Math Lab 8 MS 1

Solving Linear Momentum Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach students how to substitute values and units in linear momentum equations.
2. Teach students how to use linear impulse and linear momentum equations to solve problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises at the end of the previous subunit. Make sure that your students understand the correct answers.
2. Complete as many math activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to class. (How much is accomplished depends on the math skills that your students already have.)

   a. Sum up the explanatory material for "Activity: Solving Linear Momentum Problems."
   b. Then have students complete the Practice Exercises given at the end of the activity.

3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 8M1, "Impulse and Momentum in Linear Systems."

NOTE: Again, as explained in a teacher note several pages earlier, be sure to note that the equation $F\Delta t = \Delta(mv) = m\Delta v$ is true only so long as m remains constant while the impulse force acts. That's true, as we said, for most mechanical systems that experience an impulse and a resulting change in momentum. For special cases, such as rocket propulsion, where the mass of the rocket changes (decreases) as the rocket burns fuel and accelerates, the more general equation $\Delta(mv) = m\Delta v + v\Delta m$ is applied to the analysis of the problem.

Math Skills Laboratory

Lab 8 MS 1

MATH ACTIVITY

Solving Linear Momentum Problems

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

1. ***Substitute correct numerical values and units in momentum equations. Solve the equations for an unknown numerical value with the proper units.***
2. ***Use the following equations to solve linear momentum problems.***
   a. $P_{mom} = m \times v$ b. $Imp = F\Delta t$ c. $F\Delta t = \Delta(mv) = m\Delta v$
   d. P_{mom} ***BEFORE collision*** = P_{mom} ***AFTER collision***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

ACTIVITY

Solving Linear Momentum Problems

MATERIALS

For this activity, you'll need a calculator.

In this lab, you'll review basic units. You'll also solve problems that involve linear momentum, impulse and changes in momentum. The important equations, with typical units, are summarized.

a. Linear Momentum: $P_{mom} = mv$ ***Equation 1***

$$\left\{\begin{matrix}\text{Linear}\\ \text{Momentum}\end{matrix}\right\} = \left\{\begin{matrix}\text{Mass of Object}\\ \text{(or Fluid)}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Velocity of Object}\\ \text{(or Fluid)}\end{matrix}\right\}$$

English Units	*SI Units*
m in slugs *	m in kg
v in ft/sec	v in m/sec
P_{mom} in $\frac{\text{slug}\cdot\text{ft}}{\text{sec}}$	P_{mom} in $\frac{\text{kg}\cdot\text{m}}{\text{sec}}$

* The mass of an object in slugs is equal to the weight of the object in pounds divided by 32 ft/sec^2. In units, 1 slug = 1 lb·sec^2/ft.

NOTE: Spend some time on the review of units associated with Equation 1 (previous page) and Equations 2, 3 and 4. Have students refer to the equations and units to check their answers while working the exercises. The *Summary of Units* (paragraph "e") should provide a handy reference.

b. Impulse: Imp = F × Δt ***Equation 2***

$$\text{Impulse} = \left\{ \begin{array}{c} \text{Force Acting on} \\ \text{Object (or Fluid)} \end{array} \right\} \times \left\{ \begin{array}{c} \text{Time That} \\ \text{Force Acts} \end{array} \right\}$$

English Units	*SI Units*
F in lb	F in N
Δt in sec	Δt in sec
Imp in lb·sec	Imp in N·sec

c. Impulse and Momentum Change: F × Δt = m × Δv ***Equation 3***

$$\left\{ \begin{array}{c} \text{Force Acting} \\ \text{on Object} \end{array} \right\} \times \left\{ \begin{array}{c} \text{Time During} \\ \text{Which Force Acts} \end{array} \right\} = \left\{ \begin{array}{c} \text{Mass of} \\ \text{Object} \end{array} \right\} \times \left\{ \begin{array}{c} \text{Change in Velocity} \\ \text{Caused by the Force} \end{array} \right\}$$

English Units	*SI Units*
F in lb	F in N
Δt in sec	Δt in sec
m in slugs (same as $\frac{\text{lb·sec}^2}{\text{ft}}$)	m in kg
Δv in ft/sec	Δv in $\frac{\text{m}}{\text{sec}}$

Note: Since $F\Delta t = m\Delta v$, it's important to remember that the following relationships exist between units.

$$\text{lb·sec} = \frac{\text{slug·ft}}{\text{sec}} \quad \text{and} \quad \text{N·sec} = \frac{\text{kg·m}}{\text{sec}}$$

d. Conservation of Linear Momentum: $P_{mom\ before} = P_{mom\ after}$ ***Equation 4***

$$\left\{ \begin{array}{c} \text{Linear Momentum of Isolated} \\ \text{System BEFORE an Interaction} \end{array} \right\} = \left\{ \begin{array}{c} \text{Linear Momentum of Same} \\ \text{System AFTER the Interaction} \end{array} \right\}$$

English Units	*SI Units*
P_{mom} in $\frac{\text{slug·ft}}{\text{sec}}$	P_{mom} in $\frac{\text{kg·m}}{\text{sec}}$

e. Summary of Units

Quantity	*Symbol*	*English Units*	*SI Units*
Mass	m	slug or $\frac{\text{lb·sec}^2}{\text{ft}}$	kg
Velocity (speed)	v	ft/sec	m/sec
Change in velocity	Δv	ft/sec	m/sec
Force	F	lb	N
Time interval	Δt	sec	sec
Linear momentum	mv	$\frac{\text{slug·ft}}{\text{sec}}$	$\frac{\text{kg·m}}{\text{sec}}$
Change in linear momentum	Δmv	$\frac{\text{slug·ft}}{\text{sec}}$	$\frac{\text{kg·m}}{\text{sec}}$
Impulse	FΔt	lb·sec	N·sec

SOLUTIONS TO REVIEW OF UNITS

a. $\frac{\text{slug}\cdot\text{ft}}{\text{sec}}$

b. $\frac{\text{kg}\cdot\text{m}}{\text{sec}}$

c. mass

d. lb•sec

e. N•sec

f. $\text{lb}\cdot\text{sec} = \frac{\text{slug}\cdot\text{ft}}{\text{sec}}$

g. $\text{N}\cdot\text{sec} = \frac{\text{kg}\cdot\text{m}}{\text{sec}}$

h. $\frac{\text{lb}\cdot\text{sec}^2}{\text{ft}}$ or slugs

NOTE: Refer to Example 2.

In the solution to Part a, we used the unit (slug·ft/sec) for the change in momentum. In the solution to Part b, we used the unit (lb·sec) for impulse Both are equivalent. Nevertheless, as we mentioned earlier, it's more natural to use mass (slugs) and velocity (ft/sec) with the momentum formula ($m\Delta v$), and to use force (lb) and time (seconds) with the impulse formula ($F\Delta t$).

LET'S REVIEW UNITS

Before studying the Practice Exercises and solving the Problems, answer the following questions. They will check your understanding of the previous discussion about units. Fill in the blanks with the correct word or words.

a. The unit for linear momentum in the English system is ____ .

b. The unit for linear momentum in SI is ____ .

c. The slug is an English unit for ____ (mass; weight).

d. The unit for impulse in the English system is ____ .

e. The unit for impulse in the SI system is ____ .

f. The equation, $F\Delta t = m\Delta v$, tells us that the units for impulse and change in momentum are equivalent. That means that one lb·sec is equivalent to ____ .

g. Based on Question "f" above, we can also say that 1 N·sec is equivalent to ____ .

h. In the English system, the unit for mass is obtained by dividing the weight in pounds by g (32 ft/sec^2). This gives the unit $\frac{\text{lb·sec}^2}{\text{ft}}$ or ____ .

PRACTICE EXERCISES

Example 1: *Linear Momentum*

Given: A 20-lb hydraulic cylinder rod moves at a speed of 3 ft/sec during a certain instant of its motion.

Find: The linear momentum of the cylinder rod at this instant.

