |
| Estimated height above the table | <u>0.91</u> m |

8. Reasons for the discrepancy between the calculate height and the estimated height. *Friction, air resistance, and the loss of impact force transmitted to the ball account for the loss in height.*

Calculating the Horizontal Displacement

3. Calculate the horizontal displacement.

$$\begin{aligned}\text{Horizontal displacement} &= v_x \times \text{flight time} \\ &= 3.45 \text{ m/s} \times 0.63 \text{ s} \\ &= \underline{2.1 \text{ m}}\end{aligned}$$

Data Table 8

| | |
|------------------------------------|-----------------|
| Horizontal velocity, v_x | <u>3.45</u> m/s |
| Average flight time | <u>0.63</u> s |
| Calculated horizontal displacement | <u>2.1</u> m |
| Measured horizontal displacement | <u>1.12</u> m |

5. Compare the measured horizontal displacement to the calculated horizontal displacement. *Some friction was not taken into consideration, the measured horizontal displacement was less than the calculated horizontal displacement. Not all of the energy was transferred to the ball from the spring.*

Questions and Interpretations

1. A basketball player throws a ball from the same distance to the basket with the same projection velocity. For angles other than 45° there are two possible angles. Will the low angle shot take more or less time than a high angle shot? A low angle shot will take less time than a high angle shot.
2. You are playing deep center field and you need to make a throw to home plate. However, you have very little strength. At what angle do you throw the ball to get the most distance? Neglect air resistance? at a 45° angle
3. Neglecting air resistance, if somebody dropped a sheet of paper and a bowling ball from a very tall building which would hit the ground first? They will hit the ground at the same time.