Math Skills Laboratory

Lab 7 MS 1

MATH ACTIVITIES

Activity 1: Understanding Ratio and Proportion

Activity 2: Using Ratio and Proportion in Linear Mechanical Force Transformer Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Define ratio. Give an example.***
2. ***Define proportion. Give an example.***
3. ***Show that the work equation, $F_i \times D_i = F_o \times D_o$, leads to a useful proportion, $\frac{F_o}{F_i} = \frac{D_i}{D_o}$.***
4. ***Solve linear mechanical force transformer problems for mechanical advantage. Make use of ratios and proportions.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Work the problems.***

ACTIVITY 1

Understanding Ratio and Proportion

Understanding how force transformers work is a lot easier if you understand ratio and proportion. A ***ratio*** is the quotient of one number divided by another, like "one divided by two" or ½. In terms of letters, a ratio is one letter divided by another, like F_i/F_o. Think of a ratio as a fraction.

A ***proportion,*** on the other hand, is simply an equation that relates two equal ratios, such as $\frac{1}{2} = \frac{4}{8}$ or $\frac{F_i}{F_o} = \frac{D_o}{D_i}$.

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WHAT ARE RATIOS?

Some ratios you've already studied in *Principles of Technology* are (1) angle measure in radians and (2) specific gravity. Notice that each ratio is ***dimensionless.*** That means that units always cancel in the final answer.

(1) *Angle measure in radians:*

$$\theta = \frac{\text{D (arc length along circumference)}}{\text{r (length of radius)}}$$

For example, if D = 1.5 cm and r = 1 cm,

$$\theta = \frac{D}{r} = \frac{1.5\text{ cm}}{1.0\text{ cm}} = 1.5 \text{ (Dimensionless!)}$$

The ratio is $^{1.5}/_{1}$ or 1.5. The angle (θ) is 1.5 radians or $1.5 \times 57.3° = 85.9°$.

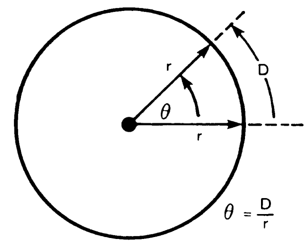

Fig. 1 Angle measure in radians.

(2) *Specific gravity (SG):*

$$SG = \frac{\text{Density of a Given Substance}}{\text{Density of Water}}$$

For example, the density of copper is 8.93 gm/cm^3 and the density of water is 1.0 gm/cm^3. The specific gravity (SG) of copper is a ratio given by:

$$SG = \frac{8.93\text{ gm/cm}^3}{1.0\text{ gm/cm}^3} = 8.93 \text{ (Dimensionless!)}$$

The ratio is $^{8.93}/_{1.0}$ (or 8.93, of course).

WHAT ARE PROPORTIONS?

A proportion connects two equal ratios. For example, each of the equations below involves a proportion. Notice that the left side and right side are each written as ratios.

$$\frac{1}{2} = \frac{4}{8}$$

This proportion is obvious. The fraction $^1/_2$ is a ratio; the fraction $^4/_8$ is a ratio. And they are equal. Each ratio means the same amount. The amount is simply stated in a different way. Often, a proportion ($^1/_2 = {}^4/_8$) is read as "one is to two as four is to eight."

Now consider a different proportion problem.

$$\frac{1}{5} = \frac{a}{10}$$

This proportion has three numbers and one unknown. In this case, the unknown is the letter "a." You know that "a" can only be equal to 2 if the proportion is to be true. That's because 1 is to 5 as 2 is to 10 ($^1/_5 = {}^2/_{10}$).

Now consider a problem where there are two unknowns.

$$\frac{1}{2} = \frac{a}{b}$$

This proportion simply says that whatever a and b are, their ratio ($^a/_b$) must always be equal to $^1/_2$. So if a = 5, then b = 10. Or if a = 10, then b = 20, and so on.

Now consider a proportion problem in which there are four unknowns.

$$\frac{a}{b} = \frac{c}{d}$$

This proportion has four letters. Their values are unknown. As it stands, the proportion simply tells us that "a is to b as c is to d." This isn't very useful. But if a, b, and c stand for something, the proportion tells you something important about how they are related and helps you solve for the value of the fourth letter (d).

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HOW DO YOU USE RATIO AND PROPORTION IN FORCE TRANSFORMER EQUATIONS?

For all ideal force transformers, where no friction or resistance is present, it's true that: Work In = Work Out. Or, expressed another way,

$$F_i \times D_i = F_o \times D_o$$

where:
F_i = input force
F_o = output force
D_i = distance input force moves
D_o = distance output force moves

The equation $F_i \times D_i = F_o \times D_o$ can be changed to one that involves a proportion and useful ratios, as follows:

$F_i \times D_i = F_o \times D_o$ — Always true if Work Input = Work Output.

$\dfrac{F_i \times D_i}{F_i \times D_o} = \dfrac{F_o \times D_o}{F_i \times D_o}$ — Divide each side by the product of $F_i \times D_o$. Cancel appropriately.

$\dfrac{D_i}{D_o} = \dfrac{F_o}{F_i}$ — A proportion results that involves ratios of forces and of distances.

The equation, $\dfrac{F_o}{F_i} = \dfrac{D_i}{D_o}$, is the same as $F_i \times D_i = F_o \times D_o$. It's just written in another form. But it's written as a proportion that says: "Output force F_o is to input force F_i as input displacement D_i is to output displacement D_o."

The ideal mechanical advantage (IMA) is always equal to the ratio D_i/D_o. But the proportion tells us that this ratio is the same as the ratio F_o/F_i.

Therefore, IMA = $\dfrac{D_i}{D_o}$ and IMA = $\dfrac{F_o}{F_i}$, too!

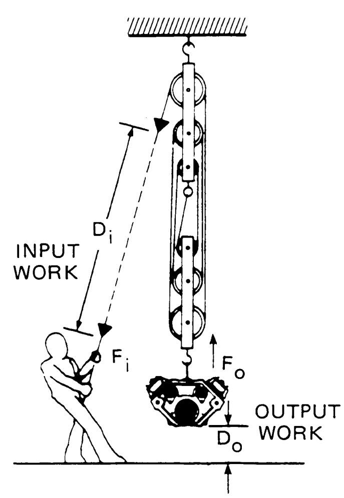
Remember that this is true only when Work In = Work Out—the case of *no friction.* Now solve a problem using ratio and proportion as they might show up in a force transformer problem.

Example A: *Ratio and Proportion*

Given: The block and tackle shown is a force transformer. The important values are:

F_i = 100 lb
D_i = 6 ft
D_o = 1 ft

There is no friction.



Find:
a. Output force F_o.
b. Force ratio F_o/F_i.
c. Displacement ratio D_i/D_o.
d. Ideal mechanical advantage.
e. Proportion between ratios F_o/F_i and D_i/D_o.

Solution:
a. Since there's no friction, Work Out = Work In.

$$F_o \times D_o = F_i \times D_i$$

Rearrange the equation to solve for F_o.

$$F_o = \frac{F_i \times D_i}{D_o}$$

By substituting given values for F_i, D_i and D_o, we get:

$$F_o = \frac{100 \text{ lb} \times 6 \text{ ft}}{1 \text{ ft}} = \left(\frac{100 \times 6}{1}\right)\left(\frac{\text{lb}\cdot\cancel{\text{ft}}}{\cancel{\text{ft}}}\right)$$

$$F_o = 600 \text{ lb}$$

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b. Force ratio F_o/F_i:

$$\frac{F_o}{F_i} = \frac{600\text{ lb}}{100\text{ lb}} = \frac{6}{1}$$ (Notice that the ratio is dimensionless!)

The block and tackle is a "6-to-1" force amplifier.

c. Displacement ratio D_i/D_o:

$$\frac{D_i}{D_o} = \frac{6\text{ ft}}{1\text{ ft}} = \frac{6}{1}$$ (Notice, no units—no dimension.)

The trade-off is clear. To amplify force by a factor of 6, six times as much rope has to be moved at the input end.

d. Because there is no friction, the ideal mechanical advantage is either

$\text{IMA} = \frac{F_o}{F_i}$ or $\text{IMA} = \frac{D_i}{D_o}$, found in b and c.

e. And now, let's figure the proportion. Since $F_i \times D_i = F_o \times D_o$, the proportion between the force ratio and the distance ratio becomes:

$$\frac{F_o}{F_i} = \frac{D_i}{D_o}, \text{ or } \frac{6}{1} = \frac{6}{1}$$

For this problem, each ratio is equal to $^6\!/\!_1$. This tells you that the "output force is to the input force" as the "input displacement is to the output displacement." Each one equals $^6\!/\!_1$, or "six times."

Now let's move to Activity 2. You can solve transformer problems where both ratio and proportion are used.

ACTIVITY 2

Using Ratio and Proportion in Linear Mechanical Force Transformer Problems

For this activity, you'll solve problems that commonly occur in industrial, commercial and construction applications of linear force transformers.

Use the equations for mechanical advantage and the relationships of work input to work output to solve the problems. Assume that the transformers are ***IDEAL,*** unless stated otherwise. The equations and relationships are as follows:

1. Work Input = Work Output

   $F_i \times D_i = F_o \times D_o$ (no friction)

2. Two important ratios:

   $\text{IMA} = \frac{D_i}{D_o}$ and $\text{IMA} = \frac{F_o}{F_i}$

3. An important proportion:

   $\frac{F_o}{F_i} = \frac{D_i}{D_o}$ (no friction)

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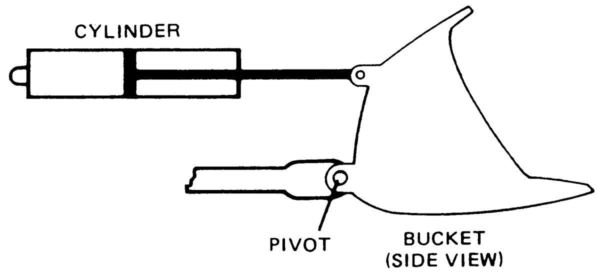
Problem 1: Given: A "front loader" is a type of earth-moving machine that has a scoop bucket, known as a "loader bucket," attached to the front of the machine. The bucket can be rotated forward and backward by a hydraulic cylinder. The bucket pivots on a lifting-arm mechanism. This bucket pivot forms a first-class lever. Assume no friction.



Find:

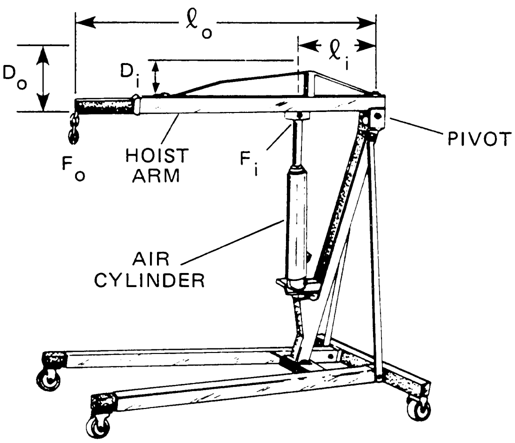
a. Draw a picture of the forces, the pivot and the displacements as they apply to the front loader.

b. The bucket lever arm has an ideal mechanical advantage of 4. The hydraulic cylinder applies 20,000 lb of force to the input end of the lever. What's the output force applied by the lever on the load?

Note: IMA $= \dfrac{D_i}{D_o} = 4$ and Work In = Work Out.

Solution:

Problem 2: Given: A portable engine hoist uses an air cylinder to raise and lower the hoist arm (a third-class lever). The height the load is raised (D_o) is more important than the lifting capacity (F_o). The hydraulic cylinder exerts a 6000-lb force (F_i) on the arm. The hoist arm has a 1000-lb lifting force (F_o) at the lifting hook. Assume no friction, so that IMA = AMA = $F_o/F_i = D_i/D_o$.



Find:

a. The mechanical advantage of the hoist (either IMA or AMA).

b. The distance (D_o) the lifting hook raises the load when the piston arm in the cylinder extends outward 6 inches from the closed position.

Solution:

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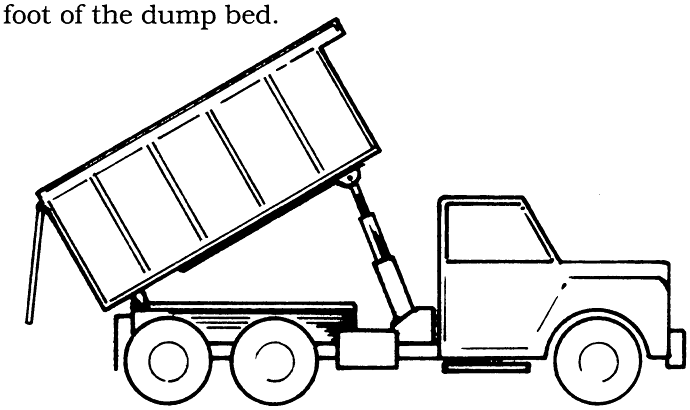
Problem 3: Given: A dump truck has a lift cylinder that exerts 7 tons of force at the front of the 12-foot-long dump bed. The pivot point (hinge) is at the foot of the dump bed.



Find:

a. Draw a simple picture that locates the load, applied force and pivot. (***Hint:*** It's a second-class lever.)

b. Assuming the load is concentrated at the center of the dump bed, find the weight this truck can dump when fully loaded.

Solution:

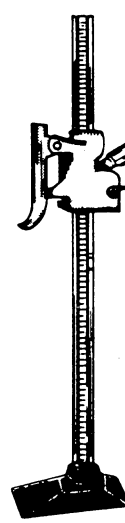
Problem 4: Given: A garage door spring exerts a force of 150 lb while doing 300 ft·lb of work on the frictionless lever mechanism of the door. The ideal mechanical advantage of this mechanism is 0.33.

Find: The output force required to raise the door 6 ft (under ideal conditions). Remember that work input equals work output if friction/resistance is ignored.

Note: In practice, garage doors use springs and lever mechanisms to create a *counterbalance.* (A counterbalance is a condition that exists when one weight balances another weight.) Under these conditions, the torques are equal and opposite in direction. The door is in equilibrium. A slight input force changes this condition. The door opens quite easily.

Solution:

Problem 5: Given: An auto bumper jack is 80% efficient. To raise the wheel of a certain auto, an 80-pound input force moves down 12 inches. The jack applies a force that raises the wheel one inch.



Find:

a. The load force, if Work Input = Work Output (ideal conditions).

b. What is the AMA if efficiency is 80%?

c. What is the actual force applied to the load?

Note: $\text{Eff} = \dfrac{\text{AMA}}{\text{IMA}} \times 100\%$ and

$$\text{IMA} = \frac{D_i}{D_o} \text{ (ideal conditions).}$$

Solution:

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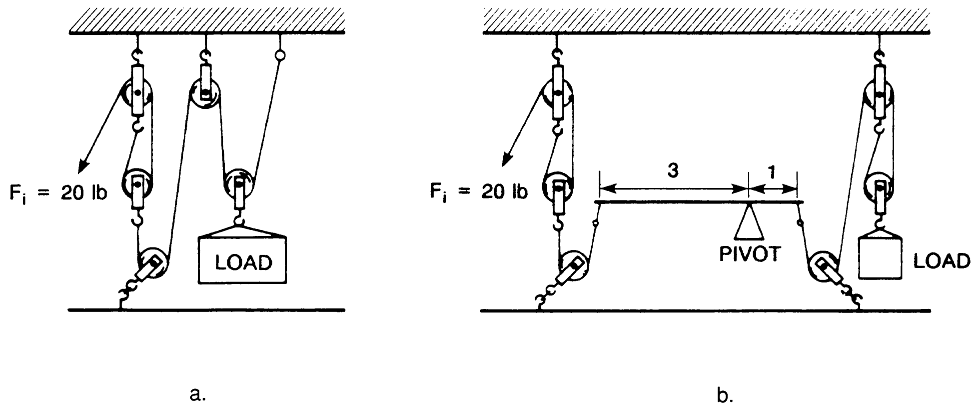
Student Challenge

Problem 6: Given: The diagrams below of two pulley systems.



Find:

a. The load that can be lifted in each arrangement if the input force is 20 lb. Assume no friction.

b. The distance the load will be lifted in each case if the input force is applied over a 24-in. distance. Justify your answer in one or two sentences.

Solution:

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