Math Skills Laboratory

Lab 14 MS 2

MATH ACTIVITY

Activity: ***Time Constants of Exponential Growth Processes***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to:

1. ***Discuss the similarities and differences between exponential growth processes and exponential decay processes.***
2. ***Analyze an exponential growth process and find the time constant.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Work the problems.***

ACTIVITY

Time Constants of Exponential Growth Processes

Many of the processes that decay in an exponential manner are reversible. That is, they can be made to go in the opposite direction. For example, you can discharge a capacitor. That's a ***decay*** process. But you can also charge it. That's the reverse process, a ***growth*** process.

When this reversing takes place, the growth process is mathematically described by the equation:

$$Q = Q_{max}\,(1 - e^{-t/\tau})$$

where: Q = quantity increasing in the growth process
Q_{max} = final value toward which "Q" is increasing
t = time during which the change takes place
τ = 1/e time constant

A graph that shows "Q" versus time (t) for exponential growth and exponential decay is shown in Figure 1. The time (t) along the horizontal axis is marked off in units of the 1/e time constant (τ)—τ, 2 τ, 3 τ, etc.

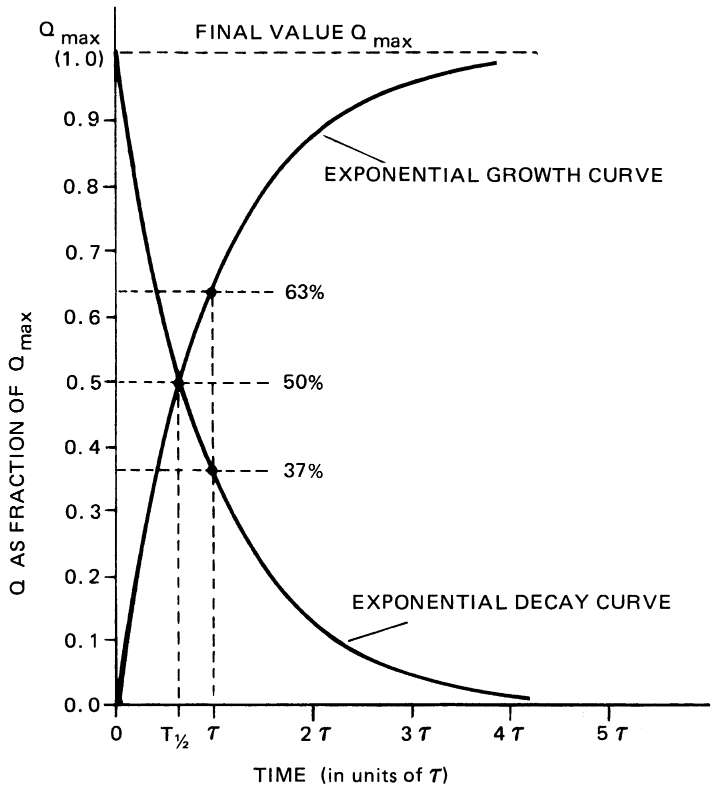

Fig. 1 Exponential growth and decay curves.

If you compare the curve for exponential growth with the curve for exponential decay, you'll see many similarities and differences. Some of the similarities between the two curves are:

- Both curves approach, but never reach, a limiting value.
- Both curves show a fast initial rate of change, but this rate gets slower as time increases.
- Both curves give the same value for $T_{1/2}$.
- Both curves have the same value for τ.
- After five *1/e time constants* (5 τ), both curves reach over 99% of their limiting value.

Some of the differences between the curves are:

- For the decay curve, $Q = 0.368\ Q_{max}$ (about 37% Q_{max}) after one time constant (τ).
- For the growth curve, $Q = 0.632\ Q_{max} = (1 - 0.368\ Q_{max})$ after one time constant (about 63% Q_{max}).
- The decay curve starts at a maximum value and approaches a minimum value. The growth curve starts at a minimum value and approaches a maximum.

Let's examine the curve of a typical growth process. The circuit shown in Figure 2 can be used to measure the voltage across a capacitor during the charging process. Consider the charging of a 7.5-μF capacitor through a 250-Ω resistor with a 100-V DC source. The voltage across this capacitor (as a function of time) is given by the equation:

$$V = V_{max}(1 - e^{-t/\tau})$$

If Vmax = 100 volts, this equation gives the data shown in Table 1.

The data in Table 1 is plotted on the graph shown in Figure 3. Note how the voltage changes rapidly at the beginning, near t = 1 msec.

By contrast, note how the voltage changes much more slowly as the charging process continues—say at t = 7 msec. Note also that the voltage approaches V_{max} = 100 volts very slowly.

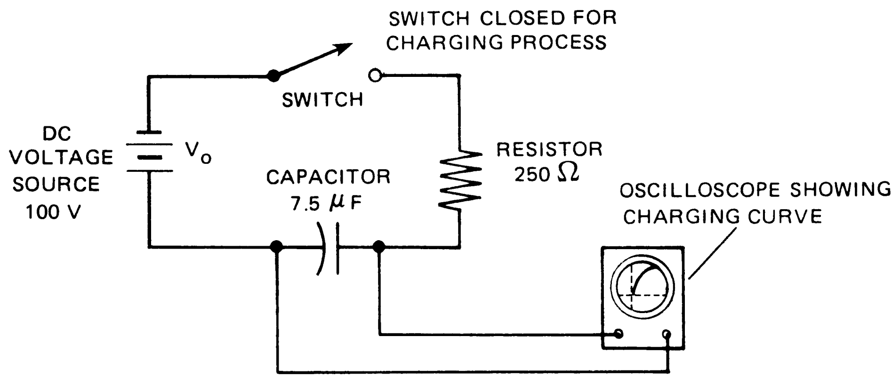

Fig. 2 Charging a capacitor.

TABLE 1. VOLTAGE ACROSS CAPACITOR (IN FIGURE 2) DURING THE CHARGING PROCESS

V	t
0 V	0 msec
41.3 V	1 msec
65.6 V	2 msec
79.8 V	3 msec
88.2 V	4 msec
95.9 V	6 msec
98.6 V	8 msec
99.5 V	10 msec

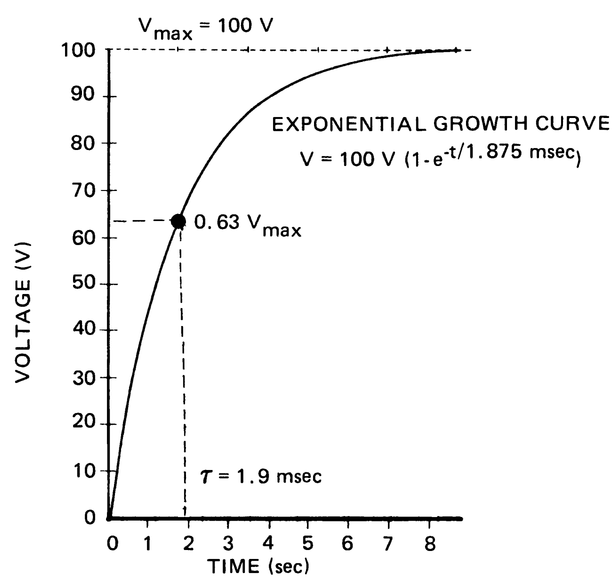

Fig. 3 Voltage versus time curve.

The 1/e time constant (τ) for the capacitor charging process is determined from the graph. A horizontal line is drawn at V = 0.63 V_{max} (63 volts) over to the curve. An intersection point is located on the curve. A vertical line is drawn from the intersection point to the time axis.

The vertical line crosses the time axis at approximately 1.9 msec. This value is the best you can "read" off the graph. (The correct value—obtained from the equation $\tau = R \times C$—is τ = 1.875 msec. So the approximate value of τ = 1.9 msec is quite acceptable.)

PRACTICE EXERCISES

Complete the following problems.

Problem 1: Given: Demitri is repairing an RC circuit that filters the output of a 100-V dc power supply. The RC circuit has a *1/e time constant* of 2 msec.

Find:
a. The voltage on the capacitor 2 msec after the power supply is turned ON.
b. The voltage on the capacitor 3 msec after the power supply is turned ON.

Solution: (***Hint:*** Solve the equation, $V = 100\,(1 - e^{-t/2\ \text{msec}})$, for V at 2 msec and 3 msec. Refer to Activity 3 of Math Lab 14MS1.)

Problem 2: Given: Amy is charging air tanks for the Scuba Club. The charging source is a storage tank that's maintained at 2,000 psi by a compressor. Amy has found that the air tanks being charged have a pressure of 1,000 psi after 5 minutes, 1,500 psi after 10 minutes, 1,750 psi after 15 minutes and 1,875 psi after 20 minutes.

Find:
a. The *1/e time constant* for the charging process.
b. The time required for the charging process to be essentially "completed."

Solution: (***Hint:*** Plot the pressure versus time on a graph and determine the time constant.)

Problem 3: Given: Austin is working in a meat-packing plant. The power to the meat-locker refrigeration system is interrupted during a thunderstorm. The temperature outside the meat locker is 75°F, and the temperature in the meat locker is maintained at 0°F. Austin notices that the air temperature in the locker is 38°F after the power has been off for 2 hours, 56°F after 4 hours, 66°F after 6 hours and 70°F after 8 hours.

Find:
a. The *1/e time constant* for the warming process.
b. The time available for the meat-packing company to restore the power if the meat can be kept at a temperature greater than 47°F for ***no more*** than 8 hours before the power (and refrigeration system) is turned on again.

Solution: (***Hint:*** Plot the temperature versus time for the warming process. Determine the *1/e time constant* [τ] from the graph.)