Math Skills Laboratory

Lab 5 MS 1

MATH ACTIVITIES

Activity 1: Reviewing Examples of Potential Energy Problems

Activity 2: Solving Problems That Involve Energy and Work in Mechanical and Fluid Energy Systems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Rearrange the equation for gravitational potential energy, $E_p = mgh$, to solve for mass (m), gravitational constant (g), or height (h).***
2. ***Rearrange the equation, $E_p = \frac{1}{2} kd^2$, to solve for a spring constant (k) or the distance (d) a spring extends or compresses.***
3. ***Rearrange the equation for a spring constant, $k = F/d$, to solve for force (F) applied to the spring, or the distance (d) the spring extends or compresses.***
4. ***Substitute correct numerical values and units in energy equations. Solve the equations for a numerical value with the proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study Table 1 and Equations 1, 2 and 3.***
3. ***Work the problems.***

ACTIVITY 1

Reviewing Examples of Potential Energy Problems

MATERIALS

For this activity, you'll need a calculator.

In Subunit 1, you learned about two types of energy: (1) ***gravitational potential energy*** and (2) ***elastic potential energy***.

Knowing about each type of energy is important. As a technician, you'll be working with machines that use these types of energy to do work. Each type of energy influences the way a machine is designed. Technicians must understand the way a machine is designed if they want to modify or repair the machine properly.

During classroom discussions, you learned some general methods of storing potential energy. You also learned that stored potential energy can be converted to do work. Activity 1 of this math lab explains methods that can be used to solve technical problems. In Activity 2, you'll solve some problems similar to those a technician might have to solve.

Let's look at the relationship of the various physical quantities in the gravitational potential energy equation and the elastic potential energy equation.

$$\begin{matrix}\text{Gravitational} \\ \text{Potential Energy}\end{matrix} = \text{Mass} \times \begin{matrix}\text{Gravitational} \\ \text{Constant}\end{matrix} \times \text{Height}$$

This relationship helps you find the value of one physical quantity if you know the value and units of the other two physical quantities. The relationship is often written with symbols rather than words, as follows.

$$E_p = mgh = wh \qquad \textit{Equation 1}$$

where: E_p = gravitational potential energy
m = mass
g = gravitational constant
w = weight
h = height

Equation 1 describes potential energy of an object in terms of the object's mass (m), the gravitational constant (g) and height (h) above a reference level.

Equation 2 describes potential energy in terms of the properties of the material. These are the material's elastic spring constant and the length a spring extends or compresses. This equation can be written as follows.

$$\begin{matrix}\text{Elastic} \\ \text{Potential Energy}\end{matrix} = \tfrac{1}{2} \times \begin{matrix}\text{Spring} \\ \text{Constant}\end{matrix} \times \begin{pmatrix}\text{Length of} \\ \text{Movement}\end{pmatrix}^2$$

This relationship helps you find the value of one physical quantity if you know the value and units of the other physical quantities. The relationship is often written with symbols rather than words, as follows.

$$E_p = \tfrac{1}{2} kd^2 \qquad \textit{Equation 2}$$

where: E_p = elastic potential energy
k = spring constant
d = length of stretch or compression

Table 1 sums up the units used with each of the physical quantities given in Equations 1 and 2. Study Table 1. Compare the units in each system.

TABLE 1. TYPICAL POTENTIAL ENERGY UNITS

		System of Units: English	System of Units: SI
Equation 1: $E_p = mgh = wh$	E_p	ft·lb	N·m
	w (or mg)	lb	N
	m	—	kg
	g	ft/sec^2	m/sec^2
	h	ft	m
Equation 2: $E_p = \frac{1}{2} kd^2$	E_p	ft·lb or in·lb	N·m or N·cm
	k	lb/ft or lb/in.	N/m or N/cm
	d	ft or in.	m or cm
	d^2	ft^2 or in^2	m^2 or cm^2

Now let's look at the physical quantities that determine the elastic spring constant of a material.

$$\text{Elastic Spring Constant} = \frac{\text{Force Applied to Spring}}{\text{Distance Spring is Extended or Compressed}}$$

This relationship is written with symbols as follows:

$$k = \frac{F}{d} \qquad \textbf{\textit{Equation 3}}$$

where:
k = elastic spring constant of the material
F = force applied to extend or compress the spring
d = displacement of spring during extension or compression

LET'S REVIEW UNITS!

Use Table 1 to answer the following questions. Fill in the blank with the correct word of words.

a. In the English system, ***gravitational potential energy*** is measured in units of ____ (ft·lb, N·m).

b. In the English system, the ***elastic spring constant*** is measured in units of ____ (lb/in., N/cm).

c. In the gravitational potential energy equation, ***height*** has units of ____ (ft, m) in SI units.

d. In the English system of units, the ***weight quantity*** (mg) is measured in ____.

PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: The gravitational potential energy that an object has at some position equals the gravitational force acting on the object times the vertical distance the object is raised above the reference point.

Given: A construction elevator raises an 800-pound load of bricks from the ground to the third floor of the building, 24 feet above the ground.

Find: The potential energy of the load of bricks when the elevator reaches the third floor.

Solution: This problem involves gravitational potential energy, so let's use Equation 1.

$$E_p = mgh, \text{ or } E_p = wh \text{ (since } w = mg)$$

where: mg = 800 lb (weight of bricks in English units)
h = 24 ft (height from ground to third floor)

$$E_p = w \times h$$
$$E_p = 800 \text{ lb} \times 24 \text{ ft} = (800 \times 24)(\text{ft} \times \text{lb})$$
$$E_p = 19{,}200 \text{ ft·lb}$$

This is the potential energy gained by the bricks with reference to the ground.

If the bricks gain 19,200 ft·lb of potential energy due to their position 24 ft above the ground, at least that much work (19,200 ft·lb) was done to raise the bricks to that height. This is an example of the relationship of stored potential energy and work done. Since machines are not 100% efficient, *more than* 19,200 ft·lb of energy was used by the machine to raise the bricks to the third floor. That's because the machine had to overcome friction while it did the work.

Problem 2: Elastic potential energy in a spring is the energy stored in the spring when the spring is stretched or compressed.

Given: A newspaper printing machine uses a set of rollers to feed paper through the machine. The rollers are held firmly against the paper by a spring. The spring exerts a force of 100 pounds when it is compressed 4 inches beyond its unstretched length.

Find:
a. The spring constant (k) that the maintenance technician should specify when ordering a spare replacement.
b. The amount of work done to compress the spring 4 inches.

Solution:
a. The value of k equals the force applied to the spring divided by the distance the spring stretches or compresses. In equation form, this is written as

$$k = \frac{F}{d} \qquad \text{(Equation 3)}$$

where: k = spring constant
F = 100 lb
d = 4 in.

Solve for k.

$$k = \frac{F}{d} = \frac{100 \text{ lb}}{4 \text{ in.}} = \left(\frac{100}{4}\right)\left(\frac{\text{lb}}{\text{in.}}\right)$$

$$k = 25 \text{ lb/in.}$$

The spring constant for this particular spring is 25 lb/in. Therefore, the technician would ask for a spare replacement of the proper dimensions with a spring constant of 25 lb/in.

b. The elastic potential energy stored in a deformed spring is equal to the work required to deform the spring. Elastic potential energy, in equation form (Equation 2) is:

$$E_p = \tfrac{1}{2} kd^2$$

where: k = 25 lb/in. (from Part a)
d = 4 in.

$$E_p = \tfrac{1}{2}\left(25\ \frac{\text{lb}}{\text{in.}}\right)(4\ \text{in.})^2$$

$$E_p = \tfrac{1}{2}(25\cdot 16)\left(\frac{\text{lb}}{\cancel{\text{in.}}} \times \text{in}^{\cancel{2}}\right) \qquad \text{(Cancel units.)}$$

$$E_p = 200\ \text{in}\cdot\text{lb}$$

$$E_p = 200\ \cancel{\text{in}}\cdot\text{lb} \times \frac{1\ \text{ft}}{12\ \cancel{\text{in.}}} \qquad \text{(Change units from in·lb to ft·lb.)}$$

$$E_p = 16.7\ \text{ft}\cdot\text{lb}$$

The work done to compress the spring 4 inches is 16.7 ft·lb. Therefore, the spring has 16.7 ft·lb of elastic potential energy stored in it.

Problem 3: The work done to stretch a spring equals the elastic potential energy stored in the spring while the spring is deforming.

Given: Five hundred inch·pounds (500 in·lb) of work is done in stretching a spring with a spring constant of 40 lb/in.

Find:
a. The distance the spring is stretched.
b. The force applied to stretch the spring.

Solution: a. This problem involves elastic potential energy, so let's use Equation 2, $E_p = \tfrac{1}{2} kd^2$, where k = 40 lb/in. and E_p = 500 in·lb.

To isolate d, first isolate d^2 by multiplying both sides of the equation by ($^2\!/_k$).

$$E_p\left(\frac{2}{k}\right) = \frac{1}{\cancel{2}}\cancel{k}d^2 \times \left(\frac{\cancel{2}}{\cancel{k}}\right) \qquad \text{(Cancel 2's and k's on right side of equation.)}$$

$$E_p\left(\frac{2}{k}\right) = d^2 \qquad \text{(Simplify.)}$$

$$d^2 = E_p\left(\frac{2}{k}\right) \qquad \text{(Rearrange equation with } d^2 \text{ isolated on the left.)}$$

$$d^2 = \frac{500\ \text{in}\cdot\text{lb} \times 2}{40\ \text{lb/in.}} \qquad \text{(Substitute in numbers. Solve for } d^2\text{.)}$$

$$d^2 = \left(\frac{1000}{40}\right)\left(\text{in}\cdot\cancel{\text{lb}} \times \frac{\text{in.}}{\cancel{\text{lb}}}\right) \qquad \text{(Cancel lb units.)}$$

$$d^2 = 25\ \text{in}^2$$

Take the square root of each side of the equation to obtain d.

$$d = \sqrt{25\ \text{in}^2} = 5\ \text{in.}$$

Thus the spring was stretched a distance of 5 inches.

b. The equation for the spring constant is $k = F/d$ where $k = 40$ lb/in. and $d = 5$ in. (from Part a).

Solving the equation $k = F/d$ for F (by rearranging and isolating F) gives:

$$F = k \times d$$

$$F = 40 \frac{\text{lb}}{\text{in.}} \times 5 \text{ in.}$$

$$F = (40 \times 5)\left[\frac{\text{lb}}{\cancel{\text{in.}}} \times \cancel{\text{in.}}\right] \qquad \text{(Cancel units.)}$$

$$F = 200 \text{ lb}$$

The force applied to the spring is 200 lb.

ACTIVITY 2

Solving Practical Problems That Involve Energy and Work in Mechanical and Fluid Energy Systems

MATERIALS

For this activity, you'll need a calculator.

Note: You've used one type of hydraulic accumulator in Lab 4F1. The following problem involves an accumulator. (You'll learn more about accumulators in Lab 5MF2.)

Problem 4: Given: A hydraulic accumulator spring has a spring constant of $k = 200$ lb/in. The spring is compressed 6 inches. The work done to compress the spring equals the potential energy stored in the spring.

Find:

a. The potential energy (E_p) stored in the spring when the spring is compressed 6 inches—in units of in·lb and ft·lb.

b. The force required to compress the spring 6 inches. (Remember: $k = F/d$ or $F = k \times d$.)

Solution:

Check the units of the solution with the units of Table 1. Are the units correct? For each problem in the Math Skills Lab, you should compare the units of the solution to the units of Table 1. Doing this will help you make sure the solution units are correct.

Problem 5: Given: Sara is a technical sales representative for a company that makes material-conveying systems. One of those systems uses a vacuum to move material from one place to another.

Sara has been talking with a production manager. The production manager is searching for a way to transfer 1000 lb of plastic pellets from a shipping container to a hopper that's 50 ft above the floor. Sara believes this work can be done with a vacuum system. She knows that the potential energy the pellets gain while being moved up to the hopper can be used to determine how much work the vacuum system must do.

Find:

a. The potential energy of the pellets in the storage hopper.

b. The work that the vacuum system must do.

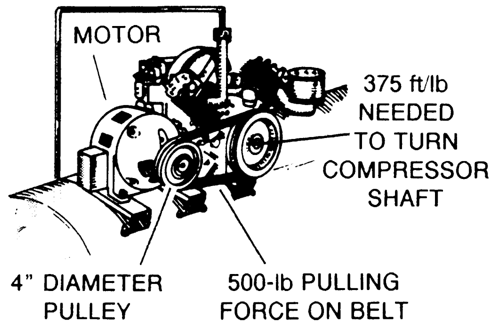
Solution:

Don't forget to compare the units obtained in your solutions to those listed in Table 1.

Student Challenge

The following problems review some concepts you have already learned. You may find it useful to refer to the equations given in Table 5-5 at the end of the summary for Unit 5.

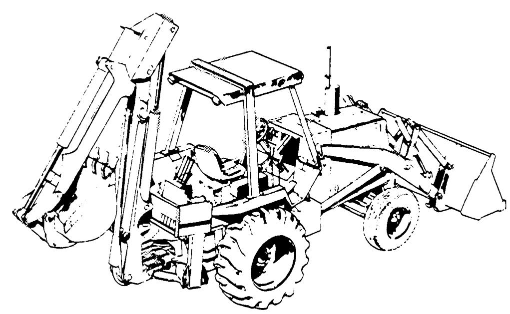
Problem 6: Given: You and another technician have installed a new electric motor on an air compressor. The motor is equipped with a 4-inch drive belt pulley. With that pulley, the motor produces 500 pounds of pulling force on the belt. The motor turns the belt in a counterclockwise direction. A torque of 375 foot·pounds is required to turn the drive shaft of the air compressor.



Find: The diameter of the pulley that is attached to the shaft of the air compressor. ***Hint:*** Remember that the diameter is 2 times the radius. The radius is the lever arm of the pulley.

Solution:

Problem 7: Given: The drawing at the right shows a hydraulically operated machine that's used to dig holes. The force required for either bucket to dig into the ground is 12,000 lb. The maximum pressure that can be developed in the hydraulic system is 1800 lb/in^2 (psi).



Find: Will a 3-in.-diameter hydraulic cylinder develop the needed digging force?

Solution: