Math Lab 12 MS 1

Solving Problems That Involve the Wavelength, Frequency, Speed and Energy of Electromagnetic Radiation

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach your students how to solve problems that involve wavelength, frequency, speed and energy of electromagnetic radiation.
2. Teach your students how to make unit conversions of values that describe EM radiation.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure your students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to this class. (How much you accomplish will depend on the math skills your students already have.) Summarize the explanatory material for the activity: "Solving Problems that Involve the Wavelength, Frequency, Speed and Energy of Electromagnetic Radiation." Then have students complete the Practice Exercises given at the end of the activity.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 12*1, "The Electromagnetic Spectrum."

Math Skills Laboratory

Lab 12 MS 1

MATH ACTIVITY

Activity: Solving Problems That Involve the Wavelength, Frequency, Speed and Energy of Electromagnetic Radiation

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. *Express wavelengths of electromagnetic radiation in angstrom units, nanometers, microns or centimeters.*
2. *Use the equation, $c = \lambda \times f$, to solve for speed (c), wavelength (λ), or frequency (f) of electromagnetic radiation.*
3. *Use the equations, $E = hc/\lambda$ or $E = hf$, to solve for the energy of electromagnetic radiation.*

LEARNING PATH

1. *Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
2. *Study the examples.*
3. *Work the problems.*

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ACTIVITY

Solving Problems That Involve the Wavelength, Frequency, Speed and Energy of Electromagnetic Radiation

MATERIALS

For this activity, you'll need a calculator.

DISCUSSION

The many forms of electromagnetic (EM) radiation are organized in a table called the "***electromagnetic spectrum.***" The table uses frequency and wavelength—two important electromagnetic radiation characteristics—to classify the different forms of EM radiation.

When the frequency (f) is low—as it is in 60-hertz AC electrical power—the wavelength (λ) is long. At high frequencies, the wavelength is short—as it is with gamma rays.

Thus, very-low-frequency, very-long-wavelength EM radiation is found at one end of the EM spectrum. At the other end of the EM spectrum, very-high-frequency, very-short-wavelength EM radiation is found. Many forms of EM radiation are classified between these extremes, according to their frequency and wavelength.

HOW DO YOU MAKE WAVELENGTH CONVERSIONS?

How do you choose units to express the wavelength of EM radiation. You follow the "conventions" that have been established. Wavelength usually is expressed in ***angstrom units*** (1 Å = 10^{-10} meters).

However, more recently, the wavelength of visible-light also is being expressed in nanometers (nm) and micrometers (μm). The unit, ***nanometer*** (nm), is equal to 10^{-9} meters. The unit, micrometer (μm), is equal to 10^{-6} meters.

No one choice is more correct than another. Blue light of λ = 400 nm is equivalent to blue light of λ = 4000 Å or λ = 0.4 μm. Table 1 can be used to convert between wavelength units of angstroms, nanometers, micrometers, centimeters and meters.

Table 1: Wavelength Conversions

λ	Å angstrom	nm nanometer	μm micrometer	cm centimeter	m meter
1 Å	1	10^{-1}	10^{-4}	10^{-8}	10^{-10}
1 nm	10	1	10^{-3}	10^{-7}	10^{-9}
1 μm	10^4	10^3	1	10^{-4}	10^{-6}
1 cm	10^8	10^7	10^4	1	10^{-2}
1 m	10^{10}	10^9	10^6	10^2	1

NOTE: If students don't understand *conversions* after Example 1 is worked, it may be necessary to go back to Figures 12-4 and 12-6 and try several other examples. One or two additional problems should improve their skills. The problems in this lab should be adequate to increase their understanding of the parts of the EM spectrum.

NOTE: In part b of Example 1, the visible spectrum is shown to extend from 4000 Å (blue) to 7000 Å (red). Have your students convert these limits to wavelengths in meters (4×10^{-7} m for blue and 7×10^{-7} m for red). Then have them verify—by examining Figure 12-4 for the overall EM spectrum—that the visible spectrum does indeed belong between the ultraviolet and infrared regions of the EM spectrum, just as depicted in Figure 12-4.

To use the table of wavelength conversions, follow the procedure explained in the following example.

