Math Lab 13 MS 2

Solving Problems That Involve Refraction of Light at a Plane Optical Boundary

Graphically Locating the Image Formed by a Concave Lens

Graphically Locating the Image Formed by a Convex Lens

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach students how to solve problems involving the refraction of light between two different optical materials.
2. Teach students how to determine an angle of refraction by applying Snell's law.
3. Teach students ray-tracing techniques for locating the image formed by a concave or convex lens.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete as many activities as time permits. Students should have read the discussion material and looked at the examples for each activity before going to this class. (How much you accomplish will depend on the math skills your students already have.) You should summarize the main points in each activity, work an example or two, and have the students complete the Practice Exercises for each activity on their own.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell students to read Lab 13*3, "Refraction of Light."

NOTE: Somewhat contrary to the policy we've been following, this math lab does not deal with the material in Subunit 2, "Wave Optics: Interference and Diffraction." Rather it deals with refraction of light, part of the material discussed in Subunit 1, "Ray Optics: Reflection and Refraction." We've done that for two reasons. First, there is too much material on reflection and refraction in Subunit 1 to cover in a single math lab. So Math Lab 13MS1 was devoted only to **reflection.** We counted on Math Lab 13MS2 to do the related math and ray tracing for **refraction.** Second, there is very little math associated with interference and diffraction. Except for drawing some "complicated" interference diagrams, there's not much math that can be done at the high school level. So ... that's why we're doing refraction and ray tracing in Math Lab 13MS2. Be sure to explain this to your students.

Math Skills Laboratory

Lab 13 MS 2

MATH ACTIVITIES:

Activity 1: ***Solving Problems Than Involve Refraction of Light at a Plane Optical Boundary***

Activity 2: ***Graphically Locating the Image Formed by a Concave Lens***

Activity 3: ***Graphically Locating the Image Formed by a Convex Lens***

MATH SKILLS LAB OBJECTIVES

1. ***Solve problems that involve refracted light at a plane boundary between two different optical materials.***
2. ***Use Snell's law to find the angle of refraction.***
3. ***Use ray-tracing techniques to locate an image formed by a concave lens.***
4. ***Use ray-tracing techniques to locate an image formed by a convex lens.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples given in Activities 1, 2 and 3.***
3. ***Work the problems for Activities 1, 2 and 3.***

NOTE: As has been the practice in *Principles of Technology* throughout the various units, the math labs and hands-on labs begin with a recap of material explained earlier in the text proper. Thus, Figure 1, Example A, and Example B are intended to refresh the students' memories on topics that they'll need to understand before they work the problems that follow. Encourage them to read the prefatory material and work through the examples before they tackle the problems assigned to them.

ACTIVITY 1

Solving Problems That Involve Refraction of Light at a Plane Optical Boundary

MATERIALS

For this activity, you'll need graph paper, a straightedge, a protractor and a calculator.

In Subunit 1, you learned about the reflection and refraction of light rays. This Math Skills Lab will show you how a light ray is refracted when it strikes and passes into a transparent material. The concepts of reflection, refraction, interference and diffraction help us understand the operation and application of optical systems.

Two things happen when an incident ray of light strikes the plane surface of a transparent material:

1. If the plane surface is polished, the incident light ray is partly reflected.
2. If the plane surface is transparent, the incident light is partly refracted—and passes into the material.

The incident ray, reflected ray, refracted ray and normal all lie in the same geometric plane. Recall that this plane is called the ***plane of incidence.*** See Figure 1.

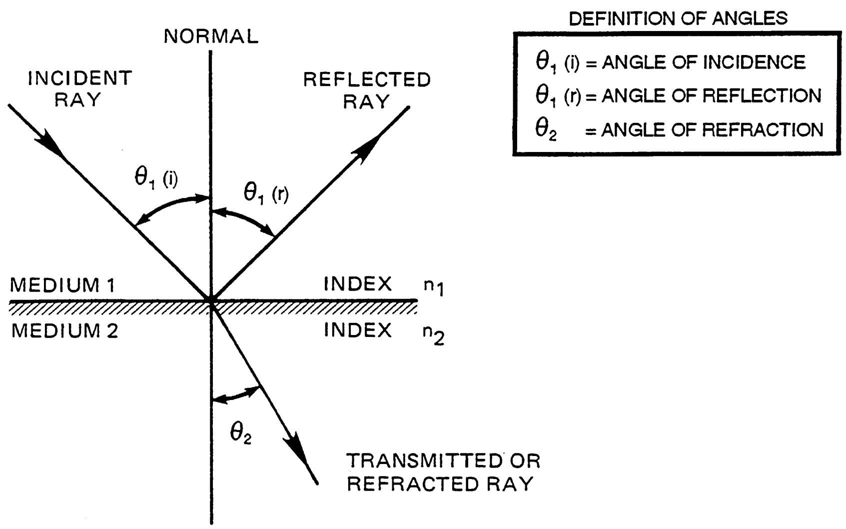

Fig. 1 Reflection and refraction at a boundary.

In the previous Math Skills Lab, you studied reflection. In this Math Skills Lab you'll study refraction. The law of refraction helps you find out what happens to a ray of light when it passes from one optical material into another. This law explains, for example, that when a light ray moves through a less dense medium (like air) into a more dense medium (like glass), the angle of refraction θ_2 is less than the angle of incidence θ_{1i}. This law also tells you that when a light ray moves from a more dense medium (glass) into a less dense medium (air), the angle of refraction is greater than the angle of incidence. The higher the index of refraction (n), the more *optically* dense the material. Thus, glass (n = 1.50) is more optically *dense* than air (n = 1.00).

Work through Example A and see what happens to a light ray heading from air into a sheet of glass. Examine the *refracted* ray only.

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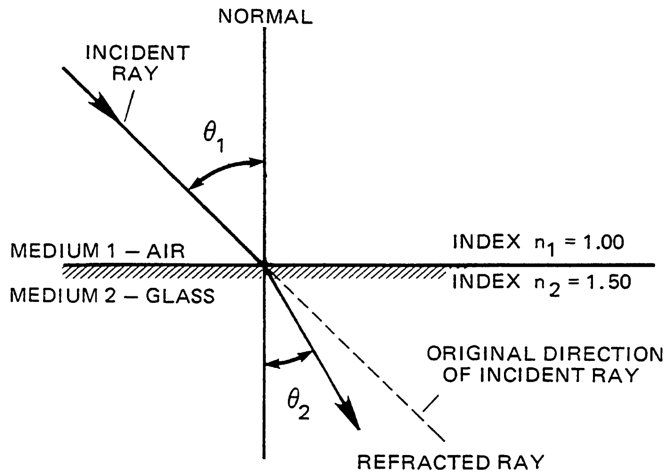
Example A: ***Refraction of a Light Ray That Passes from a Less Dense to a More Dense Optical Medium***

Given: A light ray moves through air and strikes a sheet of glass. The ray forms an angle of incidence θ_1 with the normal to the glass surface.



Find: Which way the ray bends when it enters the glass.

Solution: The ray moves from a less dense medium (air) into a more dense medium (glass). According to the law of refraction, the ray swings away from its original direction and bends *toward the normal*. (This is shown in the drawing here.) As a result, the angle of refraction (θ_2) is *less than* the angle of incidence (θ_1).

We've written that "according to the law of refraction, the ray bends toward the normal, making θ_2 less than θ_1." How does this happen? Calculate the value of the angle of refraction (θ_2). To do this, use Snell's law.

In equation form, Snell's law is written as:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where: n_1 = index of refraction for the first medium
$\sin \theta_1$ = sine of the angle of incidence
n_2 = index of refraction for the second medium
$\sin \theta_2$ = sine of the angle of refraction

Example B shows how Snell's law is used to calculate the value of the angle of refraction for the case shown in Example A.

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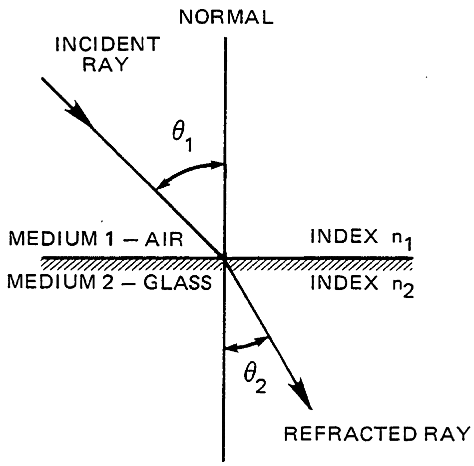
Example B: *Calculating the Value of an Angle of Refraction*

Given: A light ray moves from air into a sheet of glass. The incident ray enters the glass at an angle of incidence equal to 30° ($\theta_1 = 30°$). The index of refraction for air is 1.0 ($n_1 = 1.0$). For glass, the index of refraction is 1.50 ($n_2 = 1.50$).



Find: The angle of refraction (θ_2) in degrees.

Solution: Use Snell's law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, and the accompanying drawing.

n_1 = refractive index of air = 1.00

n_2 = refractive index of glass = 1.50

θ_1 = angle of incidence = 30°

θ_2 = angle of refraction (to be calculated)

To solve for the unknown angle (θ_2), first isolate $\sin \theta_2$ in the equation, $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Do this by dividing both sides of the equation by n_2. Then rearrange the equation to get *sin* θ_2 on the left side.

$$\frac{n_1 \sin \theta_1}{n_2} = \frac{\cancel{n_2} \sin \theta_2}{\cancel{n_2}}$$

$$\sin \theta_2 = \frac{n_1}{n_2} (\sin \theta_1)$$

$$\sin \theta_2 = \frac{1.0}{1.50} (\sin 30°)$$

To find *sin 30°* with a calculator, display 30° on the readout. Push the *sin* key. The display will read 0.5000. Thus, sin 30° (the sine of 30°) is 0.5.

Substitute 0.5 for sin 30° in the equation and complete the calculation.

$$\sin \theta_2 = \frac{1.0}{1.50} \times 0.50 = 0.3333$$

$\sin \theta_2 = 0.3333$

Now find the angle (θ_2) whose sine is equal to 0.3333. This angle is the arcsin of 0.3333. "Arcsin" is sometimes written as "$\sin^{-1}$." In equation form:

$\theta_2 = \sin^{-1} (0.3333) = \arcsin 0.3333$

To find the *arcsin*, display 0.3333 on the calculator readout. Push *inv sin or arc sin* keys. The display will read 19.47 degrees. Round this off to 19.5°.

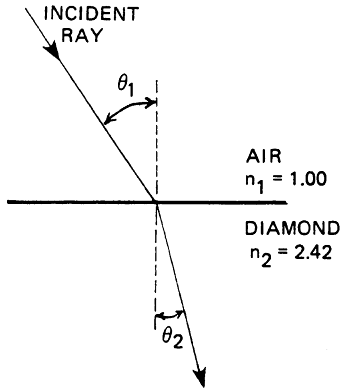
θ_2 = 19.5 degrees (the angle of refraction in glass)

Glass is denser than air. Therefore, the angle of refraction (19.5°) is less than the angle of incidence (30°). The refracted ray must indeed bend toward the normal, just as we said.

ANSWERS TO PRACTICE EXERCISES

Activity 1:

Problem 1:



θ_1 = angle of incidence = 35°
θ_2 = angle of refraction = ?

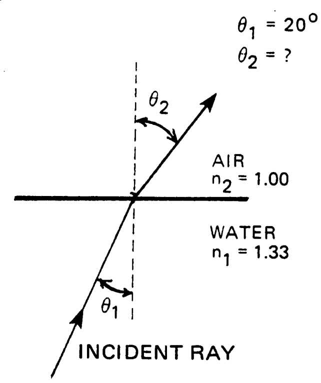
Snell's law states that $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Solve for $\sin \theta_2$ by dividing equation by n_2 and rearranging.

$$\frac{n_1 \sin \theta_1}{n_2} = \frac{\cancel{n_2} \sin \theta_2}{\cancel{n_2}}$$

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 = \frac{1}{2.42} \sin 35° \;;\; \sin 35° = 0.574$$

$$\sin \theta_2 = 0.41 \times 0.574 = 0.2353$$
$$\theta_2 = \sin^{-1} 0.2353 = 13.6°$$
$$\theta_2 \simeq 13.6°$$

Problem 2:



From the problem above, Snell's law can be written:

$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$ where:
$n_1 = 1.33$
$n_2 = 1.0$
$\sin \theta_1 = \sin 20° = 0.3420$
$\sin \theta_2 = ?$

$$\sin \theta_2 = \frac{1.33}{1.0} \sin 20°$$
$$\sin \theta_2 = 1.33 \times 0.3420 = 0.4548$$
$$\theta_2 = \sin^{-1} 0.4548 = 27.05°$$
$$\theta_2 \simeq 27°$$

Notice that $\theta_2 = 27°$ is <u>larger than</u> $\theta_1 = 20°$. The refracted ray bends <u>away</u> from the normal.

Answer to Student Challenge Problem 3 follows on Page T-63c.

ANSWERS TO PRACTICE EXERCISES, Continued

Problem 3: **Student Challenge** [In solving this problem, students will meet again the concept of "critical angles." The critical angle for this setup is 30° (Part b). So whenever the angle of incidence exceeds 30° (as in Part c, where the angle of incidence is 35°) total reflection, rather than refraction takes place. We discussed this earlier in the "Recap" at the end of Subunit 1. Refer students back to that section if further help on understanding critical angles is needed.]

Use Snell's law from Problem 1:

$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$, where: $n_1 = 2$, $n_2 = 1$

a. $\theta_1 = 28°$; $\sin 28° = 0.4694$

$\sin \theta_2 = \frac{2}{1} \sin 28° = 2 \times 0.4694 = 0.9388$

$\theta_2 = \sin^{-1} 0.9388 = 69.85° \simeq 70°$

$\theta_2 \simeq 70°$ (See sketch [a] below.)

b. $\theta_1 = 30°$; $\sin 30° = 0.5000$

$\sin \theta_2 = \frac{2}{1} \sin 30° = 2 \times 0.5000 = 1.0000$

θ_2 = arc sin 1.000 or inv sin 1.000

$\theta_2 = 90°$ (See sketch [b] below.)

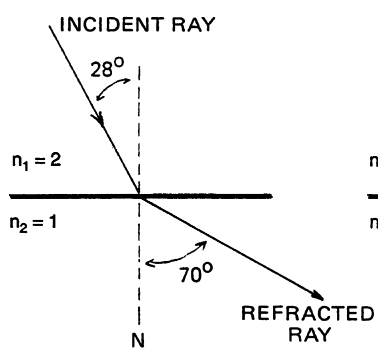
c. $\theta_1 = 35°$; $\sin 35° = 0.5735$

$\sin \theta_2 = \frac{2}{1} \sin 35° = 2 \times 0.5735 = 1.147$

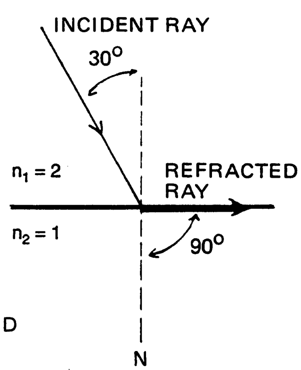
$\sin \theta_2 = 1.147$

Cannot be solved for θ_2. θ_2 does not exist since $\sin \theta$ has values only between 0.0 and 1.000. So the ray is not refracted—it's totally reflected. That's known as total internal reflection.