Solution: Use Equation 1.

$$P_{mom} = m \times v$$

$$\text{where:}\quad m = \frac{w}{g} = \frac{20\ \text{lb}}{32\ \text{ft/sec}^2} = 0.625\ \frac{\text{lb·sec}^2}{\text{ft}} = 0.625\ \text{slug}$$

$$v = 3\ \text{ft/sec}$$

$$P_{mom} = m \times v$$

$$P_{mom} = (0.625\ \text{slug})\ (3\ \text{ft/sec})$$

$$P_{mom} = (0.625 \times 3)\ (\text{slug} \times \text{ft/sec})$$

$$P_{mom} = 1.875\ \text{slug·ft/sec}$$

Example 2: *Linear Impulse and Change in Momentum*

Given: The hydraulic cylinder and rod of Example 1. The pressure in the hydraulic system causes a 7.5-lb force to act on the rod for 0.5 sec. During this time, the 20-lb hydraulic cylinder rod speeds up from zero to 6 ft/sec.

Find:
a. Change in linear momentum of the rod.
b. Impulse produced by the applied force on the rod.

Solution: a. Change in the linear momentum of the rod.

$$\Delta P_{mom} = \Delta(mv) = m\Delta v$$

$$\text{where:}\quad m = 0.625\ \text{slug (from Example 1)}$$

$$\Delta v = v_{final} - v_{initial} = 6\ \text{ft/sec} - 0\ \text{ft/sec} = 6\ \text{ft/sec}$$

$$\Delta P_{mom} = m\Delta v$$

$$\Delta P_{mom} = 0.625\ \text{slug} \times 6\ \text{ft/sec}$$

$$\Delta P_{mom} = (0.625 \times 6)\ (\text{slug} \times \text{ft/sec})$$

$$\Delta P_{mom} = 3.75\ \text{slug·ft/sec}$$

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1

Problem 1: Use Equation 1.

$$P_{mom} = mv \qquad \text{where: } m = 1500 \text{ kg}$$

$$v = 50 \frac{\text{km}}{\text{hr}} \times \frac{1000 \text{ m}}{\text{km}} \times \frac{1 \text{ hr}}{3600 \text{ sec}}$$

$$v = \frac{50{,}000}{3600} \frac{\text{m}}{\text{sec}}$$

$$v = 13.89 \text{ m/sec}$$

$$P_{mom} = 1500 \text{ kg} \times 13.89 \frac{\text{m}}{\text{sec}}$$

$$P_{mom} = 20{,}835 \frac{\text{kg}\cdot\text{m}}{\text{sec}}.$$

Problem 2: Use Equation 1.

$$P_{mom} = mv \qquad \text{where: } m = \frac{w}{g} = \frac{200 \text{ lb}}{32 \frac{\text{ft}}{\text{sec}^2}} = 6.25 \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}} = 6.25 \text{ slug}$$

$$v = 10 \text{ ft/sec}$$

$$P_{mom} = 6.25 \text{ slug} \times 10 \text{ ft/sec}$$

$$P_{mom} = 6.25 \times 10 \frac{\text{slug}\cdot\text{ft}}{\text{sec}}$$

$$P_{mom} = 62.5 \frac{\text{slug}\cdot\text{ft}}{\text{sec}}.$$

Problem 3: Use Equation 1. Solve for "v."

$$P_{mom} = mv \qquad \text{where: } P_{mom} = 10.17 \frac{\text{kg}\cdot\text{m}}{\text{sec}}$$

$$v = \frac{P_{mom}}{m} \qquad m = 0.3 \text{ kg}$$

$$v = \frac{10.17 \frac{\text{kg}\cdot\text{m}}{\text{sec}}}{0.3 \text{ kg}}$$

$$v = 33.9 \text{ m/sec}.$$

The solutions to Problems 4 and 5 are on page T-24c.

SOLUTIONS TO PRACTICE EXERCISES, ACTIVITY 1, Continued

Problem 4: Use Equation 2. Solve for "F."

$F\Delta t = m(\Delta v)$ where: $m = 0.3$ kg (from Problem 3)

$\Delta v = (V_f - v_i)$

$v_f = 33.9$ m/sec; $v_i = 0$ m/sec

$$\Delta v = (33.9 \frac{m}{sec} - 0 \frac{m}{sec}) = 33.9 \frac{m}{sec}$$

$\Delta t = 0.2$ sec

$$F = \frac{m(\Delta v)}{\Delta t}$$

$$F = \frac{0.3 \text{ kg} \times 33.9 \text{ m/sec}}{0.2 \text{ sec}}$$

$$F = (\frac{0.3 \times 33.9}{0.2}) \frac{kg \cdot m}{sec^2} = 50.85 \text{ N.}$$

Problem 5: Use Equation 2. Solve for Imp.

Imp = $F\Delta t$ where: $F = 6$ ton $= 12{,}000$ lb $= 12 \times 10^3$ lb

$\Delta t = 1/8$ sec $= 0.125$ sec $= 125 \times 10^{-3}$ sec

Imp = 12×10^3 lb $\times 125 \times 10^{-3}$ sec

Imp = $12 \times 125 \times 10^{3-3}$ lb·sec $\quad 10^{3-3} = 10^0 = 1$

Imp = 1500 lb.sec.

b. The impulse produced by the applied force

$IMP = F\Delta t$

where: $F = 7.5$ lb

$\Delta t = 0.5$ sec

$IMP = 7.5 \text{ lb} \times 0.5 \text{ sec}$

$IMP = (7.5 \times 0.5)(\text{lb} \times \text{sec})$

$IMP = 3.75 \text{ lb·sec}$

Note: Since we know that $F\Delta t = m\Delta v$, the answers to Parts "a" and "b" should be the same. They are! But they're different units. However, we know that 1 slug·ft/sec is the same as 1 lb·sec. Recall that we "proved" this in Table 8.1.

PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: Given: A car with a mass of 1500 kg is moving along a straight road at a speed of 50 km/hr.

Find: The linear momentum of the car. (Units should be in $\frac{\text{kg·m}}{\text{sec}}$.)

Solution:

Problem 2: Given: A man who weighs 200 lb is thrown forward from his seat at a speed of 10 ft/sec when the bus he's on stops suddenly.

Find: The momentum of the man. (Units should be in slug·ft/sec.)

Solution:

Problem 3: Given: The anvil on an electric nail driver has a mass of 0.3 kg. It has a linear momentum of 10.17 kg·m/sec as it strikes the nail.

Find: The velocity (speed) of the anvil in m/sec as it hits the nail.

Solution:

Problem 4: Given: The same nail driver as in Problem 3. The anvil starts at an initial speed of zero. It accelerates (speeds up) to 33.9 m/sec in 0.2 second. The acceleration is caused by a constant impulse force acting on the anvil.

Find: The force applied to the anvil.

Solution: (***Hint:*** Rearrange the equation, $F\Delta t = m\Delta v$, to isolate F. Then solve the new equation.)

Problem 5: Given: A forge hammer in a metal-shaping shop strikes a piece of metal with a force of 6 tons. The hammer is in contact with the metal for ⅛ sec (0.125 sec). The hammer weighs 2400 lb.

Find: The impulse delivered by the forge hammer.

Solution:

Problem 6: Use Equation 3. Solve for "F."

$F\Delta t = m(\Delta v)$ where: F_a = force on car without shock-absorbing bumper

F_b = force on car with shock-absorbing bumper

$$m = \frac{w}{g} = \frac{4000\ \text{lb}}{32\ \frac{\text{ft}}{\text{sec}^2}} = 125\ \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}}$$

$\Delta v = (v_f - v_i)$

$v_i = 7.3$ ft/sec

$v_f = 0$ ft/sec

$$\Delta v = (0 - 7.3\ \frac{\text{ft}}{\text{sec}}) = -7.3\ \frac{\text{ft}}{\text{sec}}$$

$\Delta v = 7.3$ ft/sec (Minus sign means auto was slowing down.)

$\Delta t_a = 0.35$ sec

$\Delta t_b = 0.78$ sec

a. $$F_a = \frac{m\Delta v}{\Delta t_a}$$

$$F_a = \frac{125\ \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}} \times 7.3\ \frac{\text{ft}}{\text{sec}}}{0.35\ \text{sec}}$$

$$F_a = \frac{125 \times 7.3}{0.35\ \text{sec}}\ \frac{\text{lb}\cdot\cancel{\text{sec}^2}\cdot\cancel{\text{ft}}}{\cancel{\text{ft}}\cdot\cancel{\text{sec}^2}} \qquad \text{(cancel units)}$$

$$F_a = 2607\ \text{lb.}$$

b. $$F_b = \frac{m\Delta v}{\Delta t_b}$$

$$F_b = \frac{125\ \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}} \times 7.3\ \frac{\text{ft}}{\text{sec}}}{0.78\ \text{sec}}$$

$$F_b = \frac{125 \times 7.3}{0.78}\ \frac{\text{lb}\cdot\cancel{\text{sec}^2}\cdot\cancel{\text{ft}}}{\cancel{\text{ft}}\cdot\cancel{\text{sec}^2}} \qquad \text{(cancel units)}$$

$$F_b = 1169.8\ \text{lb.}$$

$\Delta F = F_a - F_b = 2607$ lb $- 1169.8$ lb $= 1437.2$ lb
(difference in stopping force resulting in less damage).

Problem 7: a. $P_{mom}(\text{Juan}) = m_J v$

$$m_J = \frac{w}{g} = \frac{200\ \text{lb}}{32\ \frac{\text{ft}}{\text{sec}^2}} = 6.25\ \text{slug}$$

$$v = 8\ \text{ft/sec}$$

$$P_{mom}(\text{Juan}) = 6.25\ \text{slug} \times 8\ \text{ft/sec}$$

$$P_{mom}(\text{Juan}) = 50\ \frac{\text{slug}\cdot\text{ft}}{\text{sec}}\ .$$

b. $P_{mom}(\text{Alex}) = m_A v$

$$m_A = \frac{w}{g} = \frac{100\ \text{lb}}{32\ \frac{\text{ft}}{\text{sec}^2}} = 3.125\ \text{slug}$$

$$v = 16\ \text{ft/sec}$$

$$P_{mom}(\text{Alex}) = 3.125\ \text{slug} \times 16\ \frac{\text{ft}}{\text{sec}}$$

$$P_{mom}(\text{Alex}) = 50\ \frac{\text{slug}\cdot\text{ft}}{\text{sec}}\ .$$

c. $P_{mom}(\text{Grover}) = m_G v$

$$m_G = \frac{w}{g} = \frac{300\ \text{lb}}{32\ \frac{\text{ft}}{\text{sec}^2}} = 9.375\ \text{slug}$$

$$v = 5.3\ \text{ft/sec}$$

$$P_{mom}(\text{Grover}) = 9.375\ \text{slug} \times 5.3\ \frac{\text{ft}}{\text{sec}}$$

$$P_{mom}(\text{Grover}) = 50\ \frac{\text{slug}\cdot\text{ft}}{\text{sec}}$$

Therefore, $P_{Juan} = P_{Alex} = P_{Grover}$.

All three players have the same momentum. Therefore, Alex or Grover is equally capable of stopping Juan. However, if you examine this from a kinetic energy standpoint rather than conservation of momentum, where $E_k = 1/2\ mv^2$, the velocity is squared. **Alex is going to hurt Juan more than Grover will, since his kinetic energy is greater at impact.**

(Solutions for kinetic energy are shown on page T-25c.)

$$E_k(\text{Alex}) = \frac{1}{2} m_A v^2 = (0.5)(3.125 \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}})(16 \frac{\text{ft}}{\text{sec}})^2$$

$$= (0.5 \times 3.125 \times 256) \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}} \times \frac{\text{ft}^2}{\text{sec}^2}$$

E_k (Alex) = 400 ft•lb.

$$E_k(\text{Grover}) = \frac{1}{2} m_G v^2 = (0.5)(9.375 \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}})(5.3 \frac{\text{ft}}{\text{sec}})^2$$

$$= (0.5 \times 9.375 \times 28.09) \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}} \times \frac{\text{ft}^2}{\text{sec}^2}$$

E_k (Grover) = 131.7ft•lb.

$$E_k(\text{Juan}) = \frac{1}{2} m_J v^2 = (0.5)(6.25 \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}})(8 \frac{\text{ft}}{\text{sec}})^2$$

$$= (0.5 \times 6.25 \times 64) \frac{\text{lb}\cdot\text{sec}^2}{\text{ft}} \times \frac{\text{ft}^2}{\text{sec}^2}$$

E_k (Juan) = 200 ft•lb.

Problem 8: a. (3) 4 $m_0 v_0$.

b. $P_{mom} = mv$ (If m_0 is doubled, we have 2 m_0.)
$P_{mom} = (2m_0)(2v_0)$ (If v_0 is doubled, we have 2 v_0.)
$P_{mom} = 4\ m_0 v_0$.

Problem 9: As the boater steps forward from the boat, a force is applied to the boat, as well as to the boater. The boater pushes back on the boat, but the boat pushes forward on the boater with an equal force. These forces cause the boater of mass (m_1) to move forward with velocity (v_1) at the same time that the boat of mass (m_2) moves backward with velocity (v_2). Because momentum is conserved, the boater's momentum (m_1v_1) is equal to the momentum of the boat (m_2v_2). The initial momentum of the boater and boat was zero before the boater stepped from the boat to the dock. After the boater steps from the boat, the total momentum must still be zero, since momentum is conserved. But the boater's momentum when stepping from the boat is m_1v_1, while the boat's momentum is m_2v_2. The sum of $m_1v_1 + m_2v_2$ must remain zero. Thus, $m_1v_1 = -m_2v_2$. This simply means the boater and boat move away from each other in opposite directions, each with the same numerical value of momentum. The total momentum in the system is zero--before, during and after the boater steps from boat to dock; momentum is conserved. Applying this to our situation:

$m_1v_1 = -m_2v_2$ where: $m_1 = \frac{w}{g} = \frac{200 \text{ lb}}{32 \text{ ft/sec}^2}$

(mass of woman) = 6.25 slug

$m_2 = \frac{w}{g} = \frac{120{,}000 \text{ lb}}{32 \text{ ft/sec}^2}$

(mass of yacht) = 3750 slug

v_1 = velocity of boater

v_2 = velocity of yacht

$6.25 \text{ slug} \times v_1 = -3750 \text{ slug} \times v_2$

We don't know "v_1" or "v_2," but we do know $m_1v_1 = -m_2v_2$. So they have to have the same numerical value. This means that "m_1" is a small number compared to "m_2." Therefore, "v_1" must be a large number compared to "v_2." Let's take their ratio from the momentum equation.

$m_1v_1 = -m_2v_2$

$\frac{v_1}{v_2} = \frac{m_2}{m_1}$ (Ignore the minus sign for magnitude of "v_1" and "v_2.")

$\frac{v_1}{v_2} = \frac{3750 \text{ slug}}{6.25 \text{ slug}} = 600$ (So v_1 must be 600 times larger than "v_2.")

If we chose the woman to move at a velocity of 6 ft/sec, the yacht would move at $\frac{6}{600}\,\frac{\text{ft}}{\text{sec}}$ or $0.01\,\frac{\text{ft}}{\text{sec}}$ (a very slow, hardly noticeable movement). Let's prove it.

$$m_1v_1 = m_2v_2 \text{ (equate } \textit{magnitudes} \text{ only)}$$

$$6.25\text{ slug} \times 6\,\frac{\text{ft}}{\text{sec}} = 3750\text{ slug} \times 0.01\,\frac{\text{ft}}{\text{sec}}$$

$$6.25 \times 6\,\frac{\text{slug}\cdot\text{ft}}{\text{sec}} = 3750 \times 0.01\,\frac{\text{slug}\cdot\text{ft}}{\text{sec}}$$

$$37.5\,\frac{\text{slug}\cdot\text{ft}}{\text{sec}} = 37.5\,\frac{\text{slug}\cdot\text{ft}}{\text{sec}}.$$

For the 100-lb fiberglass boat:

$$m_1 = \text{(mass of the woman from Part 1)} = 6.25\text{ slug}$$

$$m_2 = \text{(mass of fiberglass boat)} = \frac{w}{g} = \frac{100\text{ lb}}{32\text{ ft/sec}^2} = 3.125\text{ slug}$$

$$m_1v_1 = m_2v_2 \text{ (equate } \textit{magnitudes} \text{ only)}$$

$$6.25\text{ slug} \times v_1 = 3.125\text{ slug} \times v_2.$$

"v_2" must be larger than "v_1" for the equation to be true. In fact, "v_2" must be two times as large.

Let v_1 = 6 ft/sec (as in Part 1)
v_2 = 12 ft/sec (since it must be two times as large)

$$6.25\text{ slug} \times 6\,\frac{\text{ft}}{\text{sec}} = 3.125\text{ slug} \times 12\,\frac{\text{ft}}{\text{sec}}$$

$$37.5\text{ slug}\,\frac{\text{ft}}{\text{sec}} = 37.5\text{ slug}\cdot\frac{\text{ft}}{\text{sec}}.$$

Since "v_2" is the boat speed, it moves **away** from the dock twice as fast as the woman moves **toward** the dock. (The recoil is fast.) This can easily cause the woman to fall into the water as she attempts to step onto the dock.

Problem 6: Sally Johnson of Johnson's Auto Sales is asked if installing shock-absorbing bumpers has any real advantage. To answer the question, she looks up the data on shock-absorbing bumpers. She finds that such bumpers extend the stopping time from 0.35 sec to 0.78 sec in a 5-mph collision. She then illustrates the forces involved with the following problem.

Given: When a vehicle that weighs 4000 lb backs up into a tree at 5 mph (7.3 ft/sec), it stops in 0.35 sec. A shock-absorbing bumper extends this time to 0.78 sec.

Find: The force required to stop the car with and without shock-absorbing bumpers. (What's the difference in the two forces?)

Solution:

Problem 7: Given: In spring tryouts for the football team, the coach has two new players try to tackle Juan, a 200-lb halfback who runs at 8 ft/sec. The first player, Alex, weighs 100 lb and runs at 16 ft/sec. The second player, Grover, weighs 300 lb and runs at 5.3 ft/sec.

Find: The player who would be more effective in stopping Juan in a head-on tackle. Explain your answer.

Solution:

Student Challenge

Problem 8: Given: An empty truck of mass m_o moves with a speed v_o. Its momentum is $P_{mom} = m_o v_o$. Suppose the empty truck is loaded until its mass is double its empty mass. Suppose its speed is also doubled.

Find:

a. The truck's new momentum is

(1) the same as before, $m_o v_o$.

(2) $2\ m_o v_o$.

(3) $4\ m_o v_o$.

(4) None of the above.

b. Use the equation "$P_{mom} = mv$" to explain your answer.

Solution:

Problem 9: Given: The law of conservation of linear momentum states that linear momentum is conserved for an isolated system when a 200-lb woman steps from a 60-ton yacht to the dock. When she does so, the only movement seen is the woman stepping on the dock. The yacht hardly moves. If the same woman steps from a 100-lb fiberglass boat to the dock, she may fall into the water if the boat isn't tied to the dock.

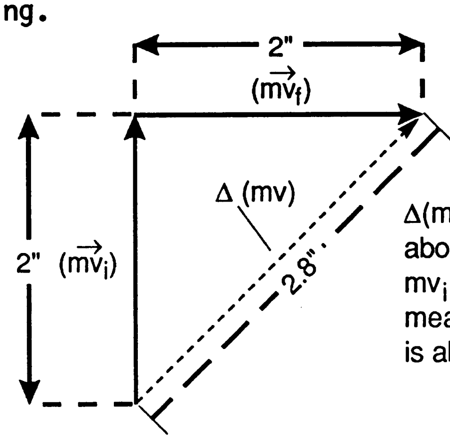
Find: Why the two situations are different. Explain by using the law of conservation of momentum.

Solution:

Problem 10: This problem does NOT involve collinear velocity vectors. Hence, we must rely on *vector* algebra. If students simply subtract the initial *speed* from the final speed as done before, they will obtain incorrect results.

a. The numerical value for $mv_i = (10\ kg)(3\ \frac{m}{sec}) = 30\ \frac{kg \cdot m}{sec}$. If you let 1 inch = $15\ \frac{kg \cdot m}{sec}$, the "$mv_i$" vector will be 2 inches long, pointing upward, as indicated on the drawing below. (Drawing is shown on a "reduced" scale.)

b. The numerical value for mv_f is **equal** to that for mv_i, but points to the right. Thus, drawn to the **same scale,** it is also 2 inches long, pointing to the right, as indicated on the drawing.



$\Delta(mv) = [\sqrt{2}\ (mv_i)]$ is the resultant, about 1.4 times the length of either mv_i or mv_f. Hence, to scale, the measured length of the resultant is about 2" × 1.4 = 2.8"

c. Change in momentum vector $\Delta(mv)$ will have a length equal to $\sqrt{2}$ times that of either mv_i or mv_f. You can understand this because you know that a right triangle with two equal sides (with the 90°∠ common) has a hypotenuse equal to $\sqrt{2}$ times the length of either side. The students should be able to measure the length of the hypotenuse (resultant) and find that it is about 2.8 inches, or 1.4 times either leg ($\sqrt{2}$ = 1.414). See diagram above.

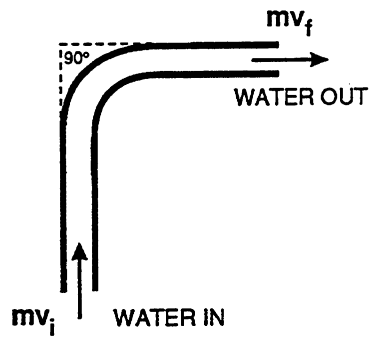
d. $$\text{Impact Force } F = \frac{\sqrt{2}\,(mv_i)}{\Delta t} = \frac{(\sqrt{2})\left(30\ \frac{kg \cdot m}{sec}\right)}{1\ sec}$$

$$F = 42.43\ \frac{kg \cdot m}{sec^2} \text{ or } 42.43\ N.$$

NOTE: In Problem 10, we have used formal vector notation--as adopted in many texts. Thus $\overrightarrow{mv_i}$ shows v_i with an arrow over it and also boldfaced. This notation indicates the *vector nature* of the velocity v_i, and thus the vector nature of the momentum $\vec{P}_{mom} = \overrightarrow{mv_i}$. The same is true for the expressions $\overrightarrow{mv_f}$ and $\overrightarrow{m\Delta v}$.

NOTE: A detailed description of the principle and math associated with Problem 10 is given in Lab 8F1/8F2, "Impulse and Momentum in Fluid Systems." Having already studied and worked through Problem 10 will help students understand Lab 8F1/8F2 better.

Problem 10: Given: Water moves through a pipe, as shown in the diagram. A 10-kg mass of water moves through a pipe at a velocity of 3 m/sec. The water is turned 90° from its original path by the 90° elbow. After the change in direction, neither the mass of the water nor the magnitude of the velocity has changed. Therefore, $mv_i = mv_f$. A vector diagram would be drawn in the following fashion:



Note: The two ***vector*** momentum quantities, $m\vec{\mathbf{v}}_i$ and $m\vec{\mathbf{v}}_f$, are ***not*** equal, since one ($m\vec{\mathbf{v}}_i$) points up and the other ($m\vec{\mathbf{v}}_f$) points to the right. But their ***magnitudes are equal***, so that $mv_i = mv_f$.

Find:

a. Length of the vector $m\vec{\mathbf{v}}_i$. Plot it on graph paper with the proper direction. Choose a convenient scale.

b. Length of the vector $m\vec{\mathbf{v}}_f$. Plot it on graph paper with the proper direction. Use the same scale.

c. Complete the vector triangle. Measure $\vec{\mathbf{F}}\Delta t$ (or $m\Delta\vec{\mathbf{v}}$).

d. If $\Delta t = 1$ sec, find the impulse force that caused the water to turn.

Solution: