

# Math Skills Laboratory

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Lab 10<sup>M</sup>3<sup>S</sup>

## **MATH ACTIVITY**

**Activity:** *Solving Energy-Conversion Problems for Electrical Energy Convertors*

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## **MATH SKILLS LAB OBJECTIVES**

*When you complete these activities, you should be able to do the following:*

- 1. Solve energy-conversion problems for electrical energy convertors.*
  - 2. Substitute correct numerical values and units in energy-conversion equations. Solve the equations for an unknown numerical value with the proper units.*
  - 3. Find the efficiency of electrical energy convertors.*
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## **LEARNING PATH**

- 1. Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
  - 2. Study the examples.*
  - 3. Work the problems.*
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## **ACTIVITY**

### **Solving Energy-Conversion Problems for Electrical Energy Convertors**

#### **MATERIALS**

For this activity, you'll need a calculator.

In this Math Skills Lab, you'll review the important formulas for work and energy. You'll review the units used to measure work, energy and power. You'll solve problems that involve electrical energy convertors.

**LET'S REVIEW FORMULAS AND UNITS!**

Examine Tables 1, 2 and 3. These are the same tables included in Math Skills Labs 10MS1 and 10MS2.

Table 1 lists the important formulas for work and energy that you've already studied. Table 2 sums up common units that you'll use in energy-conversion problems. Table 3 lists some conversions between units.

You'll find these conversions helpful when you solve the problems in this activity.

TABLE 1. REVIEW OF BASIC FORMULAS FOR WORK AND ENERGY

Energy System	Formulas	Definition of Symbols
<b>MECHANICAL</b>		
Linear Work	$W = F \times D$	F = applied force D = distance moved
Rotational Work	$W = T \times \theta$	T = torque applied $\theta$ = angular distance rotated
Gravitational Potential Energy	$E_p = w \times h$	w = weight h = height raised above reference level
Elastic Potential Energy	$E_p = \frac{1}{2} kd^2$	k = force or spring constant d = distance spring is stretched or compressed
Linear Kinetic Energy	$E_k = \frac{1}{2} mv^2$	m = mass v = speed
Rotational Kinetic Energy	$E_k = \frac{1}{2} I\omega^2$	I = moment of inertia $\omega$ = angular speed
<b>FLUID</b>		
Fluid Work	$W = p \times (\Delta V);$ $W = (\Delta p) \times V$	p; $\Delta p$ = pressure or pressure difference $\Delta V; V$ = volume of fluid moved
Fluid Kinetic Energy	$E_k = \frac{1}{2} (\rho V)v^2$	$\rho$ = mass density V = volume v = speed
<b>ELECTRICAL</b>		
Electrical Work	$W = q \times \Delta V$	q = electrical charge moved $\Delta V$ = voltage difference
Electrical Energy	$E_{elec} = P_{elec} \times t$	$P_{elec}$ = electrical power used t = time power is used
<b>THERMAL</b>		
Thermal Energy	$H = mc\Delta T$	m = mass c = specific heat $\Delta T$ = temperature difference

TABLE 2. SUMMARY OF COMMON UNITS USED IN ENERGY CONVERSION CALCULATIONS

Quantity	Symbol	English Unit	SI Unit
Mass	m	slug or (lb·sec <sup>2</sup> /ft)	kg
Weight	w	lb	N
Force	F	lb	N
Work	W	ft·lb	N·m or J
Kinetic Energy	E <sub>k</sub>	ft·lb	N·m or J
Potential Energy	E <sub>p</sub>	ft·lb	N·m or J
Power	P	(ft·lb)/sec; hp	(N·m)/sec or watt
Spring Constant	k	lb/ft; oz/in.	N/m; kgf/m *
Angular			
Displacement	$\theta$	rad	rad
Angular Speed	$\omega$	rad/sec	rad/sec
Linear Speed	v	ft/sec	m/sec
Mass Density	$\rho$	slug/ft <sup>3</sup> ; lbm/ft <sup>3</sup> **	kg/m <sup>3</sup>
Moment of Inertia	I	slug·ft <sup>2</sup>	kg/m <sup>2</sup>
Pressure or			
Pressure Difference	p; $\Delta p$	lb/ft <sup>2</sup> ; oz/in <sup>2</sup>	N/m <sup>2</sup>
Electrical Charge	q	—	coulomb (C)
Torque	T	lb·ft; lb·in.	N·m
Fluid Volume	V; $\Delta V$	ft <sup>3</sup>	m <sup>3</sup>
Electrical Current	I	—	ampere (A)
Electrical Voltage	V; $\Delta V$	—	volt (V)
Electrical Resistance	R	—	ohm ( $\Omega$ )
Heat Energy	H	Btu	cal
Temperature	T	°F	°C
Temperature Difference	$\Delta T$	F°	C°
Specific Heat	c	$\frac{\text{Btu}}{\text{lb}\cdot\text{F}^\circ}$	$\frac{\text{kcal}}{\text{kg}\cdot\text{C}^\circ}$

\* kgf  $\Rightarrow$  kilogram force

\*\* lbm  $\Rightarrow$  pound mass

TABLE 3. USEFUL CONVERSIONS FOR UNITS

1 slug	=	32 pound mass (32 lbm)
1 kilogram force (kgf)	=	9.8 newtons
1 newton-meter	=	1 joule
1 volt-coulomb	=	1 joule
1 ampere-second	=	1 coulomb
1 radian	=	57.3°
1 horsepower	=	550 ft·lb/sec
1 horsepower	=	746 watts
1 watt	=	1 newton-meter per second
1 watt	=	1 joule per second
1 watt	=	1 volt-ampere
1 watt·sec	=	1 joule = 0.7376 ft·lb
1 kilowatt-hour (kWh)	=	$3.6 \times 10^6$ joules
1 calorie	=	3.086 foot-pounds
1 calorie	=	4.184 joules
1 Btu	=	778 foot-pounds
1 Btu	=	1055 joules

Now that you've reviewed the contents of Tables 1, 2 and 3, use them to help you answer the following questions.

- The formula for electrical work in electrical systems is: \_\_\_\_.
- The formula for electrical energy used, given the electrical power and time of operation, is: \_\_\_\_.
- The formula for rotational mechanical energy is: \_\_\_\_.
- The formula for fluid kinetic energy is: \_\_\_\_.
- The formulas for fluid work are: \_\_\_\_.
- The formula for thermal energy is: \_\_\_\_.
- In Question 3 and Table 2, the units of moment of inertia (I) in SI units are \_\_\_\_.
- In Question 6, specific heat (c) in English units (from Table 2) is: \_\_\_\_.
- In Question 2, electrical energy would be in units of \_\_\_\_.
- An electrical energy convertor changes 1100 watt·sec of electrical energy into 1 Btu of heat energy.
  - One Btu equals \_\_\_\_ joules.
  - One watt·sec of heat energy equals \_\_\_\_ joule.
  - One Btu equals \_\_\_\_ watt·sec.
- In Question 10, the electrical energy convertor is \_\_\_\_ % efficient.
- Five coulombs of charge are moved by 12 volts. All energy expended is changed to electrical work. Energy expended is \_\_\_\_ joules.
- One kilowatt-hour of electrical energy is changed to  $3.5 \times 10^6$  joules of mechanical energy. One kWh equals \_\_\_\_ joules.
- In Question 13, the electrical energy convertor is \_\_\_\_ % efficient.

### **ENERGY-CONVERSION PROBLEMS THAT INVOLVE ELECTRICAL ENERGY CONVERTORS**

Two examples of energy conversion follow. Study them, and then solve the exercises.

#### **Example A: Efficiency of an Electric Motor** **(Electrical-to-Mechanical Energy Convertor)**

**Given:** An electric motor converts 0.5 kWh of electrical energy into  $1.56 \times 10^6$  J of rotational mechanical energy at the motor output shaft.

**Find:** The efficiency of the motor.

**Solution:** The efficiency is given as:

$$\text{Eff}(\%) = \frac{E_{\text{OUT}}}{E_{\text{IN}}} \times 100\% \quad \text{where: } E_{\text{OUT}} = 1.56 \times 10^6 \text{ J (output shaft mechanical energy)}$$

$$E_{\text{IN}} = 0.5 \text{ kWh (electrical energy)}$$

The energy values are in different units. Efficiency can't be calculated. Therefore, first change  $E_{\text{IN}}$  from kWh to joules. From Table 3, find that 1 kWh =  $3.6 \times 10^6$  J.

$$E_{\text{IN}} = 0.5 \text{ kWh} \times \left[ 3.6 \times 10^6 \frac{\text{J}}{\text{kWh}} \right]$$

$$E_{\text{IN}} = (0.5 \times 3.6 \times 10^6) \left[ \cancel{\text{kWh}} \times \frac{\text{J}}{\cancel{\text{kWh}}} \right] \quad \text{(Cancel units.)}$$

$$E_{\text{IN}} = 1.8 \times 10^6 \text{ J}$$

Now calculate the efficiency.

$$\text{Eff}(\%) = \frac{1.56 \times 10^6 \cancel{\text{J}}}{1.8 \times 10^6 \cancel{\text{J}}} \times 100\% \quad \text{(Cancel units.)}$$

$$\text{Eff}(\%) = 87\% \text{ (rounded)}$$

**Example B: Efficiency of an Electric Heating Element  
(Electrical-to-Thermal Energy Converter)**

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**Given:** An electric heating element, often called a “calrod unit,” is 94% efficient in changing electrical energy to thermal energy. Twenty gallons of water (167 lbm) are raised in temperature from 80°F to 105°F by the calrod unit. The specific heat of water is 1 Btu/(lbm·F°).

- Find:**
- The thermal energy required to raise the water from 80°F to 105°F.
  - The electrical energy (in joules) used by the calrod unit to change the electrical energy to thermal energy.
  - The electrical energy in kWh.

**Solution:** a. To find the thermal energy needed to heat the water, use the formula

$$H = mc\Delta T \quad \text{where:} \quad \begin{array}{l} m = 167 \text{ lbm} \\ c = 1 \text{ Btu}/(\text{lbm}\cdot\text{F}^\circ) \\ \Delta T = 105^\circ\text{F} - 80^\circ\text{F} = 25 \text{ F}^\circ \end{array}$$

$$H = 167 \text{ lbm} \times \left[ 1 \frac{\text{Btu}}{\text{lbm}\cdot\text{F}^\circ} \right] \times 25 \text{ F}^\circ$$

$$H = (167 \times 1 \times 25) \left[ \cancel{\text{lbm}} \times \frac{\text{Btu}}{\cancel{\text{lbm}}\cdot\cancel{\text{F}^\circ}} \times \cancel{\text{F}^\circ} \right] \quad \text{(Cancel units.)}$$

$$H = 4175 \text{ Btu}$$

b. To find the electrical energy input to the calrod unit, use the efficiency formula.

$$\text{Eff}(\%) = \frac{E_{\text{OUT}}}{E_{\text{IN}}} \times 100\% \quad \text{where:} \quad \begin{array}{l} \text{Eff}(\%) = 94\% \\ E_{\text{OUT}} = H = 4175 \text{ Btu} \\ E_{\text{IN}} = \text{electrical input energy} \\ \text{in joules} \end{array}$$

Rearrange the equation to isolate  $E_{\text{IN}}$ .

$$E_{\text{IN}} = \frac{E_{\text{OUT}}}{\text{Eff}(\%)} \times 100\%$$

Substitute in values, converting  $E_{\text{OUT}}$  to joules. From Table 3, 1 Btu = 1055 J.

$$E_{\text{IN}} = \frac{4175 \text{ Btu} \times 1055 \frac{\text{J}}{\text{Btu}}}{94\%} \times 100\%$$

$$E_{\text{IN}} = \left[ \frac{4175 \times 1055 \times 100}{94} \right] \left[ \frac{\cancel{\text{Btu}} \times \text{J} \times \cancel{\%}}{\cancel{\%} \times \cancel{\text{Btu}}} \right] \quad \text{(Cancel units.)}$$

$$E_{\text{IN}} = 4,685,771 \text{ J (rounded)}$$

c. To convert the electrical energy to kWh, use the relationship, 1 kWh =  $3.6 \times 10^6$  J from Table 3.

$$E_{\text{IN}} = 4,685,771 \text{ J} \times \frac{1 \text{ kWh}}{3.6 \times 10^6 \text{ J}}$$

$$E_{\text{IN}} = \left[ \frac{4,685,771 \times 1}{3.6 \times 10^6} \right] \left[ \frac{\cancel{\text{J}} \times \text{kWh}}{\cancel{\text{J}}} \right] \quad \text{(Cancel units.)}$$

$$E_{\text{IN}} = 1.3 \text{ kWh (rounded)}$$

Thus, the calrod unit consumed 1.3 kWh of electrical energy to heat the water.

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**PRACTICE EXERCISES**

**Problem 1:** **Given:** An electric motor is rated at 137.5 ft·lb/sec ( $\frac{1}{4}$  hp) shaft output. This means the shaft has 137.5 ft·lb of energy available to do work each second. The electrical input energy to the motor is 203.5 watt·sec (about 150 ft·lb; see Table 3).

**Find:** The motor efficiency.

**Solution:**

**Problem 2:** Given: The solenoid in an electrical relay opens and closes electrical contacts. When twelve volts (12 V) of electricity are applied, 4 coulombs of charge move through the solenoid coil. The resulting plunger movement uses 40 joules of energy to close a set of electrical contacts.

Find: The efficiency of the solenoid.

Solution: (**Hint:** Use the relationship that 1 volt-coulomb = 1 joule.)

**Problem 3:** Given: A remote water-line cut-off valve located in the ceiling of a building is operated by a 110-volt solenoid. The plunger that seats the valve in the closed position is held in place by a spring. It takes 550 N·m of energy to unseat the valve by compressing the spring. The solenoid assembly is 90% efficient in changing electrical energy to the linear mechanical energy that is used to compress the spring.

Find: The charge moved in operating the solenoid.

Solution: (**Hint:**  $E_{IN} = W_{IN} = q \times \Delta V$ .)

**Problem 4:** Given: An electrical motor has the following data plate on it:

AJAX MOTOR CO.			
Power Summary			
INPUT		OUTPUT	
220	volts	3.0	hp
12.5	amperes	1800	rpm (shaft speed)
1/60	phase/Hz	20°C	temp rise (cont)

Find: The efficiency of this motor.

Solution: (**Hint:** The output is given in units of power. Input power can be calculated easily by  $\Delta V \times I$ . Therefore, use the efficiency formula

$$\text{involving power, namely } \text{Eff}(\%) = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\%.)$$

### Student Challenge

**Problem 5:** An electric induction heater (an energy convertor) is used to heat the inner race (ring) of ball bearings or roller bearings. This expands the inside diameter of the inner race of the bearing, allowing a slip-fit of the bearing onto a close-tolerance shaft or axle. When the bearing cools, the inner race shrinks to its original size, fits snugly on the shaft, and doesn't slip.

Given: Bearing inner races are heated to 120°C to allow the slip-fit under normal conditions. Room temperature is 30°C. The specific heat of steel is 0.113 kcal/(kg·C°).

- Find:
- The heat energy that must be added to the inner race of a bearing of 0.5-kg mass to heat the bearing from room temperature to 120°C.
  - The electrical energy that must be supplied to the induction heater if the induction heater is 75% efficient.

**Problem 6:** Given: A coal-fired boiler in an electric power plant uses a draft-damper control to regulate the fire. The control motor that rotates the draft-damper has a data plate, as shown below.

SPECIALTY MOTORS, INC.	
Hercules Motor Damper, Model 120-1A	
MOTOR INPUT	SHAFT OUTPUT
48 volts DC	120 lb-in torque
0.83 amperes	SHAFT ROTATION
20°C temp rise (cont)	90/1.57 degrees/radians

- Find:
- The electrical energy converted each second into rotational mechanical energy of the output shaft. (**Hint:** Use  $E_{IN} = P_{IN} \times t = [\Delta V \times I] \times t$ .)
  - The work done (energy used) each second by the output shaft while it spins at constant speed through an angle of  $90^\circ$  (1.57 radians). (**Hint:** Use the formula,  $W_{OUT} = E_{OUT} = T \times \theta$ .)
  - The efficiency of the motor/damper unit as an energy convertor.

Solution: