PREPARATORY MATH SKILLS LAB

Lab PMS 8

MATH ACTIVITY

Substituting in Formulas

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. ***Substitute into formulas. Solve for the unknown quantity.***
2. ***Check answer for correct number and unit by "checking units"—doing a dimensional analysis.***

MATERIALS

For this activity, you'll need a pencil, paper and a hand calculator.

The following three examples involve equations that have to be solved by substituting in numbers (with units). You will use the formulas for percent efficiency and for work—concepts introduced in Unit 2, *Work*.

Example A shows how to calculate the efficiency of a block-and-tackle system. (You'll have a chance to review what you learned about "percents" in Activity 7.)

Example B shows how to substitute values in the "work" equation, $W = F \times D$.

Example C shows how to "check units" to be sure the equation you're using isn't wrong, or that the correct units have been used in the substitution process.

Study Examples A, B and C before you try the Practice Exercises.

Example A: ***Calculating Efficiency of a Pulley System***

Given: A block-and-tackle pulley system lifts a weight. The pulley system does 980 N·m of work (Work Out) to lift a weight. To cause the pulley system to operate, 1000 N·m of work (Work In) are required.

Find: Efficiency of the pulley system.

Solution:

Step 1: Write the formula for percent efficiency.

$$\text{Eff} = \left[\frac{\text{Work Out}}{\text{Work In}}\right] \times 100\%$$

Step 2: Identify what's given and what needs to be found.

Given: Work Out = 980 N·m
Work In = 1000 N·m

Find: Eff

Step 3: Substitute given values for ***Work In*** and ***Work Out*** in the formula. Include both the numerical value and the units, as follows:

$$\text{Eff} = \left[\frac{\text{Work Out}}{\text{Work In}}\right] \times 100\%$$

$$\text{Eff} = \left[\frac{980\ \text{N·m}}{1000\ \text{N·m}}\right] \times 100\%$$

Step 4: Use a calculator to perform the calculation. Write out the answer.

$$\text{Eff} = 98\%$$

Conclusion: The pulley system is 98% efficient, or it has an efficiency of 98%.

Always take care to substitute correct numbers and units for known symbols in an equation. Only then can you calculate the number and unit of the unknown (isolated) symbol correctly.

Example B: *Substituting in the Work Equation $W = F \times D$ to Find Force*

Given: The formula $W = F \times D$, where $W = 10$ N·m and $D = 2$ m.

Find: Force, F

Solution: Step 1: The equation isn't in the form with F isolated. So first isolate F. Write down the given equation:

$$W = F \times D$$

Divide each side by D.

$$\frac{W}{D} = \frac{F \times \cancel{D}}{\cancel{D}} \qquad \text{(Cancel D's.)}$$

$$\frac{W}{D} = F$$

Reverse order of equation.

$$F = \frac{W}{D}$$

Now we have the equation with "F" isolated.

Step 2: Substitute given numerical values—with units—for W and D. From the given information, W = N·m, and D = 2 m.

$$F = \frac{10 \text{ N·}\cancel{\text{m}}}{2 \cancel{\text{m}}} \qquad \text{(Cancel m's.)}$$

Step 3: Divide the numbers. The result with the remaining units is:

$$F = 5 \text{ N}$$

The force is 5 newtons.

A good way to be sure that the equation you're using is set up correctly is to check the units. Do this for each physical quantity (or symbol) in the equation. Check to see if the units are correct and are the same on either side of the equation. Example C shows how to do this.

Example C: *Checking Accuracy of an Equation by Checking Units*

Given: The formula $W = F \times D$, where W is in newton·meters, F is in newtons, and D is in meters.

Find: F, and verify the units.

Solution: Step 1: Rewrite the given equation so F is at the left of the equal sign. Everything else should be at the right. (Refer to Example B if necessary.)

$$F = \frac{W}{D}$$

Step 2: Substitute units for F, W and D in this equation.

$$\text{newtons} = \frac{\text{newton·}\cancel{\text{meters}}}{\cancel{\text{meters}}} \qquad \text{(Cancel meters.)}$$

$$\text{newtons} = \text{newtons}$$

This leaves "newtons = newtons." The units "check out." Therefore, the equation has been set up correctly.

PRACTICE EXERCISES

In the problems that follow, you need to:

- isolate the required physical quantity to derive a "new" formula.
- substitute the numerical values of the known physical quantities into the formula. Use correct units with the numerical value.
- check the formula by checking units, as in Example C.
- solve for the unknown quantity.

Problem 1: This problem has been solved for you. Follow through the solution to make sure you understand each step.

Given: The formula $T = F \times L$, where $T = 20$ lb·ft and $F = 5$ lb.

Find: L.

Solution:

Step 1: Write down basic equation as the first step in "isolating" L.

$$T = F \times L$$

Step 2: To "free" L on the right side, divide both sides of the equation by F.

$$\frac{T}{F} = \frac{\cancel{F} \times L}{\cancel{F}}$$ (F's cancel out on right side of equation.)

$$\frac{T}{F} = L$$ (L has been isolated.)

Step 3: Reverse sides of equation to put L on the left.

$$L = \frac{T}{F}$$

Step 4: Substitute known values for T and F.

$$L = \frac{T}{F}$$ where: $T = 20$ lb·ft, $F = 5$ lb

$$L = \frac{20\ \cancel{\text{lb}}\cdot\text{ft}}{5\ \cancel{\text{lb}}}$$ (Cancel lb units.)

$$L = 4\ \text{ft}$$

Step 5: Check units in equation.

$$L = \frac{T}{F}$$ where: L is in ft, T is in lb·ft, F is in lb

So, $$\text{ft} = \frac{\cancel{\text{lb}}\cdot\text{ft}}{\cancel{\text{lb}}}$$ (Cancel lb units.)

$$\text{ft} = \text{ft}$$

The units check; so this equation is correct.

Now follow the same steps to solve Problem 2. The steps you should follow are indicated.

Problem 2:

Given: The formula $W = T \times \theta$, where $\theta = 6.28$ rad and $W = 62.8$ ft·lb.

Find: T.

Solution:
Step 1: Isolate T in the equation.
Step 2: Reverse sides of equation. This will put T on the left.
Step 3: Substitute known values for W and θ. Solve for T.
Step 4: Do "unit check" on equation.

Given the formula, Efficiency = $\left[\frac{\text{Work Out}}{\text{Work In}}\right] \times 100\%$, solve each system listed below for its efficiency.

Problem 3: Calculate the efficiency for a block and tackle that lifts a load of 600 pounds under the following conditions:

Work Out = 1200 ft·lb
Work In = 1400 ft·lb

Problem 4: Calculate the efficiency for a winch hauling in a load of 1000 newtons under the following conditions:

Work Out = 3000 N·m
Work In = 3150 N·m

Problem 5: Calculate the efficiency for an ideal "frictionless" pulley system that raises an automobile under the following conditions:

Work Out = 3000 ft·lb
Work In = 3000 ft·lb