Example 1: *Wavelength Conversions*

Given: In the visible-light spectrum, red light (with a wavelength of $\lambda = 0.7\ \mu m$) is on one end of the visible-light band. Blue light (with a wavelength $\lambda = 0.4 \times 10^{-4}$ cm) is on the other end. The eye is most sensitive to visible light of wavelength $\lambda = 555$ nm.

Find:

a. **The wavelength in angstrom units of $\lambda = 0.7\ \mu m$, $\lambda = 0.4 \times 10^{-4}$ cm and $\lambda = 555$ nm.**

b. The probable color for the $\lambda = 555$ nm light. (Do this by constructing a visible-light spectrum band. Use the fact that the wavelength of orange light is 6000 Å, and the wavelength of green light is 5000 Å. Arrange from left to right as red, orange, yellow, green, blue and violet.)

Solution:

a. To use Table 1 for converting 0.7 μm to Å, proceed as follows:

1. First, find the ***row*** labeled "1 μm" in the left-hand column under "λ." Then move to the right along this row until you reach the ***column*** for angstrom units (Å). Use the number found there (10^4) as the conversion factor and write 1 $\mu m = 10^4$ Å.
2. Then use this information to make the conversion from "μm" to "Å" as follows:

$$0.7\ \mu m \times \frac{10^4\ \text{Å}}{1\ \mu m} = \frac{0.7 \times 10^4}{1} \frac{\cancel{\mu m} \cdot \text{Å}}{\cancel{\mu m}} = 7000\ \text{Å}$$

$\lambda = 7000$ Å (red light)

3. Use the same procedure to convert "cm" to "Å." From the table, find that 1 cm $= 10^8$ Å.

Then convert 0.4×10^{-4} cm to "Å" as follows:

$$0.4 \times 10^{-4}\ \text{cm} \times \frac{10^8\ \text{Å}}{1\ \text{cm}} = \left(\frac{0.4 \times 10^{-4} \times 10^8}{1}\right) \frac{\cancel{\text{cm}}\ \text{Å}}{\cancel{\text{cm}}} = 0.4 \times 10^4\ \text{Å}$$

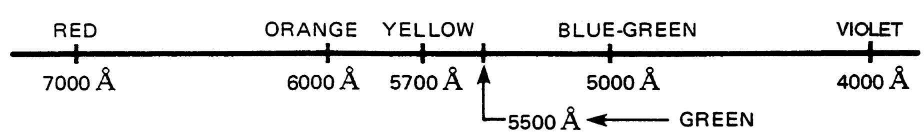
$\lambda = 0.4 \times 10^4$ Å $= 4000$ Å (blue light)

4. Use the same procedure to convert "nm" to "Å." From the table, find that 1 nm = 10 Å. Then convert 555 nm to "Å" as follows:

$$555\ \text{nm} \times \frac{10\ \text{Å}}{1\ \text{nm}} = \frac{555 \times 10}{1} \frac{\cancel{\text{nm}}\ \text{Å}}{\cancel{\text{nm}}} = 5550\ \text{Å}$$

$\lambda = 5550$ Å.

b. A spectrum band with these visible-light wavelengths would have violet on the right at 4000 Å and red on the left at 7000 Å, with orange, yellow, green and blue between—from left to right. According to the spectrum band the 5550-Å light is between the yellow and the blue-green. It turns out to be green.



NOTE: The speed of light in a vacuum is given as approximately 2.998×10^8 meters/second. For ease in calculation, this is almost always rounded to read 3×10^8 meters/second. In addition, the speed of light through air is generally taken to be very close to that in a vacuum (although air at sea level is clearly not a vacuum), so that one finds common use of 3×10^8 meters/second as the speed of light in vacuum and air.

HOW DO YOU CALCULATE WAVELENGTH, FREQUENCY, SPEED AND ENERGY OF EM RADIATION?

It's often important to know the frequency of electromagnetic radiation. You can find frequency if you know speed and wavelength. The three (λ, v and f) are related for any part of the electromagnetic spectrum by the equation:

$$v = \lambda f$$

where: v = speed of the wave in m/sec
λ = wavelength in m
f = frequency in cycles/sec or Hz

In free space or a vacuum, $v = c \approx 3.0 \times 10^8$ meters/second. The value changes for each type of medium the radiation moves through. For light traveling in air, "c" also equals approximately 3×10^8 meters/second.

The energy associated with EM radiation is related to the frequency and a constant (h) known as Planck's constant. The energy can be expressed by the equation:

$$E = hf$$

where: E = energy in J
h = Planck's constant, equal to 6.63×10^{-34} J·sec
f = radiation frequency in cycles/sec or Hz

Another useful energy equation results when "f" in the energy equation, E = hf, is replaced by "c/λ."

$$E = hc/\lambda$$

where: E = energy in J
h = Planck's constant (6.63×10^{-34} J·sec)
c = speed of light in a vacuum or air (3×10^8 m/sec)
λ = wavelength in m

The following example shows how to use these equations to solve electromagnetic radiation problems.

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Example 2: ***Finding Values for Electromagnetic Radiation Characteristics***

Given: The light from a helium-neon laser has a wavelength of 6328 Å.

Find:
a. The frequency of the laser beam when it's traveling in a vacuum where $c = 3.0 \times 10^8$ m/sec.
b. The energy associated with the laser beam.

Solution: a. The frequency can be found from the equation, $c = \lambda f$.

Isolate "f" by dividing both sides of the equation, $c = \lambda f$, by "λ."

$$\frac{c}{\lambda} = \frac{\cancel{\lambda} f}{\cancel{\lambda}} \quad \text{(Cancel like values.)}$$

Rearrange the canceled equation to give:

$$f = \frac{c}{\lambda} \qquad \text{where: } c = 3.0 \times 10^8 \frac{\text{m}}{\text{sec}}$$

Since "c" is in meters/second, λ must be in meters. First change 6328 angstroms to meters. From Table 1, Å = 10^{-10} m. Therefore,

$$6328\ \text{Å} \times \frac{10^{-10}\ \text{m}}{1\ \text{Å}} = 6328 \times 10^{-10} \frac{\cancel{\text{Å}}\cdot\text{m}}{\cancel{\text{Å}}} = 6.328 \times 10^{-7}\ \text{m}.$$

Substitute values in $f = c/\lambda$ and solve:

$$f = \frac{3 \times 10^8\ \text{m/sec}}{6.328 \times 10^{-7}\ \text{m}} = \frac{3 \times 10^8}{6.328 \times 10^{-7}} \frac{\cancel{\text{m}}}{\text{sec}\cdot\cancel{\text{m}}}$$

$$f = \frac{3}{632.8} \times 10^{8+7} \frac{1}{\text{sec}} = 4.741 \times 10^{14} \frac{1}{\text{sec}}$$

But $\frac{1}{\text{sec}} = 1$ Hz. Therefore,

$$f = 4.741 \times 10^{14}\ \text{Hz} \approx 4.74 \times 10^{14}\ \text{Hz}.$$

The frequency of helium-neon laser light is about 4.74×10^{14} Hz.

b. You can find the energy from the equation, $E = hf$. Or you can use the equation, $E = hc/\lambda$, without knowing the frequency if you know both speed and wavelength. Since the frequency was found in "a" above, let's use the simpler equation, $E = hf$.

$$E = hf$$

where: $h = 6.63 \times 10^{-34}$ J·sec
$f = 4.74 \times 10^{14}$ cycles/sec

$$E = 6.63 \times 10^{-34}\ \text{J}\cdot\text{sec}\ (4.74 \times 10^{14}\ \text{cycles/sec})$$

$$E = (6.63 \times 4.74 \times 10^{-34+14}) \times \frac{\text{J}\cdot\cancel{\text{sec}}\ \text{cycles}}{\cancel{\text{sec}}}$$

$$E = 31.4 \times 10^{-20}\ \text{J}$$

$$E = 3.14 \times 10^{-19}\ \text{J}.$$

The energy per photon of helium-neon laser light is equal to 3.14×10^{-19} J.

SOLUTIONS TO PRACTICE EXERCISES

Problem 1: $c = \lambda f$ (Solve for "f" by dividing both sides of the equation by "λ".)

$\frac{c}{\lambda} = \frac{\cancel{\lambda} f}{\cancel{\lambda}}$ (Cancel like terms. Rearrange the equation.)

$f = \frac{c}{\lambda}$ where: $c = 3.0 \times 10^8$ m/sec
$\lambda = 3$ m

$$f = \frac{3.0 \times 10^8 \text{ m/sec}}{3 \text{ m}} = \frac{3.0 \times 10^8}{3} \frac{\cancel{\text{m}}}{\text{sec}\cdot\cancel{\text{m}}}$$

$$f = 1.0 \times 10^8 \frac{1}{\text{sec}} = 100 \times 10^6 \frac{\text{cycles}}{\text{sec}} \text{ or } 100 \times 10^6 \text{ Hz}$$

$f = 100$ megahertz (midpoint frequency range for FM radio band).

Problem 2: $E = \frac{hc}{\lambda} = hf$ (The frequency is known from Problem 1, so we'll use $E = hf$.)

$E = hf$ where: $h = 6.63 \times 10^{-34}$ J•sec
$f = 100 \times 10^6$ cycles/sec

$$E = (6.63 \times 10^{-34} \text{ J}\cdot\text{sec})(100 \times 10^6 \text{ cycles/sec})$$

$$E = (6.63 \times 100 \times 10^{-34+6}) \text{ J}\cdot\cancel{\text{sec}} \frac{\text{cycles}}{\cancel{\text{sec}}}$$

$E = 663 \times 10^{-28}$ J (The word "cycles," is dropped.)

$E = 6.63 \times 10^{-26}$ J.

Problem 3: Use the equation, $c = \lambda f$, and isolate "f". (See Problem 1.)

(1) $f = \frac{c}{\lambda}$ where: $c = 3.0 \times 10^8$ m/sec
$\lambda = 1 \times 10^{-4}$ m

$$f = \frac{3.0 \times 10^8 \text{ m/sec}}{1 \times 10^{-4} \text{ m}} = \left(\frac{3.0 \times 10^8}{1 \times 10^{-4}}\right)\left(\frac{\cancel{\text{m}}}{\text{sec}\cancel{\text{m}}}\right)$$

$$f = 3.0 \times 10^{8+4} \frac{\text{cycles}}{\text{sec}} = 3 \times 10^{12} \text{ Hz.}$$

(2) $f = \frac{c}{\lambda}$ where: $c = 3.0 \times 10^8$ m/sec
$\lambda = 10^{-6}$ m

$$f = \frac{3.0 \times 10^8 \text{ m/sec}}{1 \times 10^{-6} \text{ m}} = \left(\frac{3.0 \times 10^8}{1 \times 10^{-6}}\right) \frac{\cancel{\text{m}}}{\text{sec}\cancel{\text{m}}}$$

$$f = 3.0 \times 10^{8+6} \frac{\text{cycles}}{\text{sec}} = 3 \times 10^{14} \text{ Hz.}$$

SOLUTIONS TO PRACTICE EXERCISES, Continued

Problem 4:

Use the equation, $\lambda_{glass} = \dfrac{c_{glass}}{f_{glass}}$.

where: $f_{glass} = f_{air} = 5.45 \times 10^{14}\,\text{Hz}$

$c_{glass} = 2 \times 10^{8}\,\text{m/sec}$

$$\lambda_{glass} = \frac{c_{glass}}{f_{glass}} = \frac{2 \times 10^{8}\,\text{m/sec}}{5.45 \times 10^{14}\,\text{cycles/sec}}$$

$$\lambda_{glass} = \left(\frac{2}{5.45} \times 10^{8-14}\right)\left(\frac{\text{m} \cdot \cancel{\text{sec}}}{\cancel{\text{sec}} \cdot \text{cycle}}\right)$$

$$\lambda_{glass} = 3.67 \times 10^{-7}\,\frac{\text{m}}{\text{cycle}} = 367 \times 10^{-9}\,\frac{\text{m}}{\text{cycle}}$$

$\lambda_{glass} = 367$ nm. (Drop the word, "cycle.")

The wavelength of the light in glass is shorter. In air it was 550 nm; in glass it's decreased to 367 nm. But since the frequency (f) is the same in air and glass, and since E = hf, the energy remains unchanged.

SOLUTIONS TO PRACTICE EXERCISES

Problem 5: a. Use the equation, E = hf, and solve for "f".

To isolate "f", divide both sides of the equation by "h."

$$\frac{E}{h} = \frac{\cancel{h}f}{\cancel{h}}$$ (Cancel like terms and rearrange the equation.)

Then solve for "f":

$$f = \frac{E}{h} \quad \text{where: } E = 5 \times 10^{-19}\ \text{J}$$
$$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{sec}$$

$$f = \frac{5 \times 10^{-19}\ \text{J}}{6.63 \times 10^{-34}\ \text{J}\cdot\text{sec}} = \left(\frac{5}{6.63}\right)\left(10^{-19+34}\right)\left(\frac{\text{J}}{\text{J}\cdot\text{sec}}\right)$$

$$f = 0.754 \times 10^{15}\ \frac{1}{\text{sec}} = 7.54 \times 10^{14}\ \frac{\text{cycles}}{\text{sec}}$$

$$f = 7.54 \times 10^{14}\ \text{Hz.}$$

b. Use the equation v = λf to find λ. Since the photon is moving in air, v = c = 3 x 10^8 m/sec. Rearrange the equation c = λf to isolate λ. You should obtain λ = c/f. Then substitute values for c and f.

$$\lambda = \frac{c}{f} \text{ where } c = 3 \times 10^8\ \text{m/sec and } f = 7.54 \times 10^{14}\ \text{cycles/sec}$$

$$\lambda = \frac{3 \times 10^8\ \frac{\text{m}}{\text{sec}}}{7.54 \times 10^{14}\ \frac{\text{cycles}}{\text{sec}}} = \frac{3}{7.54} \times (10^{8-14}) \times \frac{\text{m}}{\cancel{\text{sec}}}\ \frac{\cancel{\text{sec}}}{\text{cycle}}$$; drop "cycle"

$$\lambda = 0.398 \times 10^{-6}\ \text{m}$$

Use Table 1 to convert meters to nanometers.

$$\lambda = 0.398 \times 10^{-6}\ \text{m} \times \frac{1\ \text{nm}}{10^{-9}\ \text{m}} = 0.398 \times 10^{-6+9}\ \text{nm}$$

$$\lambda = 0.398 \times 10^{3}\ \text{nm} = 398\ \text{nm}$$

This is light with a wavelength barely shorter than the blue light (400 nm) at the end of the visible spectrum.

PRACTICE EXERCISES

Problem 1: Given: AM radio waves are electromagnetic radiations with frequencies between 530 and 1600 khz and with wavelengths in the 10 to 1000-meter range. FM radio broadcasting stations use electromagnetic radiation with frequencies between 85 to 110 megahertz (mega = 10^6) and wavelengths of about 3 meters.

Find: The midpoint frequency (in megahertz) of the FM band if the midpoint wavelength is 3 meters and the speed is $c = 3.0 \times 10^8$ meters/second.

Solution: (***Hint:*** Start with the equation, $c = \lambda f$.)

Problem 2: Given: The conditions given in Problem 1 and Planck's constant, $h = 6.63 \times 10^{-34}$ J·sec.

Find: The energy associated with the electromagnetic radiation of an FM radio wave.

Solution: (***Hint:*** Use the equation, $E = hc/\lambda$, or $E = hf$.)

Problem 3: Given: The infrared region of the electromagnetic spectrum has radiation with wavelengths between 10^{-4} and 10^{-6} meters.

Find: The frequencies that correspond to these end-point wavelengths for infrared radiation. Assume that $c = 3.0 \times 10^8$ meters/second.

Solution:

Problem 4: Given: When light passes from air into another medium such as glass, its wavelength and speed change. But its frequency remains the same. For example, when light of wavelength 550 nm and frequency 5.45×10^{14} Hz passes from air—where its speed is about 3.0×10^8 m/sec—into a piece of thin glass, the wavelength changes. The speed is reduced to 2×10^8 m/sec in the glass. But the frequency remains constant at 5.45×10^{14} Hz.

Find: The wavelength in glass, given that $v = 2 \times 10^8$ m/sec, and the frequency does not change.

Solution:

Problem 5: Given: The energy of a particular photon of light is found to be 5×10^{-19} joules. It is moving in air.

Find:
a. The frequency of the light.
b. The wavelength of the light.

Solution: (***Hint:*** Planck's constant is equal to 6.63×10^{-34} J·sec.)