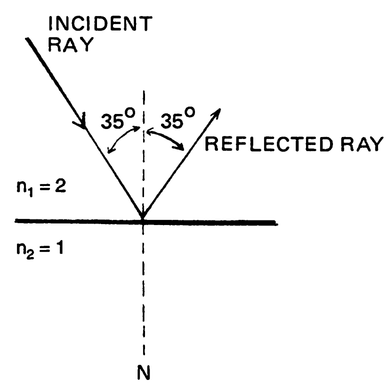
d. Drawings for a, b, and c.



a. Refracted ray is bent away from the normal as it should be.



b. Refracted ray is bent away from the normal; it heads along the boundary.



c. There is no refracted ray. Instead, the incident ray is totally reflected-- at the same angle. That is, angle of incidence(35°) equals angle of reflection (35°)

PRACTICE EXERCISES

Problem 1: Given: A light ray traveling in air strikes the surface of a diamond. The angle of incidence is 35°. The index of refraction for air (n_1) is 1.0. The index of refraction for diamond (n_2) is 2.42.

Find: The angle of refraction (θ_2) for the light that enters the diamond.

Solution: (Use Snell's law to find θ_2. Follow the procedure outlined in Example B.) Make a drawing that shows the *incident ray, normal, interface*, and the *refracted ray*. Label the *angles* and write the appropriate *index of refraction* for each medium.

Problem 2: Given: An underwater light is installed in a swimming pool. A ray of light slants upward through the water and makes an angle of incidence of 20° at the surface of the pool. The ray continues into the air above the pool. The index of refraction for water (n_1) is 1.33. The index of refraction for air (n_2) is 1.0.

Find: The angle of refraction (θ_2) for the light ray that enters the less dense medium of air.

Solution: (***Hint:*** Use Snell's law to find θ_2. Follow the procedure outlined in Example B. Since light is moving from a more dense medium [water] into a less dense medium [air], you should expect to find that θ_2 [refraction angle] is *greater* than θ_1 [incidence angle].) Make a drawing that shows the *incident ray, normal, interface*, and the *refracted ray*. Label the *angles* and write the appropriate *index of refraction* for each medium.

Student Challenge

Problem 3: Given: A light ray travels from a transparent medium, where $n_1 = 2$, to a less dense medium where $n_2 = 1$.

Find:

a. θ_2 when $\theta_1 = 28°$

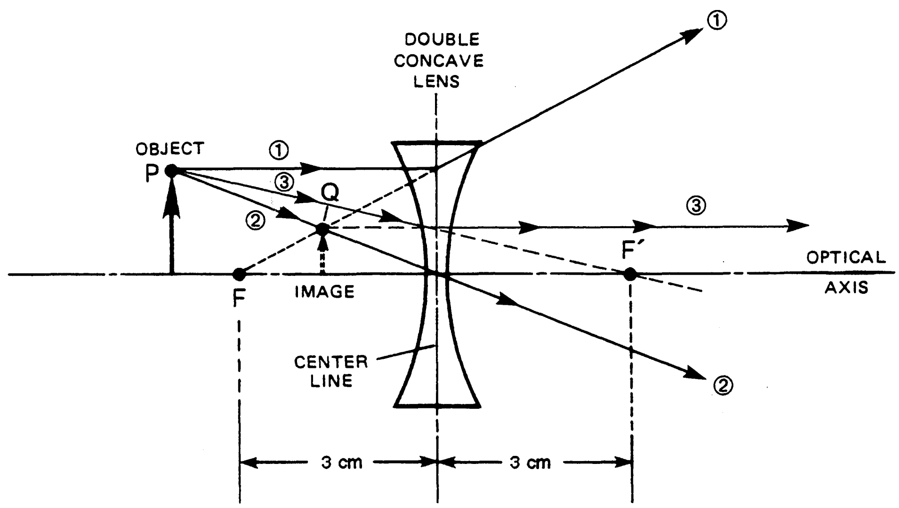
b. θ_2 when $\theta_1 = 30°$

c. θ_2 when $\theta_1 = 35°$

d. For Parts a, b and c, draw a diagram of the incident and refracted rays. On each diagram, label the angle of incidence and the angle of refraction.

Solution:

NOTE: For Example C, there is a third ray that can be drawn. It involves the use of the focal point F'. So you can use it as a ray to check the accuracy of the image point. Here's the way you do it. (See Figure below.) Draw a ray from P straight toward F'—as a solid line from P to the center line of the double concave lens, and as a dotted line from the center line on to F'. Next, start at the point where the line from P to F' intersects the center line. From this point, draw a solid line PARALLEL to the optical axis, extending to the right. Then extend it BACKWARDS as a dotted line. The backward extension should intersect the point Q on the image, providing a check of the correct image location. The **solid line** drawn from P to the center line of the lens, and then forward as a ray parallel to the optical axis, is the true path of the light ray. You can see that, together with rays 1 and 2 shown in the drawing for Example C, the three rays, 1, 2, and 3, form a set of diverging rays as they exit the double concave lens.



ACTIVITY 2

Graphically Locating the Image Formed by a Concave Lens

MATERIALS

For this activity, you'll need graph paper, a straightedge and a compass.

A lens is made from an optically transparent material such as glass or plastic. Since light goes through a lens, the shape of the front and back surfaces of the lens is important. Frequently, lenses have a combination of shapes. (See Figure 2.)

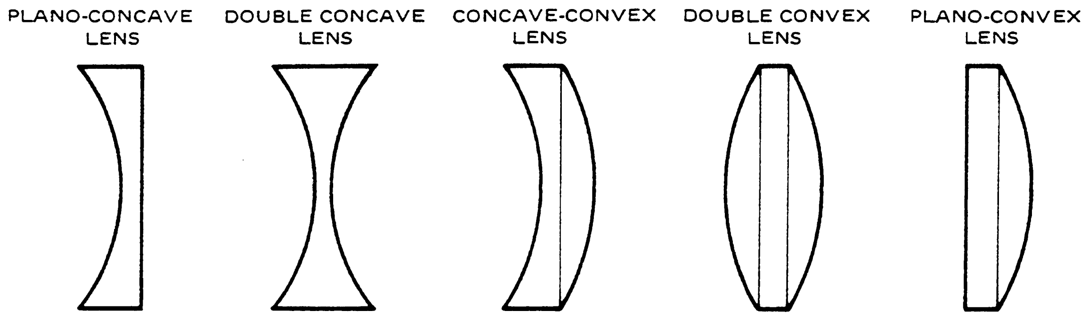


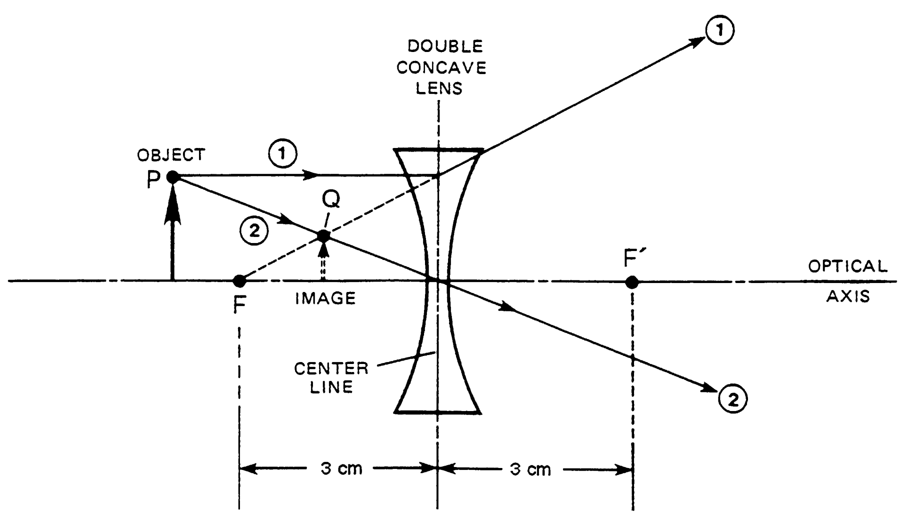
Fig. 2 Examples of lens shapes.

For example, one surface can be concave and the other surface flat. This combination is called a *plano-concave* lens. A *double concave* lens has two concave surfaces. If a lens has one concave and one convex surface, it's called a *concave-convex* lens. Two convex surfaces form a *double convex* lens. One flat and one convex surface make up a *plano-convex* lens.

You know how to find the image formed by a concave mirror. Locating the image formed by a concave lens is very similar to the ray-tracing technique for a concave mirror. Let's work through an example to see how this is done.

Example C: *Locating the Image Formed by a Concave Lens*

Given: Sunlight is reflected from an object (such as an arrow) and passes through a double concave lens. (See the following sketch.) The lens has two focal points, F and F′, each located 3 centimeters from the center of the lens. The focal point F is in front of the lens. The focal point F′ is behind the lens.



NOTE: Based on the discussion presented on the previous Teacher Page, you can draw ray 3, just as described there, for each of the two drawings shown on the adjoining student page. You will draw a ray from P towards F', bending it as it strikes the center line of the lens, so that it proceeds forward in a direction parallel to the optical axis. Extending the parallel line backwards should cause it to intersect the image point Q'—and Q" for the second drawing.

Find: The image point of the arrowhead formed by the concave lens.

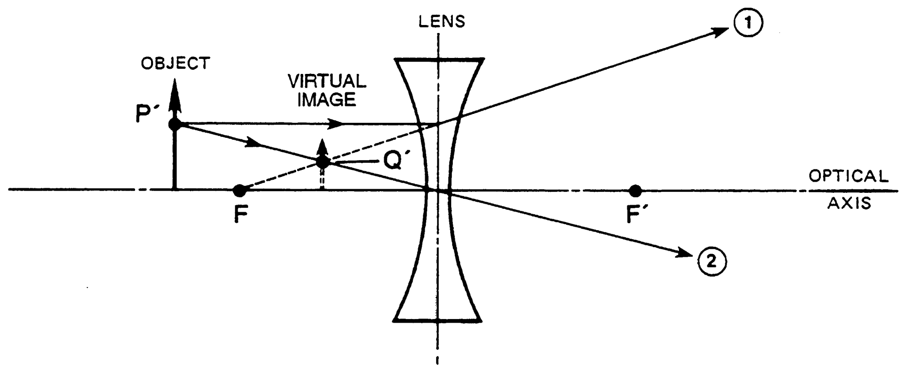
Solution: The image point can be found by using the ray-tracing technique. The optical axis divides the lens in half, top to bottom. A vertical center line (broken) is shown in the middle of the lens. All bending of light rays will be shown as occurring at this line, even though the bending actually takes place at the front and rear surfaces of the lens. (This gives the same result—and it's simpler doing it this way.)

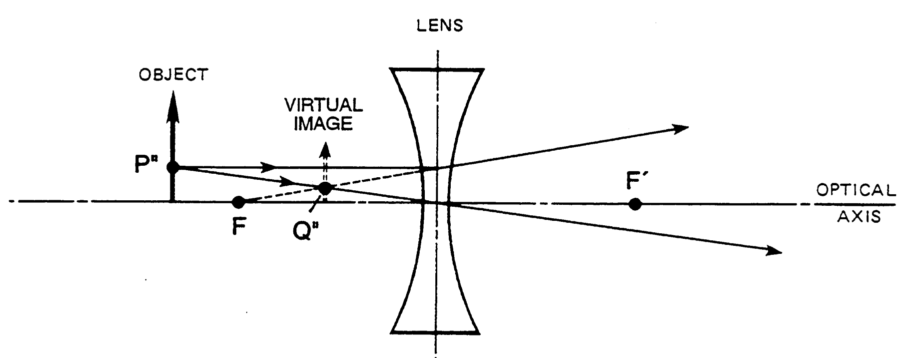
Step 1: Locate a point P at the tip of the arrow. From this point draw an incident ray (ray 1) that's parallel to the optical axis and stops at the center line. From focal point F, draw a (dotted) line to the point where the parallel ray meets the vertical center line. From that point, continue drawing a solid, rather than dotted, line. The solid line—to the right of the lens—represents the refracted ray leaving the lens in an upward direction. This is ray ①.

Step 2: Starting again at the tip of the arrow, (point P), drawn an incident ray (ray 2) that goes straight through the lens at its center point. This point is the intersection point between the vertical center line and the optical axis. This is ray ② .

Step 3: The image point (Q) of the arrow tip is the point where ray 2 and the dashed line of refracted ray 1 intersect. This point is located in front of the lens. (See sketch.)

Step 4: By repeating the ray-tracing procedure for other points along the object (arrow), a series of image points (P′ and P″) are located, as shown on the next page. The points Q, Q′, Q″, all lie along a vertical line. They form a "virtual image" of the arrow. The image is right-side-up and much smaller than the object itself. The image is "virtual" because you can't find the image on a screen if you place it there. Nevertheless, the refracted rays leaving the lens appear to come from the virtual-image point. The image **would be seen** by a person on the right side of the lens, looking back through the concave lens.

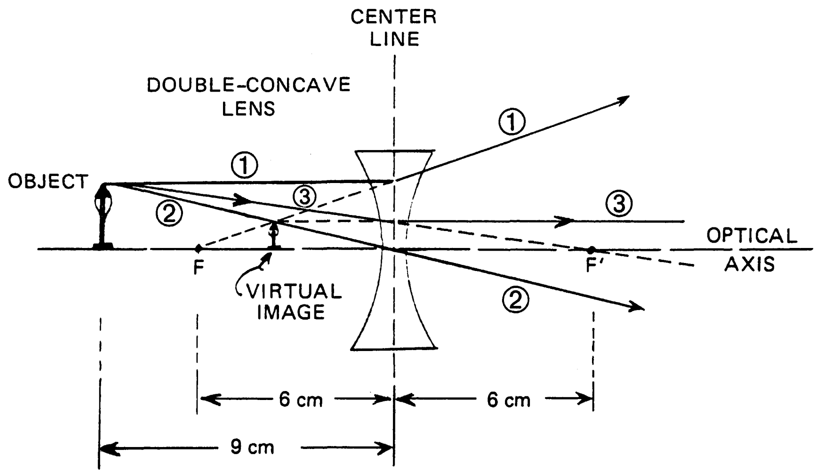




ANSWERS TO PRACTICE EXERCISES, Continued

Activity 2:

Problem 4:



NOTE: We've added the *third ray*, as described on the two previous Teacher Guide pages, to provide the "check." Unless you've taught your students how to draw the third ray, labeled 3 above, they will show only rays 1 and 2. But these two are sufficient to locate the image point.

NOTE: For Example D, there is also a third ray that can be drawn as a check, just as we showed in Example C. In this case we make use of the object point P and the focal point F, both on the left side of the lens. Draw a line straight from point P through point F onto the center line of the double convex lens. At the point where this line meets the center line, bend it and extend it forward as a line PARALLEL to the optical axis. The parallel line should pass through point Q, providing the desired check.

Problem 4:	Given:	A concave lens with a focal length of 6 centimeters. The focal point F is 6 centimeters in front of the lens. The focal point F′ is 6 centimeters behind the lens. An object 3 centimeters high is placed on the same side of the lens as the focal point F. The object is 9 centimeters from the center line of the lens.
	Find:	The image of the object formed by the concave lens.
	Solution:	Using graph paper, straightedge and compass, draw a scale diagram of the optical system. Follow the procedure outlined in Example C. You should locate a *virtual* image, somewhere between the focal point F and the front sides of the lens. It should be smaller in size than the object.

ACTIVITY 3

Locating the Image Formed by a Convex Lens

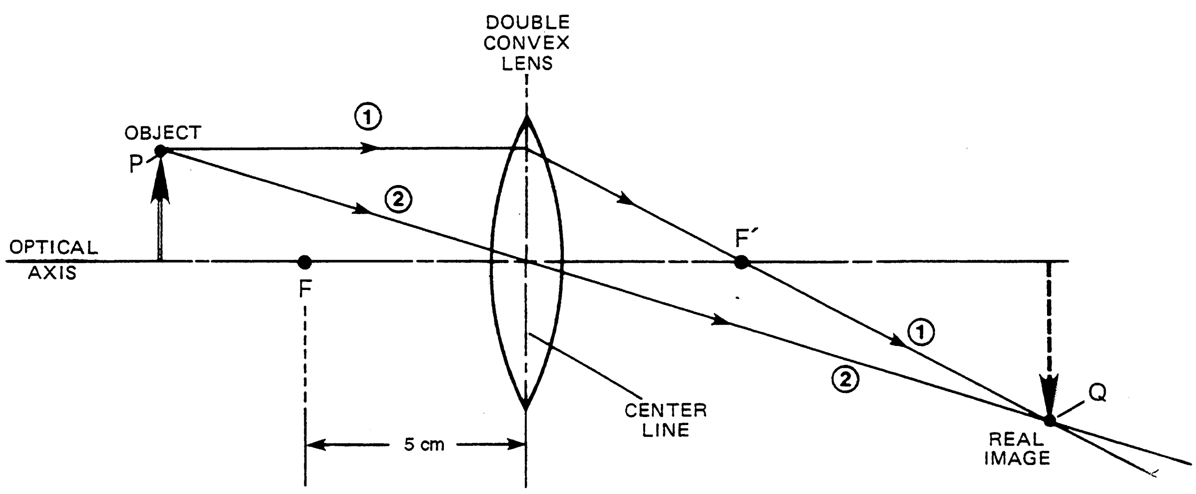
MATERIALS

For this activity, you'll need graph paper, a straightedge and a compass.

You've just learned how to locate the image formed by a concave lens. The method for locating the image formed by a convex lens is similar. Let's work through an example to see how this is done.

Example D: ***Locating the Image Formed by a Convex Lens***

Given: Sunlight is reflected from an object (such as an arrowhead) and, after reflection, passes through a convex lens. (See the following illustration.) The convex lens has a focal length equal to 5 centimeters. The focal points F and F′ are each 5 centimeters from the vertical center line of the lens.

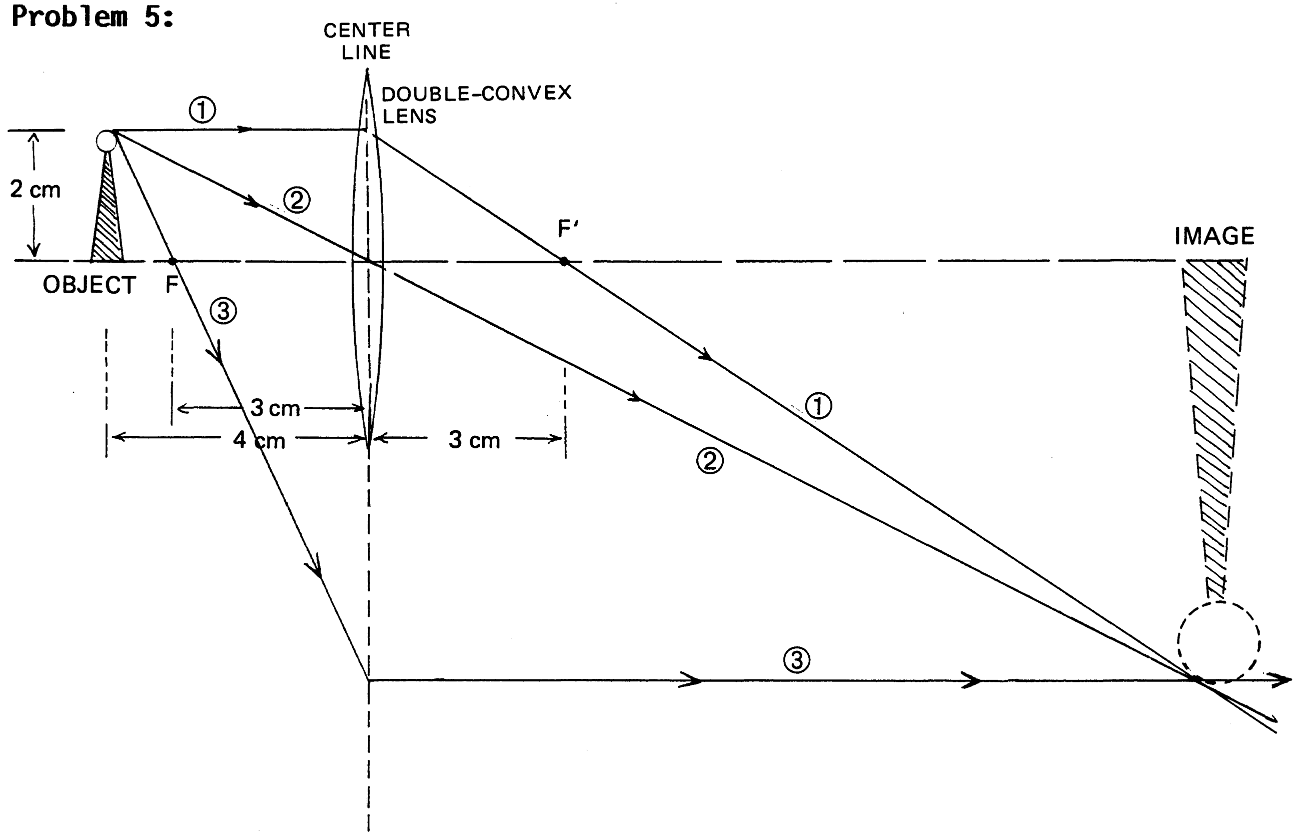


Find: The image of the arrow tip formed by the convex lens.

ANSWERS TO PRACTICE EXERCISES, Continued

Activity 3:

Problem 5:



The image formed by the lens is real and inverted. It is larger than the object.

NOTE: If one tried to draw ray 3, from the tip of the object to the focal point and on to the center line of the double convex lens, one would "miss" the lens, since it is not "large" enough. But for ray tracing purposes, this doesn't matter. One simply extends the center line of the lens up or down as much as needed, so ray 3 can be drawn to the center line, bent, and then directed forward, parallel to the optical axis. It should then pass through the image point. All of this is shown above as ray 3.

Answer to Student Challenge Problem 6 follows on Page T-67c.

ANSWERS TO PRACTICE EXERCISES, Continued

Problem 6:

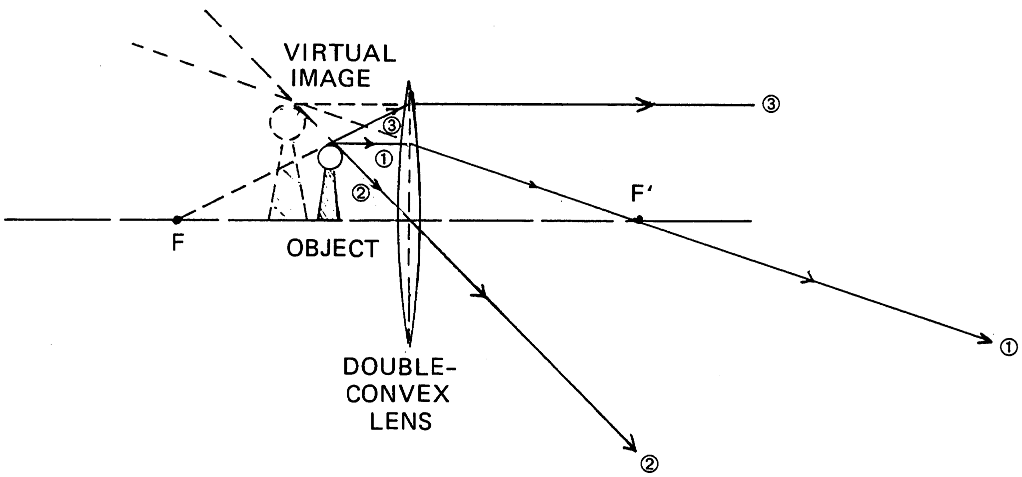


Image is virtual,
right side up and
larger than the object.

NOTE: You may have to assist students who accept the challenge to solve this problem. After they draw rays 1 and 2 as shown above, they will see that the rays spread apart on the right side of the lens. Thus the rays can never intersect and form a real image, as they did in Problem #5. If the two rays (1 and 2) on the right side are traced back along themselves, they will intersect behind the lens, as shown, and locate the tip of the virtual image. The image is virtual because it cannot be formed on a screen placed where we've drawn it. But a person, looking into the convex lens from the right side, trying to see the object, will see, instead, the vertical image, just where it's located. This, by the way, is how a simple magnifying lens works. It forms a virtual image of the object you're looking at through a lens—like a splinter in your finger—and forms an enlarged image so that you can see it more easily.

NOTE: Again a third ray can be used as a check. That's ray 3 shown above. Solid lines depict the actual light ray. Dotted lines help one see how to draw the actual ray, while involving the object point (tip) and focal point F. Note dotted line from F to tip of object. That establishes the "ray" which leaves the focal point on the left and emerges as a parallel ray after bending by the lens.

Solution: You can find the image of the arrowhead tip by using the ray-tracing technique as follows.

Step 1: Locate a point P at the tip of the arrow. From this point, draw an incident ray (ray 1) that's parallel to the optical axis and stops at the center line. Then, from the stopping point, draw a line that goes through the focal point F′ and beyond.

Step 2: Starting again at the same point P on the object, draw an incident ray (ray 2) that goes through the lens at its center and continues on, along a straight line, until it intersects ray 1.

Step 3: The image point Q of the arrow tip is the point where ray 1 and ray 2 intersect, on the right side of the lens. By repeating the ray-tracing procedure for other points along the object (arrow), a series of image points can be located, just as was done in Example C. These points form the image of the entire arrow.

Notice that these points form a *real image*. This image is upside down and larger than the object itself. It is a *real image* because it will actually *form* on a screen placed in the plane of the image points.

Problem 5: Given: A convex lens is 6 centimeters in diameter (from rim to rim). Its focal points, F and F′, are each located 3 centimeters from the vertical center line of the lens. An object that is 2 centimeters tall is placed on the same side of the lens as the focal point F. The object is located 4 centimeters from the center of the lens.

Find: The image of the object formed by the lens.

Solution: Using graph paper, a straightedge, compass and the ray-tracing technique, draw a diagram of this optical system. On the diagram, label the lens, optical axis, focal points and object. Locate the image by following the solution given in Example D.

Student Challenge

Problem 6: Given: A convex lens is 5 centimeters in diameter (from rim to rim). Its focal points, F and F′, are each located 6 centimeters from the vertical center line of the lens. An object that is 2 centimeters tall is placed on the same side of the lens as focal point F. The object is 2 centimeters from the center of the lens.

Find: The image of the object formed by the lens.

Solution: Using graph paper, a straightedge, compass and the ray-tracing technique, draw a diagram of this optical system. On the diagram, label the lens, optical axis, focal points and object. Locate the image of the object by following the solution given in Example D.

(***Hint:*** The image will be virtual. It will be located on the *same* side of the lens as the object.)