Math Skills Laboratory

MATH ACTIVITIES

Activity 1: ***Determining Amplitude, Frequency, Period, Wavelength, and Wave Speed of a Traveling Sine Wave***

Activity 2: ***Adding Two Interfering Waves in a Medium to Obtain the Resultant Wave***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Identify amplitude, frequency, period and wavelength of a sine wave.***
2. ***Find numerical values for amplitude, frequency, period, wavelength and wave speed of given traveling sine waves.***
3. ***Find the resultant waveform for two waves in the same medium as a result of wave interference.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

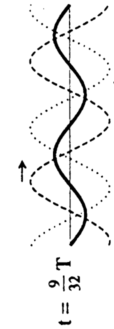
ACTIVITY 1

Determining Amplitude, Frequency, Period, Wavelength and Wave Speed of a Traveling Sine Wave

MATERIALS

For this activity, you'll need a scaled ruler, pencil and paper.

Let's begin the Math Skills Lab with a review of the terms with which we'll work. They're defined here in terms of their relationship to a sinusoidal (sine) wave.



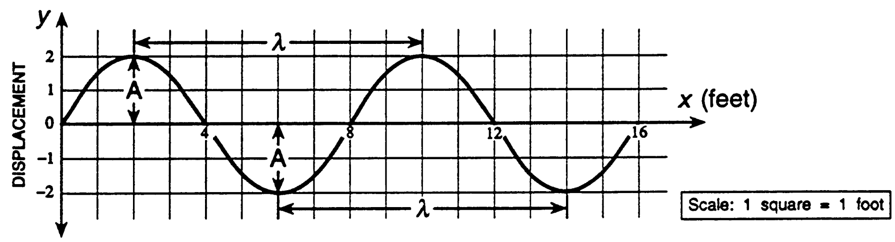
- ***Amplitude*** (A) — The maximum distance that particles or molecules are displaced from their neutral position.
- ***Frequency*** (f) — The number of wavelengths that pass by a point along the wave each second.
- ***Period*** (T) — The number of seconds for each complete wave—one wavelength—to pass by a given point along the direction of wave propagation.

- ***Wavelength*** (λ) The distance between adjacent particles or molecules that are moving in phase. Particles between adjacent crests or between adjacent troughs—for example—are one wavelength apart.
- ***Wave speed*** (v) The wave speed (v) is equal to the product of the wavelength (λ) and the frequency (f). It tells us how fast energy is "carried" by the wave.

These definitions are used in Example A to identify the characteristics of a sine wave.

Example A: *Characteristics of a Traveling Sine Wave*

Given: The traveling transverse sine wave shown in the sketch is moving from left to right. The sketch shows a "snapshot" of the moving wave. Wave displacement is measured vertically. Instantaneous wave position is measured horizontally, along the x-axis.



Find:
a. The amplitude (A) in feet.
b. The wavelength (λ) in feet.
c. The wave frequency (f) if 3 wavelengths pass the point x = 8 every half-second.
d. The period (T). Use the formula, $T = 1/f$.
e. The wave speed (v). Use the formula, $v = \lambda \times f$.

Solution:
a. The wave amplitude (A) in the drawing is 2 squares. Therefore, A = 2 ft.
b. The wavelength from crest to crest or trough to trough is 8 squares. Therefore, the wavelength $\lambda = 8$ feet.
c. If 3 waves (cycles) pass every half-second, 6 cycles pass each second. Therefore, the frequency f = 6 cycles/sec.
d. $T = 1/f$. Therefore,

$$T = \frac{1}{6\,\frac{\text{cycle(s)}}{\text{sec}}} = \frac{1}{6} \times \frac{\text{sec}}{\text{cycle(s)}} = 0.167 \text{ sec/cycle}$$

e. Wave speed $v = \lambda \times f$. Therefore,

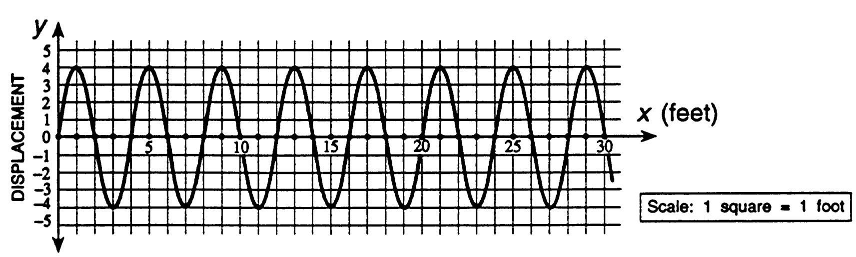
$$v = 8\,\frac{\text{ft}}{\text{cycle}} \times 6\,\frac{\text{cycle(s)}}{\text{sec}}$$

$$v = (8 \times 6)\left[\frac{\text{ft}}{\cancel{\text{cycle}}} \times \frac{\cancel{\text{cycle(s)}}}{\text{sec}}\right]$$

$$v = 48 \text{ ft/sec}$$

Use the definitions just given to complete the following exercises.

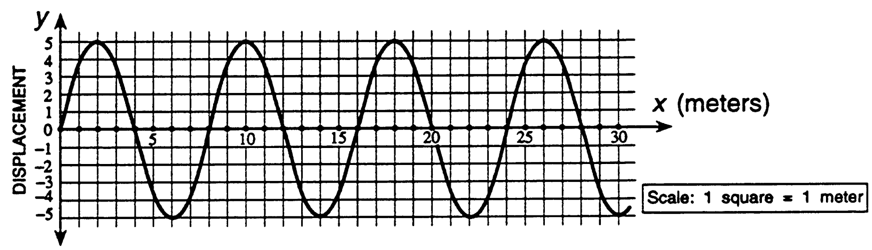
Problem 1: Given: The traveling transverse sine wave shown here in a "snapshot." The vertical (y) axis shows wave displacement. Instantaneous wave position is plotted along the horizontal (x) axis.



Find:
- a. The wave amplitude (A) in feet.
- b. The wavelength (λ) in feet.
- c. The wave frequency (f) if 9 wavelengths (cycles) pass by point x = 10 every 2 seconds.
- d. The wave period (T) in seconds/cycle. Use the formula, $T = \frac{1}{f}$.
- e. The wave speed (v) in feet/second.

Solution:

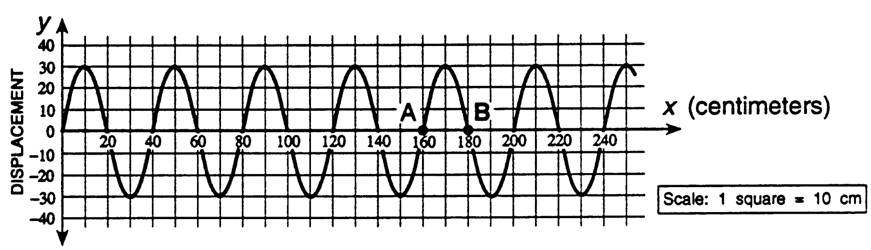
Problem 2: Given: The traveling transverse sine wave shown below in a "snapshot." The vertical (y) axis shows wave displacement. Instantaneous wave position is plotted along the horizontal (x) axis.



Find:
- a. The period (T) if it takes 0.5 second for one cycle to pass the point x = 15.
- b. The frequency (f) in cycles/second. Use the formula, $f = \frac{1}{T}$.
- c. The wave amplitude (A) in meters.
- d. The wavelength (λ) in meters.
- e. The wave speed (v) in meters/second.

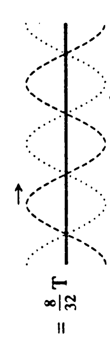
Solution:

Problem 3: Given: The traveling transverse sine wave shown below in a "snapshot," with points **A** and **B**, as indicated. The vertical (y) axis shows wave displacement. Instantaneous wave position is plotted along the horizontal (x) axis.



Find:
- a. The wavelength (λ) in centimeters.
- b. The number of wavelengths from the origin to point A along the x-axis.
- c. The number of wavelengths from the origin to point B along the x-axis.
- d. The number of wavelengths from point A to point B.

Solution:



ACTIVITY 2:

Adding Two Interfering Waves in a Medium To Obtain the Resultant Wave

Materials

For this activity, you'll need a scaled ruler, pencil and paper.

When two or more waves exist at the same point in a medium, the resultant displacement of the wave at this point is the sum of the displacements of each wave. Waves that are in phase will add their amplitudes and produce a larger resultant amplitude. Waves that are out of phase will subtract their amplitudes and produce a smaller resultant amplitude. Example B indicates how in-phase and out-of-phase waves add to form a resultant waveform.

Example B: *Adding Two In-phase Waves*

Given: Two *in-phase* sine waves with the following wave characteristics.

Wave #1: wavelength = 12 cm
amplitude = 2 cm

Wave #2: wavelength = 12 cm
amplitude = 3 cm

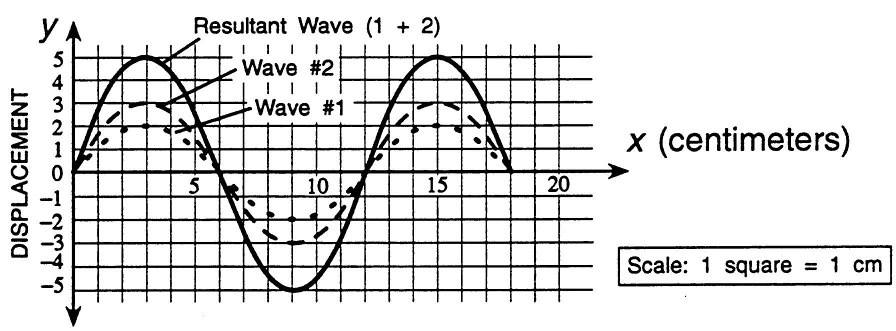
Find:

a. The resultant waveform obtained by adding the two waves from x = 0 cm to x = 18 cm.

b. The *displacement* of the resultant waveform at the points x = 0 cm, 3 cm, 6 cm, 9 cm, 12 cm, and 14 cm.

Solution:

a. To find the resultant waveform, first graph the two sine waves as shown below. (Be sure to draw them so that they are *in phase*—corresponding crests occur at the *same x-value* on the horizontal axis.) Then add the corresponding wave displacements due to each sine wave at appropriate positions along the horizontal axis to get the resultant waveform. The drawing below shows the two individual sine waves and the resultant waveform.



b. The displacement of the resultant waveform at any position is equal to the *algebraic sum* of the displacements of the two in-phase waves at the same position:

x = 0 cm: displacement = 0 cm + 0 cm = 0 cm
x = 3 cm: displacement = 2 cm + 3 cm = 5 cm
x = 6 cm: displacement = 0 cm + 0 cm = 0 cm
x = 9 cm: displacement = −2 cm + (−3 cm) = −5 cm
x = 12 cm: displacement = 0 cm + 0 cm = 0 cm
x = 14 cm: displacement = about (1.8 + 2.5 cm) or = 4.3 cm

Example C: *Adding Two Out-of-phase Waves*

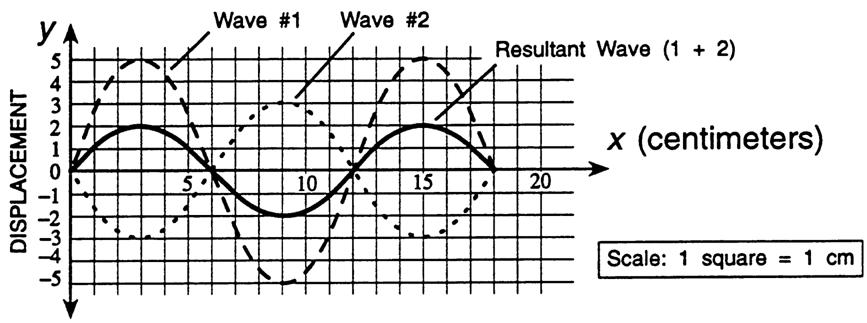
Given: Two sine waves that are 180° out of phase, with the following wave characteristics:

Wave #1: wavelength = 12 cm
amplitude = 5 cm

Wave #2: wavelength = 12 cm
amplitude = 3 cm

Find:
a. The resultant waveform obtained by adding the two waves.
b. The *displacement* of the resultant waveform at the points x = 0 cm, 3 cm, 6 cm, 9 cm, 12 cm, 14 cm,

Solution:
a. To find the resultant waveform, first graph the two sine waves as shown below. (Be sure to draw them so that they are 180°-out-of-phase-peaks with positive values aligned with peaks of negative values on the ***opposite side*** of the horizontal axis.) Then add the corresponding wave displacements due to each sine wave at appropriate positions along the horizontal axis to get the resultant waveform. The drawing below shows the two individual sine waves and the resultant wave.



b. The displacement of the resultant waveform at any position is equal to the *algebraic* sum of the displacements of the two 180° out-of-phase waves at the same x-position.

x = 0 cm; displacement = 0 cm + 0 cm = 0 cm

x = 3 cm; displacement = +5 cm + (−3 cm) = +2 cm

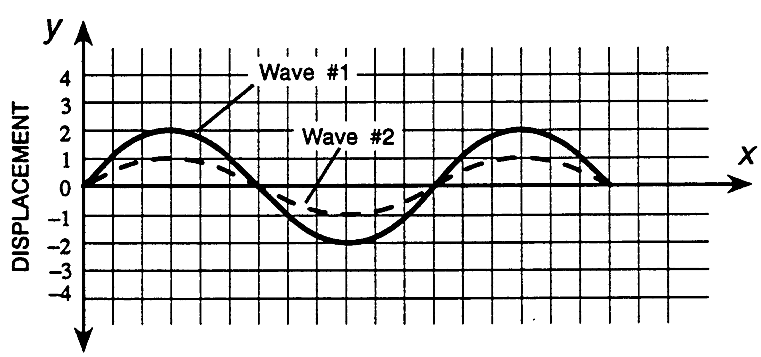
x = 6 cm; displacement = 0 cm + 0 cm = 0 cm

x = 9 cm; displacement = −5 cm + (+3 cm) = −2 cm

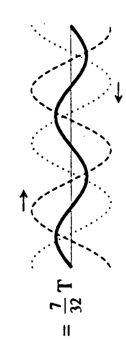
x = 12 cm; displacement = 0 cm + 0 cm = 0 cm

x = 14 cm; displacement = +4.4 cm + (−2.6 cm) = 1.8 cm

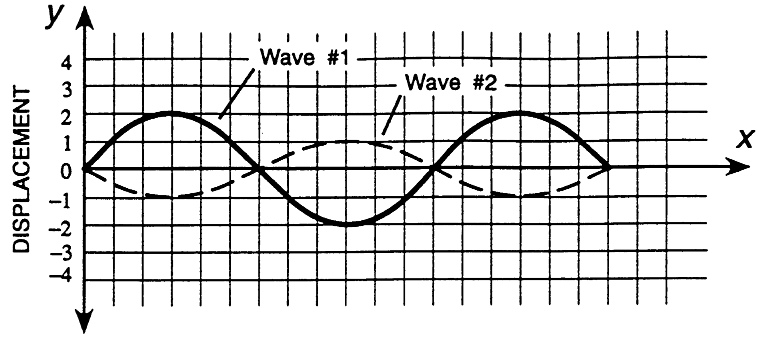
Problem 4: Copy the two in-phase waves shown below. Add the two waves and draw the resultant wave on your copy.



This wave pair results in ____ (constructive, destructive) interference. (Complete the sentence with the correct word.)



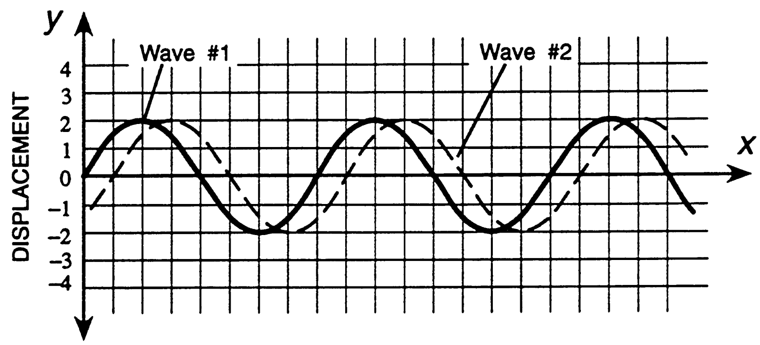
Problem 5: Copy the two waves shown below that are 180° out of phase. Add the two waves and draw the resultant wave on your copy.



This wave pair results in ____ (constructive, destructive) interference. (Complete the sentence with the correct word.)

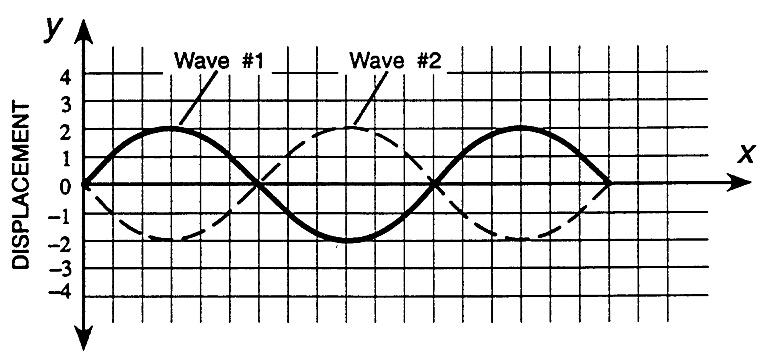
Student Challenge

Problem 6: Copy the two waves shown below that are 45° out of phase. Add the two waves and draw the resultant wave on your copy.



This wave pair results in ____ (constructive, destructive) interference. (Complete the sentence with the correct word.)

Problem 7: Copy the two waves shown below that are 180° out of phase. Add the two waves and draw the resultant wave on your copy.



This wave pair results in ____ (constructive, destructive) interference. (Complete the sentence with the correct word.)