Math Lab 13 MS 1

Solving Problems That Involve Light Rays Reflected from the Surface of a Plane (Flat) Mirror

Calculating the Focal Length of Concave and Convex Mirrors

Locating Images Formed by Concave and Convex Mirrors

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach students ray-tracing techniques to determine how light rays are reflected from plane, concave, and convex mirrors.
2. Teach students how to locate the focal point of a convex or concave mirror.
3. Teach students how to locate the image points of light rays reflected from a concave or convex mirror.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete the activities. Students should have read the discussion material and looked at the examples for each activity before coming to this class. (How much you accomplish will depend on the math skills your students already have.)
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell students to read Lab 13*1/2, "Reflection of Light."

Math Skills Laboratory

Lab 13 MS1

Math Activities

Activity 1: ***Solving Problems That Involve Light Rays Reflected from the Surface of a Plane (Flat) Mirror***

Activity 2: ***Calculating the Focal Length of Concave and Convex Mirrors***

Activity 3: ***Locating Images Formed by Concave and Convex Mirrors***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you'll be able to do the following:

1. ***Use ray-tracing techniques and the law of reflection to find how light rays reflect from a plane (flat) mirror.***
2. ***Use ray-tracing techniques and the law of reflection to find how light rays reflect from a concave or convex mirror.***
3. ***Locate the focal point of a concave or convex mirror.***
4. ***Use ray-tracing techniques to locate the real image formed by light rays that reflect from a concave mirror.***
5. ***Use ray-tracing techniques to locate the virtual image formed by light rays that reflect from a convex mirror.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skill Lab Objectives.***
2. ***Study the examples given in Activities 1, 2 and 3.***
3. ***Work the problems for Activities 1, 2 and 3.***

NOTE: Because the introduction of student text material dealing with *reflection*, *refraction*, and *lenses* was rather concentrated, and because the topic of "geometrical" optics is new for many students, we have repeated, in the prefatory pages of both the math labs and the hands-on labs, many of the basic ideas treated earlier in the text. Since review is good for learning and retention, have your students read through the review material included here.

ACTIVITY 1

Solving Problems That Involve Rays Reflected From the Surface of a Plane (Flat) Mirror

MATERIALS

For this activity you'll need graph paper, a protractor, a compass and a straightedge or ruler.

Light rays strike the reflecting surface of an object and bounce off at an angle. A light ray that strikes the surface is called an ***incident ray.***

The ray that has bounced off the surface is called a ***reflected ray.*** (See Figure 1.)

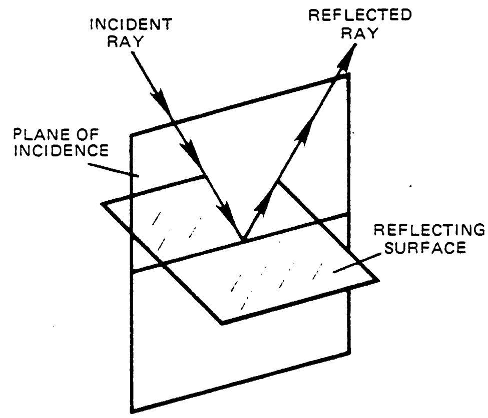


Fig. 1 Incidence and reflected ray.

A ***normal line,*** shown in Figure 2, is used to measure the angle at which an incident ray strikes a surface and the reflected ray leaves the surface. The ***normal line*** is drawn through the point of reflection and is perpendicular to the reflecting surface at the point where the incident ray strikes the surface.

The reflected light obeys the law of reflection. This law states that the angle formed between the incident ray and the normal line is equal to the angle formed between the normal line and the reflected ray. In short, angle A = angle B.

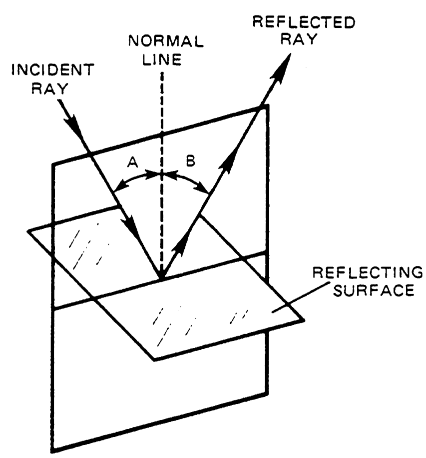


Fig. 2 Law of reflection: angle of incidence A equals angle of reflection B.

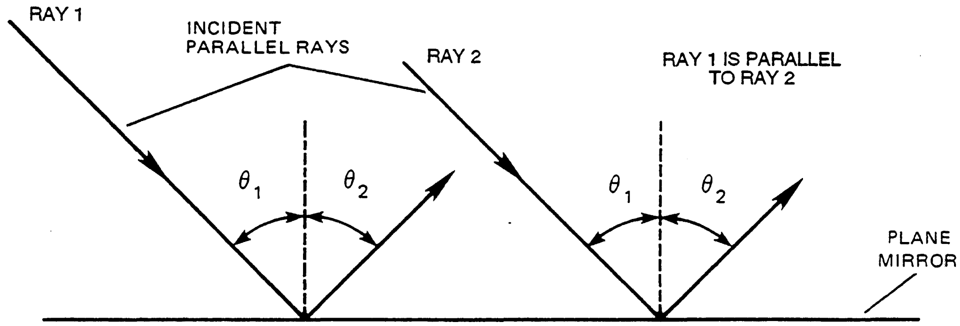
NOTE: Students should conclude that in Example A, incident parallel rays reflect as parallel rays. In Example B, they should see that incident rays that are not parallel will reflect as rays that are likewise nonparallel. At the same time, for any one ray, these examples reinforce the meaning of the law of reflection—ANGLE OF REFLECTION EQUALS ANGLE OF INCIDENCE.

Let's use the law of reflection, and the fact that light travels in a straight line, to find the angle of incidence and the angle of reflection for light rays reflected from a plane mirror.

Example A: *Parallel Light Rays Reflecting from a Plane Mirror*

Given: Rays of light coming from a point on a distant source are considered to be parallel rays. These rays strike the reflecting surface of a plane (flat) mirror. Two of these rays are shown in the sketch here. Each of the rays forms an angle of incidence (θ_1) with the normal line. The angle θ_1 is the same for both rays.

Find: The value, in degrees, for the angle of incidence (θ_1) and the angle of reflection (θ_2) for the rays.

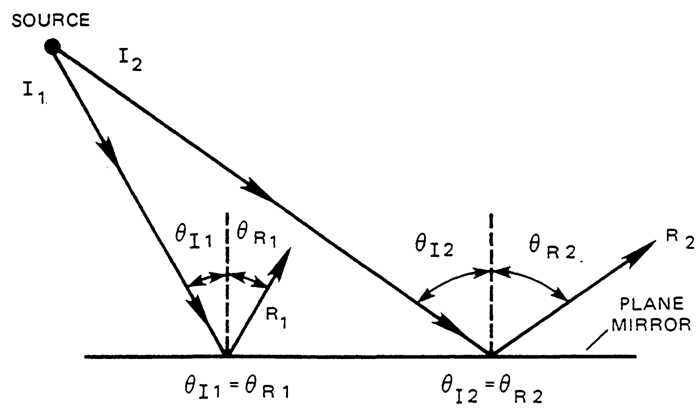


Solution: Use a protractor to measure the angle formed between the incident ray and normal line (θ_1), and the normal line and reflected ray (θ_2). For each ray, the angle of incidence equals the angle of reflection ($\theta_1 = \theta_2$). Careful measurements show that $\theta_1 = 45°$ and $\theta_2 = 45°$, for both figures.

Example B: *Nonparallel Light Rays Reflecting from a Plane Mirror*

Given: Light rays that diverge from a nearby source are ***not*** parallel rays. These rays strike a plane mirror and form angles of incidence (θ_{I1} and θ_{I2}) with the normal lines, as shown.

Find: The value, in degrees, of the angle of reflection (θ_{R1}) for ray R_1, and the angle of reflection (θ_{R2}) for ray R_2.

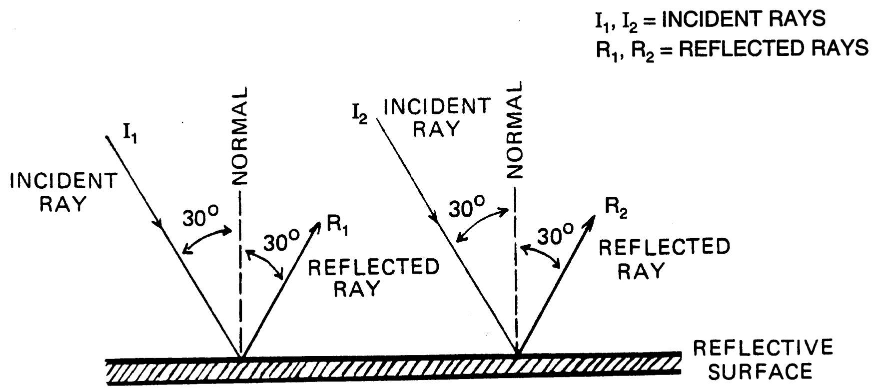


Solution: Use a protractor to measure the angle formed between the ***normal line*** and ***reflected ray.*** Since the incident rays are not parallel, the angle of reflection is different for ray R_1 and R_2. But notice that the angle of incidence for ray I_1 equals the angle of reflection for ray R_1 ($\theta_{I1} = \theta_{R1}$). Also, the angle of incidence for ray I_2 equals the angle of reflection for ray R_2. ($\theta_{I2} = \theta_{R2}$.) Careful measurements show that $\theta_{I1} = \theta_{R1} = 30°$; that $\theta_{I2} = \theta_{R2} = 55°$.

ANSWERS TO PRACTICE EXERCISES

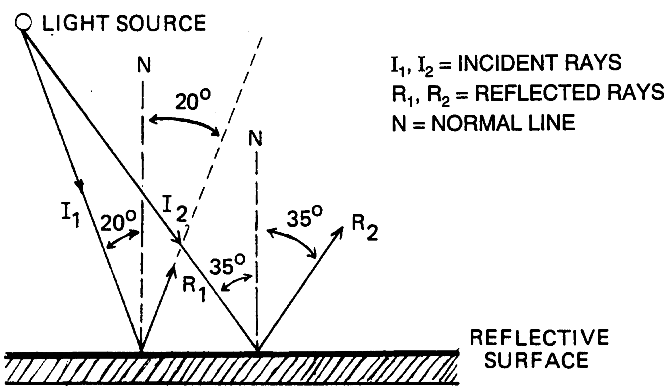
Activity 1:

Problem 1:



Reflected rays R_1 and R_2 <u>are parallel</u> to each other.

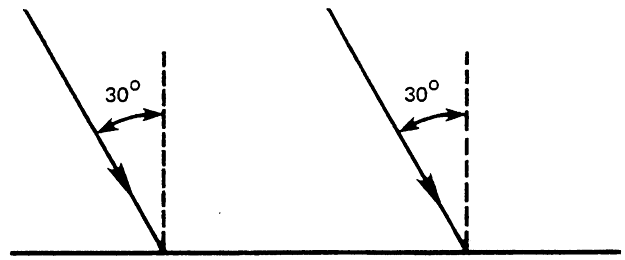
Problem 2:



Reflected rays R_1 and R_2 are <u>not parallel</u> to each other.

PRACTICE EXERCISES

Problem 1: Given: Two parallel, incident rays of light from a point on a distant source, strike the surface of a plane mirror. At the points of reflection, the angle of incidence is 30 degrees with the normal line.



Find: Draw and label a diagram showing the incident ray, the normal line and the angle of reflection for each ray. (Use graph paper, a ruler and a protractor.)

Solution:

Problem 2: Given: Two nonparallel, incident rays of light from a point on a nearby source strike the surface of a plane mirror. The angle of incidence for ray one is 20°. The angle of incidence for ray two is 35°.

Find: Draw and label a diagram showing the incident ray, the normal line and the angle of reflection for each ray. (Use graph paper, a ruler and protractor.)

Solution:

ACTIVITY 2

Calculating the Focal Point of Concave and Convex Mirrors

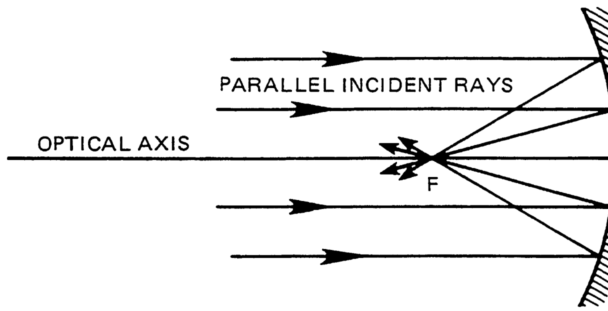
MATERIALS

For this activity, you'll need graph paper and a ruler.

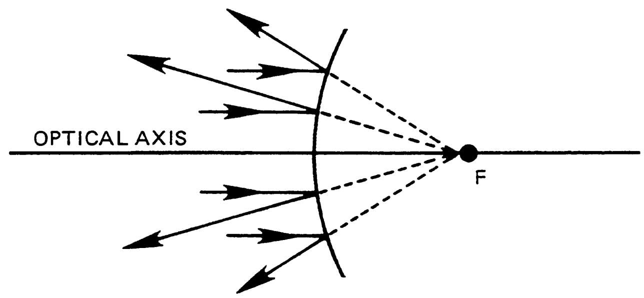
Concave and convex mirrors each have a ***focal point.*** Parallel rays that strike a concave mirror converge to the focal point F in *front* of the mirror. (See Figure 3a.)

Parallel rays that strike a convex mirror reflect and spread out appearing to diverge from a focal point F *behind* the mirror. (See Figure 3b.)

NOTE: We reemphasize that parallel light striking a concave mirror, as in Figure 3a, will not reflect to a single point F (the focal point) unless the transverse dimension of the mirror is a small portion of the sphere it's a part of. The same caution holds for the light rays shown in Figure 4b. The condition that describes **how small** the transverse dimension has to be is beyond the scope of this introductory treatment. But rest assured that designers of lenses and mirrors are all aware of the condition, and try to design lenses and mirrors accordingly. If the transverse dimension of a mirror has to be large, to collect more light, for example, then special grinding of the surface must be done to ensure clean, well-defined images. The Hubble Telescope episode is an excellent example of this. The initial special grinding/polishing of the outer mirror surface was not done correctly. Fuzzy images resulted that could not be tolerated. As a result, astronauts had to fly to outerspace to "electronically" correct the "surface error."



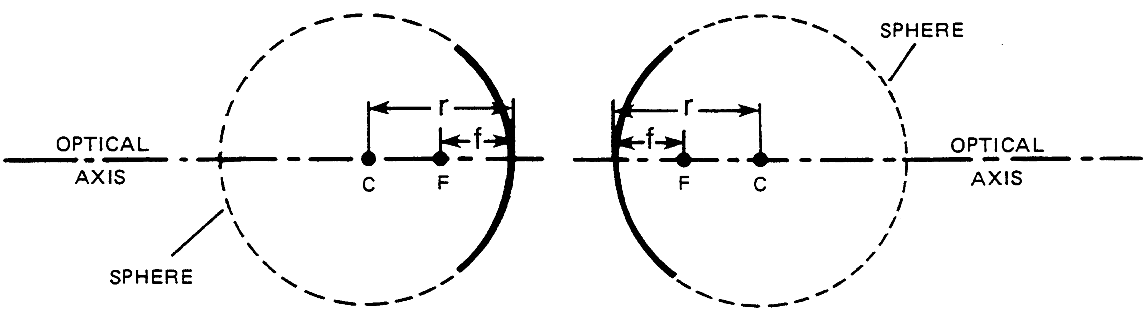
a. Rays converge at focal point F
of concave mirror.



b. Rays reflect and appear to diverge from
focal point F of convex mirror.

Fig. 3 Illustration of focal point for concave and convex mirrors.

The distance from the focal point to the mirror surface is called the ***focal length.*** The focal length (f) for a concave or convex mirror is equal to one-half of the radius of curvature (r) of the mirror. (See Figure 4.)



a. Concave mirror

b. Convex mirror

Fig. 4 Focal length f and radius of curvature r for mirrors.

In equation form, the relationship between the focal length (f) and the radius of curvature (r) is written as:

$$f = \frac{1}{2} r$$

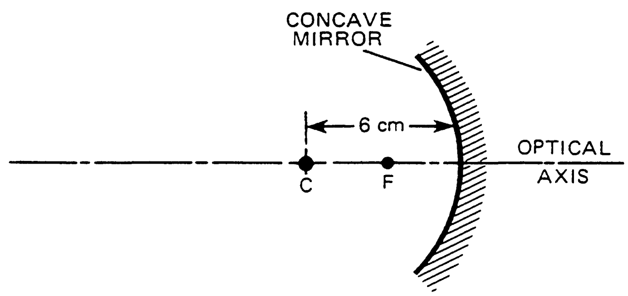
The radius of curvature (r) of the mirror is found by treating the surface of the mirror as if it were part of a ***complete sphere.*** The radius of curvature (r) is the distance from the center of the sphere to any point on the surface of the sphere. From the equation $f = \frac{1}{2} r$, the focal point is located halfway between the center of the sphere and the surface of the sphere. Examples C and D use the equation to determine the focal length of concave and convex mirrors.

(This page is intentionally blank.)

Example C: ***Calculating the Focal Length of a Concave Mirror***

Given: The radius of curvature of a concave mirror is 6 centimeters.

Find: The focal length of the concave mirror.



Solution: To find the focal length, use the equation:

$f = \frac{1}{2} r$, where $r = 6$ cm.

So $f = \frac{1}{2} \times 6$ cm,

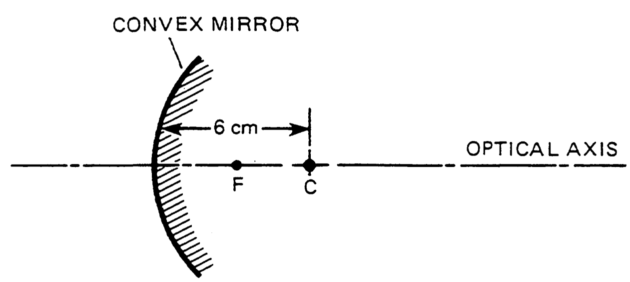
or $f = 3$ cm.

The focal length is 3 centimeters. The focal point is located halfway between point C and the mirror surface in the illustration shown here. Therefore, the focal point (F) is 3 centimeters from the mirror surface, to the front (or left) of the mirror surface.

Example D: ***Calculating the Focal Length of a Convex Mirror***

Given: The radius of curvature of a convex mirror is 6 centimeters.

Find: The focal length of the convex mirror.



Solution: To find the focal point, use the equation:

$f = \frac{1}{2} r$, where $r = 6$ cm.

So $f = \frac{1}{2} \times 6$ cm,

or $f = 3$ cm.

The focal length is 3 centimeters. The focal point is located halfway between point C and the mirror in the illustration shown here. The focal point is, therefore, 3 centimeters from the mirror surface—to the ***rear*** (or right) of the mirror surface.

ANSWERS TO PRACTICE EXERCISES

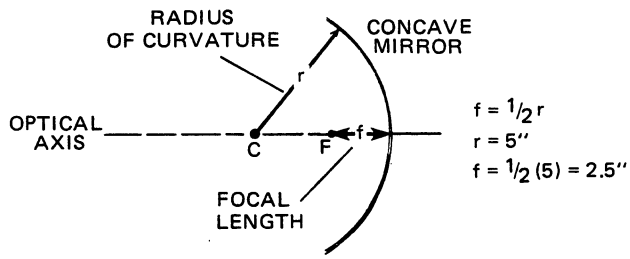
Activity 2:

Problem 3: Use the equation f = 1/2 r where r = 5 in.

$$f = \frac{1}{2} r = \frac{1}{2} (5\text{ in}) = 2.5 \text{ inches}$$

f = 2.5 inches

The focal point F is 2.5 inches in front of the mirror surface.

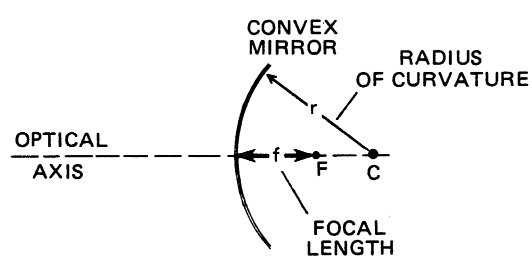


Problem 4: Use the equation f = 1/2 r, where r = 3 in.

$$f = \frac{1}{2} r = \frac{1}{2} (3\text{ in}) = 1.5 \text{ inches}$$

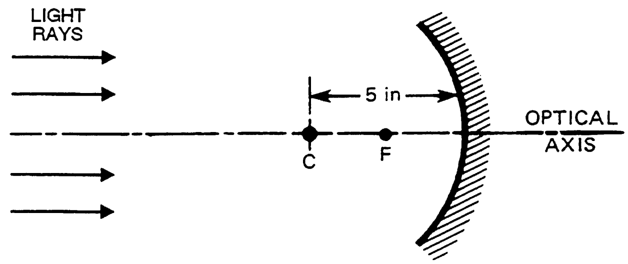
f = 1.5 inches

The focal point F is 1.5 inches behind the mirror surface.



PRACTICE EXERCISES

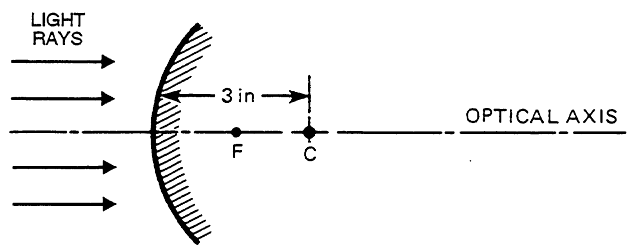
Problem 3: Given: The radius of curvature of a concave mirror is 5 inches.



Find: The focal length of the mirror. Draw a diagram of the mirror. On it, label the center of curvature C, the radius of curvature r, the focal *point,* F, and the focal *length,* f.

Solution:

Problem 4: Given: The radius of curvature of a convex mirror is 3 inches.



Find: The focal length of the convex mirror. Draw a diagram of the mirror. On it, label the center of curvature C, the radius of curvature r, the focal *point,* F and the focal *length,* f.

Solution:

ACTIVITY 3

Locating Images Formed by Concave and Convex Mirrors

MATERIALS

For this activity, you'll need graph paper, a straightedge and a compass.

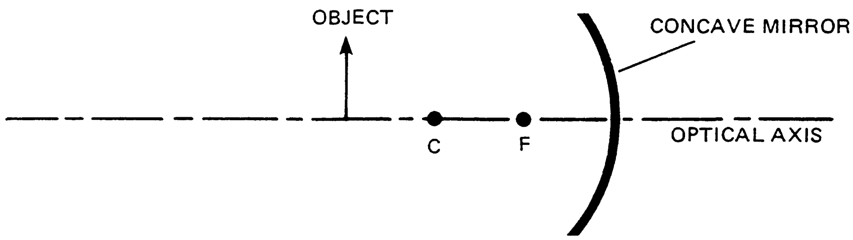
Let's use the law of reflection and the fact that light travels in a straight line to understand a technique called ***ray tracing.*** Ray tracing is a way to determine the paths that light rays take as they travel through an optical system.

NOTE: Be sure to study the figure for Example 3 on the adjoining text page in conjunction with the two figures that follow on the next page. Taken together, the three figures show how to trace light rays, from point-by-point on the object to the corresponding points on the image, thereby developing the entire image.

Example E shows how ray tracing is used to find the image point of rays reflected from a concave mirror.

Example E: *Finding the Image Point of Light Rays Reflected from a Concave Mirror*

Given: Sunlight is reflected from an object (such as an arrow) onto a concave mirror. (See the following sketch.) The radius of curvature (r) of the mirror is 6 centimeters. Therefore, the focal point (F) is located 3 centimeters from the surface of the mirror.

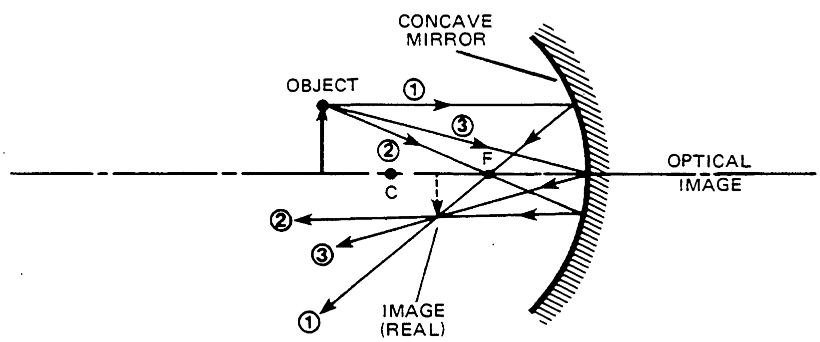


Find: The image point created by the light rays after reflection from the mirror.

Solution: The image point can be found by using the ray-tracing technique. Point C represents the center of the spherical surface of the mirror. It's called the ***center of curvature***. Any line drawn from point C to the surface of the mirror is a normal line. That's because all such lines are perpendicular to the surface. The focal point F is located halfway between the center of curvature C and the mirror surface.

A special normal line, called an ***optical axis,*** divides the mirror surface in half, top to bottom.

Between any point on the object (such as the tip of the arrow) and the surface of the mirror, trace an incident ray (ray 1) that is parallel to the optical axis. See the sketch. From the point where the ray strikes the mirror, draw a reflected ray that passes ***through the focal point F.***



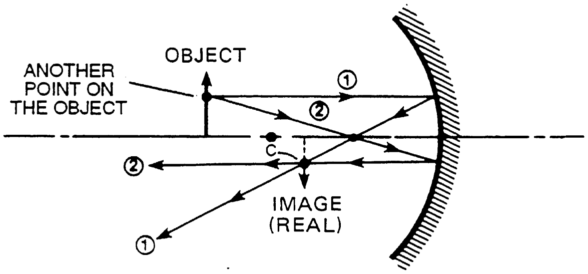
Starting again at the same point on the object (the tip of the arrow), trace an incident ray (ray 2) that goes through point F and strikes the mirror. The ray then is reflected back below and parallel to the optical axis.

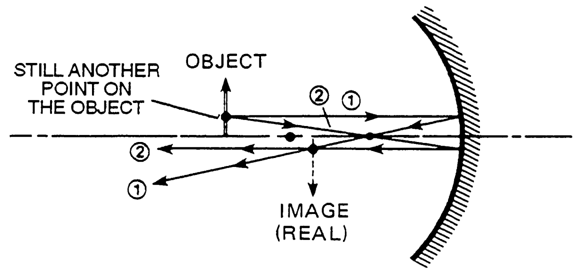
As a check, a third line can be drawn. This third line starts at the arrow's head and is drawn to the point where the optical axis and reflecting surface intersect. Since the optical axis is a normal line and the law of reflection holds true—ray 3 is reflected at the same angle as it is incident. Reflected ray 3, ray 1 and ray 2 all should meet at a common point of intersection.

The point where rays 1, 2 and 3 intersect is the ***image*** point of the arrowhead.

By repeating the ray-tracing procedure for other points along the object (arrow), a series of image points would be located, as shown on the following page. The points all would lie along the same line. If a screen were placed where the line is formed, you would see an image of the object. This image is called the ***real image.*** It would appear to be upside down and smaller than the object itself.

NOTE: In the lower figure in Example F, adjoining text page, ask your students how the third ray would be drawn. They should reply that it is the ray drawn from the object point to the vertex (where convex surface and optical axis intersect) and thereafter reflecting off with angle of reflection equal to angle of incidence. If this is done, extending the reflected ray backwards, as was done for ray 2, the backward extended line should cross at the top of the virtual image determined by rays 1 and 2.

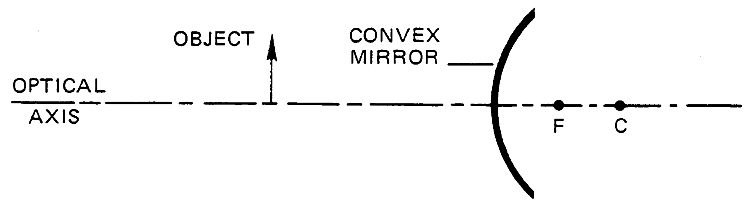




Now let's see how ray tracing is used to find the image points of rays reflected from a convex mirror.

Example F: ***Finding the Image Point of Light Rays Reflected from a Convex Mirror***

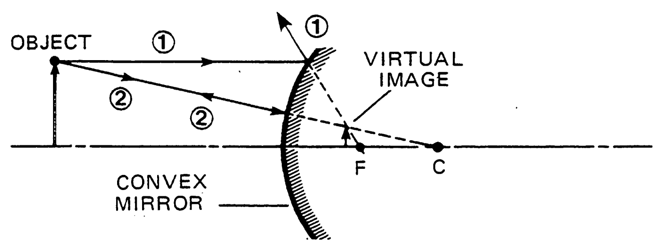
Given: Sunlight is reflected from an object (such as an arrow) onto a convex mirror. (See the sketch.) The radius of curvature of the mirror is 6 centimeters. Therefore, the focal point F is ***behind*** the mirror, 3 centimeters from the surface of the mirror.



Find: The image point created by the light reflected from the mirror.

Solution: The image point can be found by using the ray-tracing technique. Point C represents the center of curvature of the mirror. Any line drawn from point C to the surface of the mirror is a normal line. The focal point F is located halfway between the center of curvature C and the mirror surface. Again, a special normal line—the optical axis—divides the mirror surface in half.

Between any point on the object (such as the tip of the arrow) and the surface of the mirror, trace an incident ray (ray 1) that is parallel to the optical axis. (See the sketch.) From the focal point F, trace an imaginary (dashed) line to where ray 1 hits the mirror. From that point, continue drawing a solid, rather than dashed, line. The solid line represents the reflected ray, diverging outward.



Starting again at the same point on the object (the tip of the arrow), trace an incident ray (ray 2) that appears to go through the mirror and point C. Any line drawn from point C to the surface of the mirror is a normal line. Because of this, the angle of incidence is zero and the reflected ray bounces back ***along the same path*** it followed from the object to the mirror.

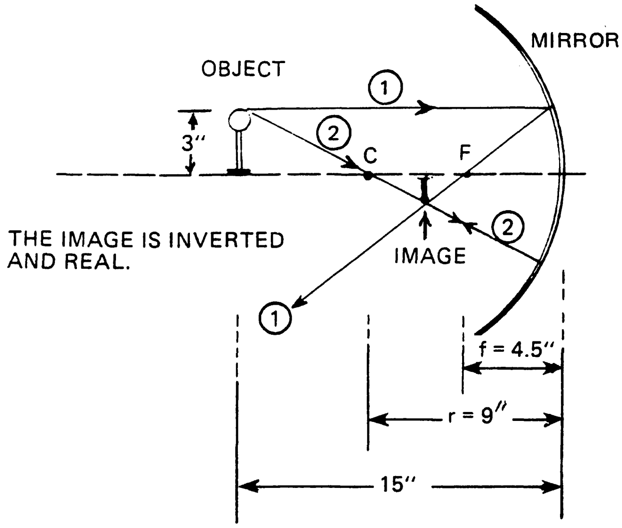
Look at the point where the extensions of ray 1 and ray 2 intersect (the dashed lines to the right of the mirror). This point is the ***image*** point of the arrowhead. By repeating the ray-tracing procedure for other points along the object (arrow), a series of image points will be located, as shown on the following page. The points all lie along a line in the same plane.

If a screen were placed in that plane, you would ***not*** see an image of the object on the screen. But you do see the image looking into the mirror. Because the image can't be detected on a screen, it's called a ***virtual*** image. The virtual image seen by someone looking into the convex mirror appears smaller than the object itself.

ANSWERS TO PRACTICE EXERCISES

Activity 3:

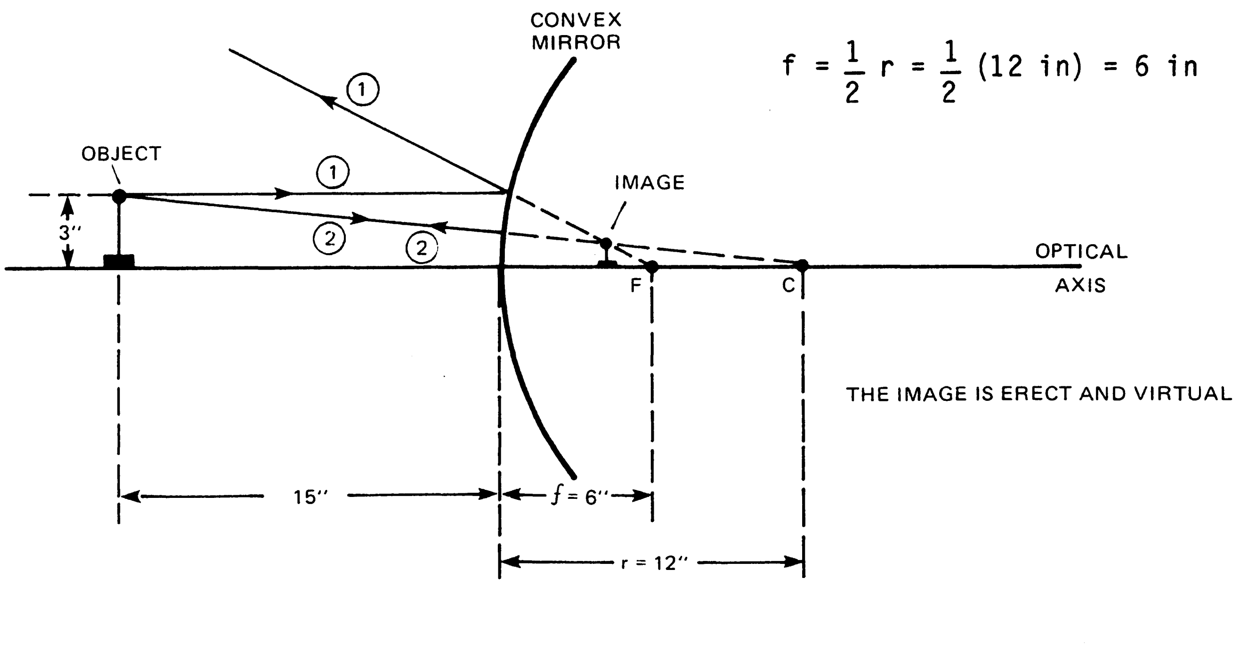
Problem 5:



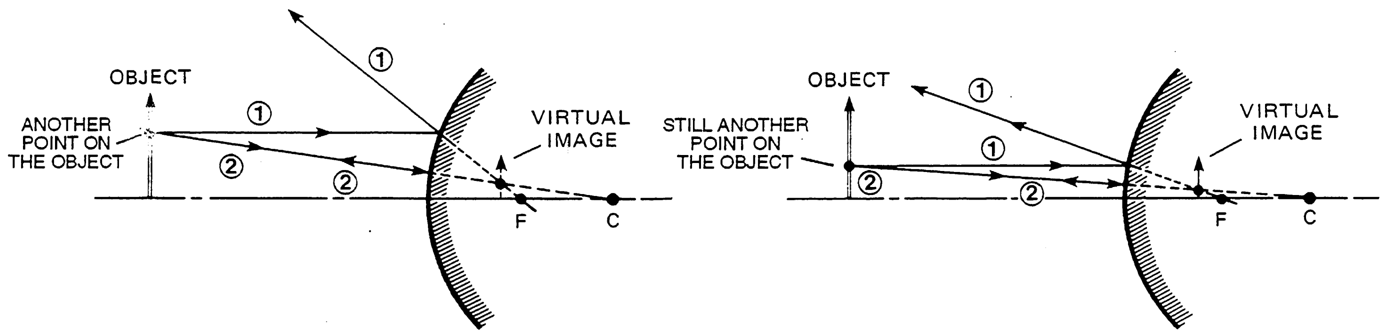
$$f = \frac{1}{2} r = \frac{1}{2} (9 \text{ in}) = 4.5 \text{ in}$$

ANSWER TO STUDENT CHALLENGE

Problem 6:



$$f = \frac{1}{2} r = \frac{1}{2} (12 \text{ in}) = 6 \text{ in}$$



PRACTICE EXERCISES

Problem 5: Given: A 3-inch-high object is placed 15 inches in front of a ***concave*** mirror. The radius of curvature of the mirror is 9 inches.

Find: The image point of the tip of the object. Using graph paper, straightedge and compass, draw a diagram of this optical system. Draw the diagram to scale, letting 1 centimeter = 3 inches. On the diagram, label the center of curvature C, focal point F and the optical axis. Use the ray-tracing technique outlined in Example E to locate the image point of the tip of the object. (***Note:*** The image will appear in front of the mirror. Hence, the image is a *real image.*)

Solution:

Student Challenge

Problem 6: Given: A 3-inch-high object is placed 15 inches in front of a ***convex mirror.*** The radius of curvature of the mirror is 12 inches.

Find: The image point of the tip of the object. Using graph paper, straightedge and compass, draw a diagram of this optical system. Draw the diagram to scale, letting 1 centimeter = 3 inches.

On the diagram, label the center of curvature (C), the focal point (F), and the optical axis. Use the ray-tracing technique outlined in Example F to locate the image point of the tip of the object. (***Note:*** The image point will appear *behind* the mirror. Hence, the image of the object is a *virtual image.*)

Solution: