Supplemental Experiment 1
Measuring Vector Forces

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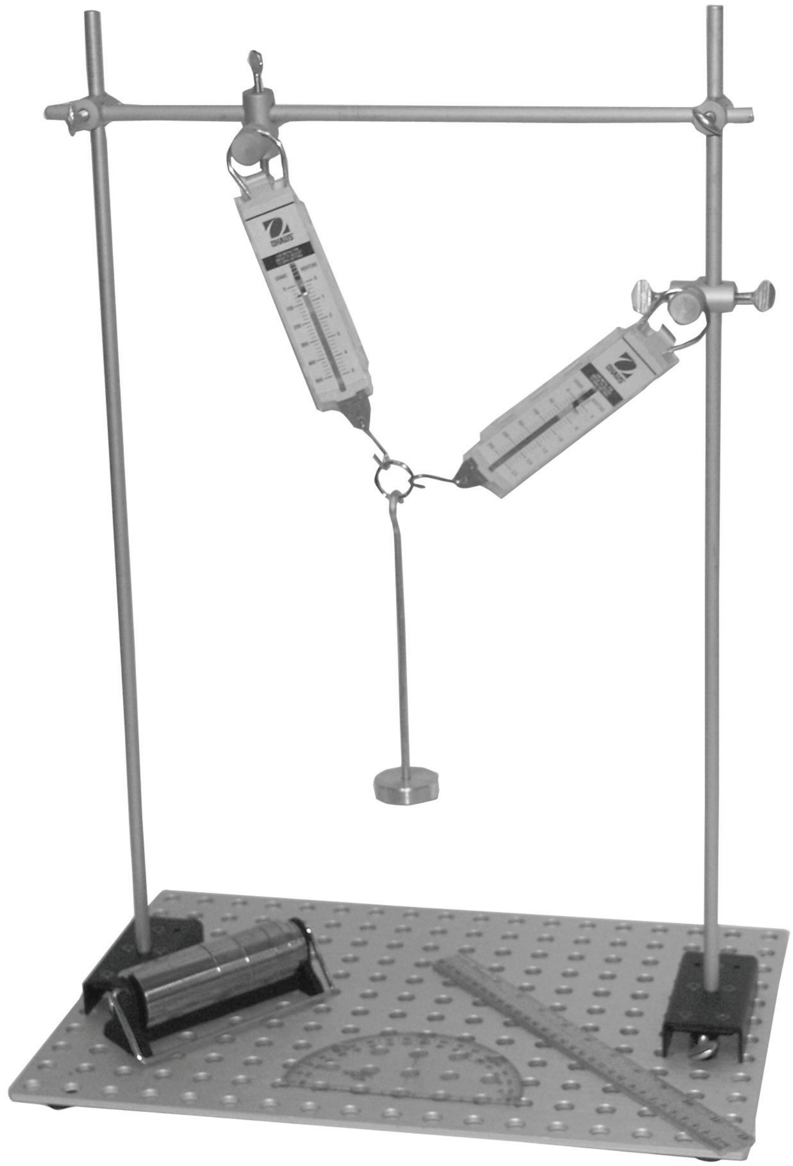

Figure 1
Setup for Supplemental Experiment 1

Supplemental Experiment 1

Measuring Vector Forces

Experiment Objectives

- Set up a system to measure forces using spring scales.
- Measure the resultant force produced by adding two forces.
- Graphically represent the resultant force by adding two or more force vectors.

Laboratory Proficiencies

- Use a spring scale to measure force in Newtons.
- Use a protractor to measure angles to the nearest degree.

Discussion

A force is the push or pull exerted by one body on another. Sometimes two or more forces act on a body at the same time. The result of this action is called the resultant force. If the resultant force is zero, then the body experiences no net force. When this happens, the body is said to be in **equilibrium**.

Vectors are used to graphically represent forces. Their angle specifies the direction of the force, and their length specifies the magnitude of the force. Vectors can be graphically combined to determine the resultant force when two or more forces are acting on a body.

In this experiment, you will place a system of forces in equilibrium. Then you will measure the forces and graphically add them to find the resultant. The resultant of forces in equilibrium should be zero. If it is not, then there must be an unmeasured force in the system that is not being taken into account. This unmeasured force is often due to friction.

Equilibrium A state of balance between opposing forces.
Vectors A quantity that has magnitude and direction. Commonly represented by a directed line whose length equals magnitude and angle equals direction.

Equipment and Materials Required

- Metal O-Ring, 1"
- Protractor
- Ruler, 30 cm
- Small Slotted Weight Set
- Small Weight Hanger
- Spring Scale, 2.5 Newton
- Spring Scale, 5 Newton
- Support Stand Set (following parts)
  - — Long Crossbar
  - — Mechanical Breadboard
  - — Rod Connectors, 4
  - — Scale Hangers, 2
  - — Support Rods with Base, 2

Procedure

Lab Setup

The lab setup is shown in Figure 1 at the beginning of the experiment. Refer to this figure and the detailed figures that follow when assembling the equipment.

❑ 1. Use the mechanical breadboard, support rods, long crossbar, rod connectors, and scale hangers to assemble the support stand. Details of the assembly are shown in Figure 2.

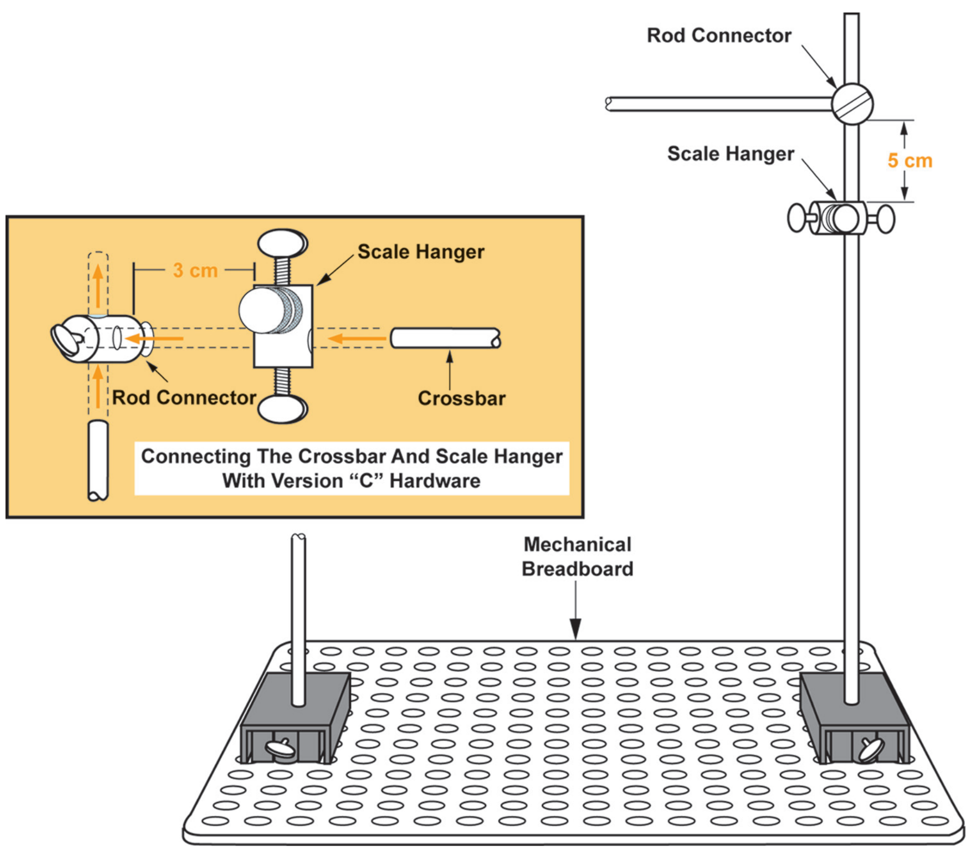

Figure 2
Assembling the support stand

❑ 2. Mount the two scale hangers. Position one on the crossbar, 3 cm from the left rod connector. Position the other hanger on the right support rod, 5 cm below the crossbar. See Figure 2.

❑ 3. Hang the 5 Newton scale from the scale hanger on the crossbar. Hang the 2.5 Newton scale from the scale hanger on the support rod. Fully extend the tongue of the 5 Newton scale. Use a pencil to draw a line down the middle of the tongue. Label the front of this scale A. Repeat for the 2.5 Newton scale and label it scale B. See Figure 3.

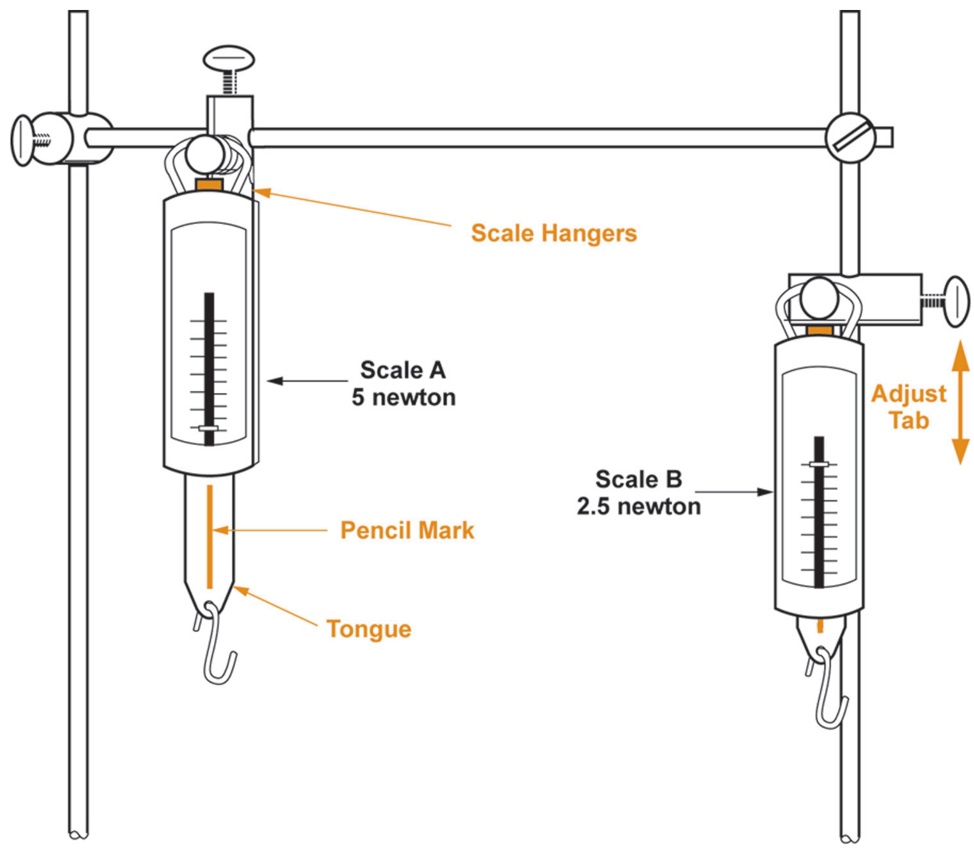

Figure 3
Marking and zeroing the spring scales

❑ 4. Adjust both scales by sliding the movable indicator plate up or down. The scales should read zero when hanging vertically. See Figure 3.

❑ 5. Attach the S-hook from each scale to the 1" O-ring. Then suspend the 50-gram weight hanger from the ring. See Figure 4.

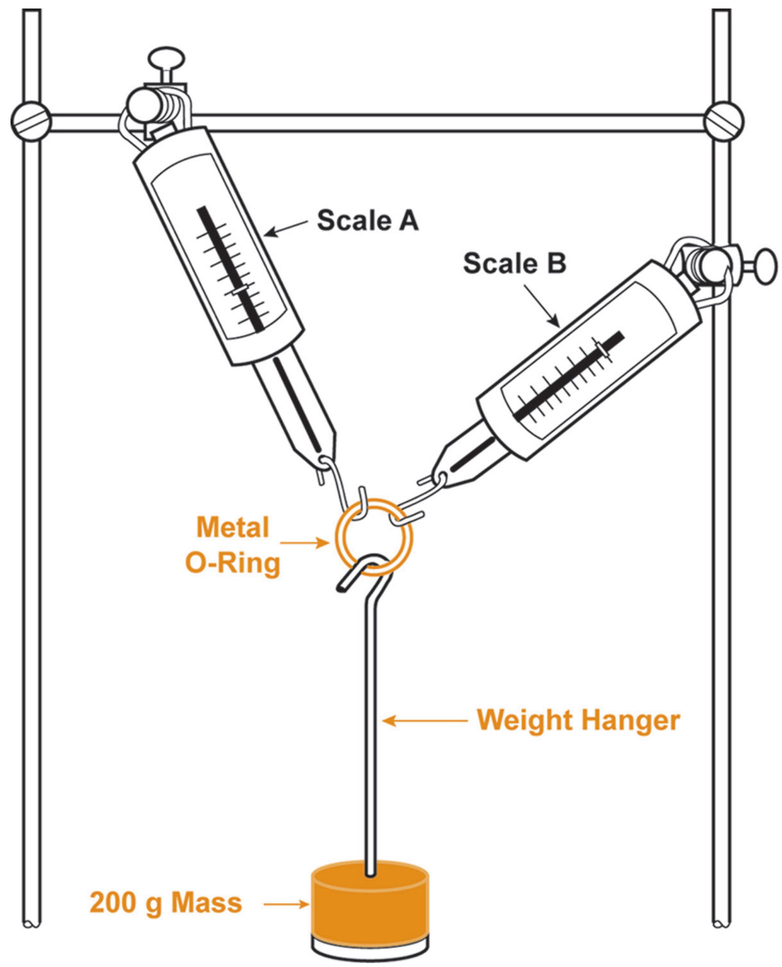

Figure 4
Weight hanger and scales setup

❑ 6. Add 200 grams of mass to the weight hanger for a total hanging mass of 250 grams.

Observations and Data Collection

The system you have assembled is in equilibrium since no motion is observed. Three forces are acting on the ring: F_A (scale A); F_B (scale B); and F_W (weight hanger.) Use the following procedure to record the forces under Observation 1, in Data Table 1, of your Student Journal.

❑ 1. Convert the mass of the hanging weights from grams to kilograms.

$$1\ \text{Gram} = \frac{1}{1000}\ \text{Kilogram}$$

Enter the value in the Student Journal.

❑ 2. Use the following conversion equation to convert the mass of the hanging weights to units of force (weight) in Newtons.

$$F\ (\text{Newtons}) = \text{Mass (Kilograms)} \times \frac{9.8\ \text{Newtons}}{\text{Kilograms}}$$

Record this as the value of Force F_W in Data Table 1.

❑ 3. Enter the scale readings in Data Table 1 as F_A (scale A) and F_B (scale B).

❑ 4. Use the protractor to measure the angle of each force with respect to the vertical axis. Position the protractor so that the center mark is aligned with the center of the O-ring. Align the baseline of the protractor along the shaft of the weight hanger. See Figure 5.

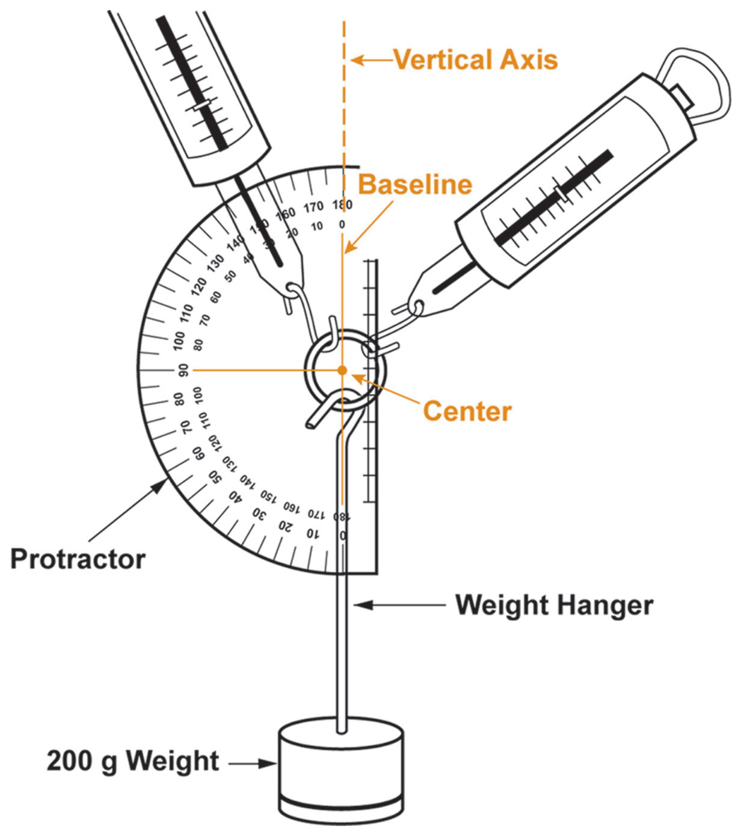

Figure 5
Positioning the protractor

❑ 5. Refer to Figure 6 for details on how to read the angles. Use a protractor to read angle *A*. Record all angles that are to the right of the vertical axis as positive angles. Record all angles that are to the left of the vertical axis as negative angles. This will produce a negative reading for angle *A*. Move the protractor to the other side and read angle *B*. Record these values in Data Table 1. Since the weight is pulling straight down on the weight hanger, angle *C* is 180°.

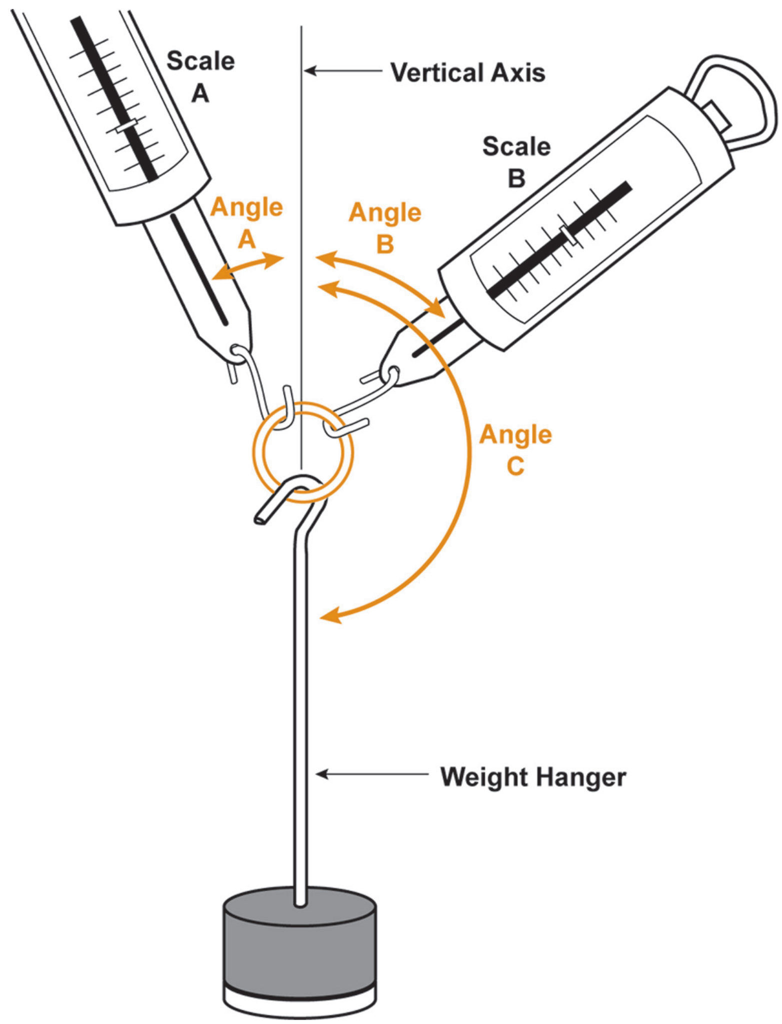

Figure 6
Force angles

Graphing the Data

You will now graphically add the forces to see if the resultant force equals zero.

❑ 1. Place the protractor center over the graph origin under Graphing the Data, Data Graph 1. Align the protractor baseline with the vertical axis of the graph. See Figure 7.

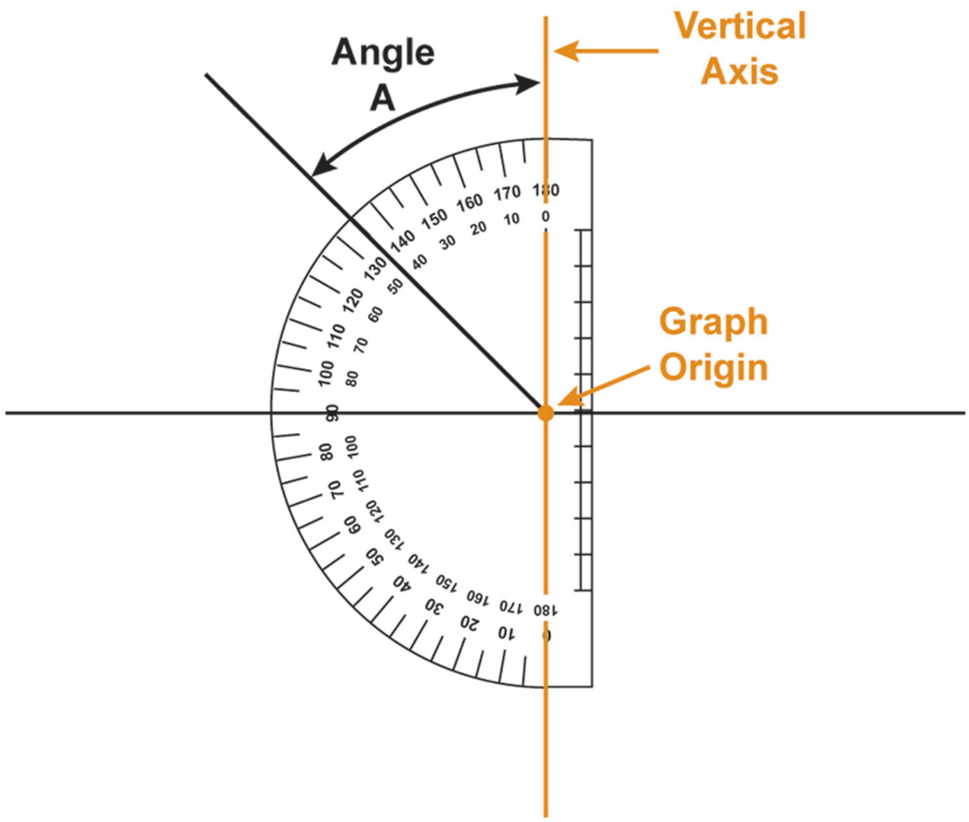

Figure 7
Plotting angle *A*

❑ 2. Locate and mark a point at the exact value of angle *A* on the graph. Since angle *A* is negative, the protractor must face to the left. See Figure 8. Use a ruler to draw a line from the origin through the point. The direction of this line represents the direction of F_A.

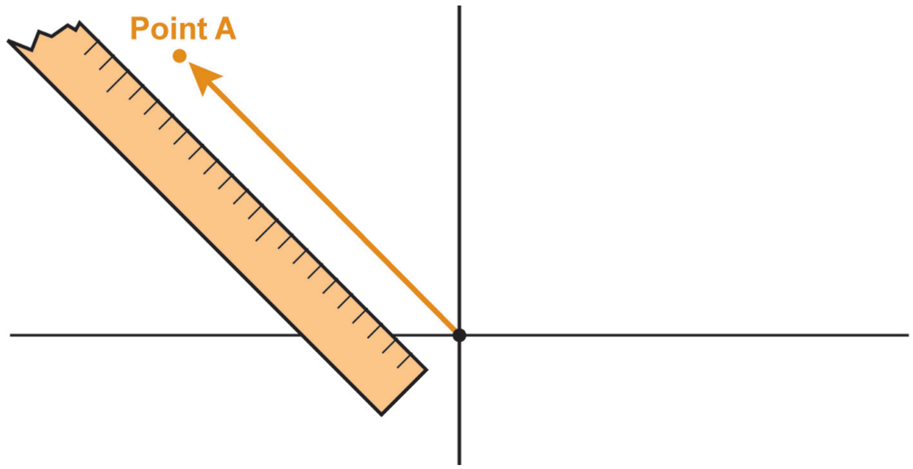

Figure 8
Graphing vector A

❑ 3. Use the following scale to find the length of this vector.

One Large Division = 1 Newton

Make a ruler from a piece of paper marked with the graph scale. Use this ruler to mark off the length of vector F_A on the line, measuring from the origin outward. Label the end point A. Draw an arrowhead at point A. Label this vector F_A. See Figure 8.

❑ 4. When adding vectors, the next vector has its origin at the arrowhead of the preceding vector. Draw a vertical axis through point A. Repeat steps 2 and 3, draw vector F_B, and label it vector F_B. See Figure 9.

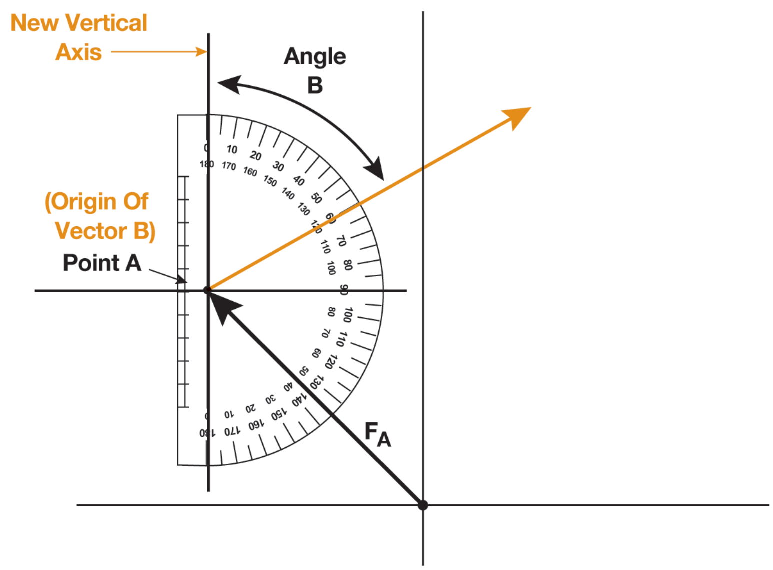

Figure 9
Locating vector B

❑ 5. Draw a vector (F_R) from the origin of vector F_A to point B. Place the arrowhead at point B. This vector is the resultant of vectors F_A and F_B. See Figure 10.

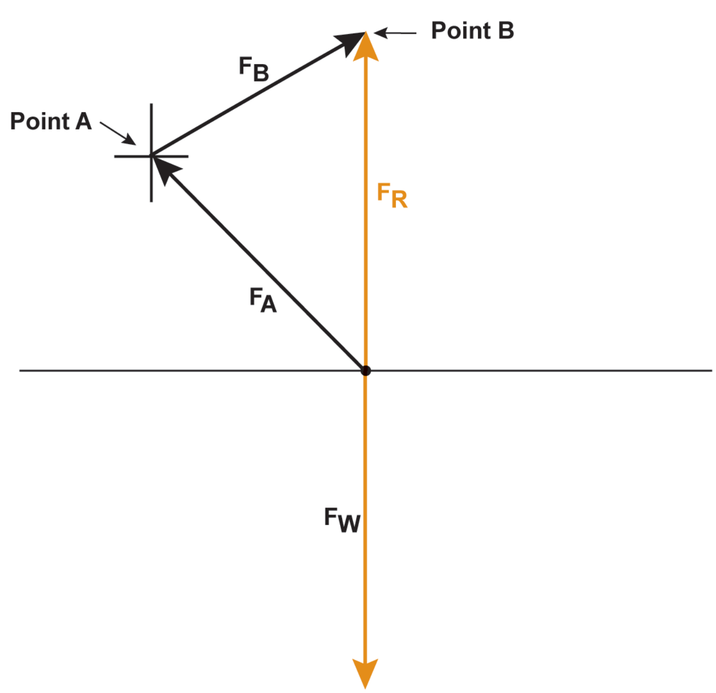

Figure 10
Locating vector C

❑ 6. Draw the vector F_W to represent the hanging weight. Start at the origin and direct this vector straight down as in Figure 11.

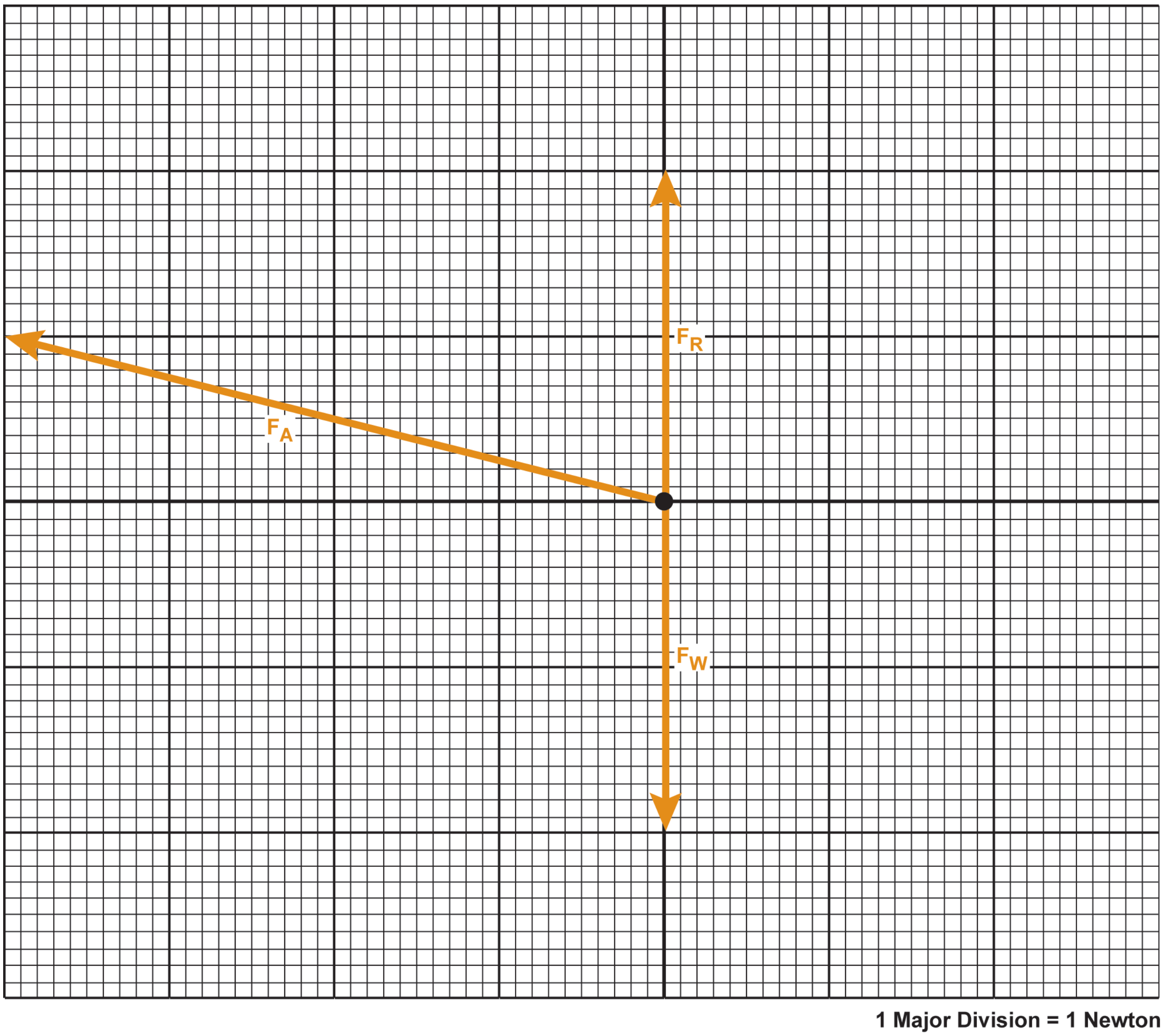

Figure 11
Graph for Question 3

❑ 7. Measure the length and angle of your resultant vector, F_R. Record the values under Graphing the Data. The length of vector F_R should be equal to the length of F_W. The angle of vector F_W is measured from the vertical axis. The angle of vector F_R should be zero degrees. However, due to friction and other factors, the length of vector F_R may not equal the length of vector F_W and neither may the angle of vector F_R be exactly zero degrees.

Questions and Interpretations

1. What should your resultant vector have been? Was it as expected?

2. Give as many reasons as you can for a non-zero resultant force.

3. Figure 11 shows vectors acting at a point, similar to the experiments you did. Vector F_R is the resultant of vectors F_A and F_B. Draw the missing vector, F_B, on the graph in your Student Journal.

4. If vector F_W in Figure 11 represents a force of 2N, what force does vector F_A represent? Record the value in your Student Journal

5. Looking at the vectors in Figure 12 below, the resultant vector can consist of the sum of vectors F_A and F_B or the sum of vectors F_C and F_D. Comparing the angles L_A and L_C, can we say that the vector (or the force that it represents) increases or decreases as the angle increases?

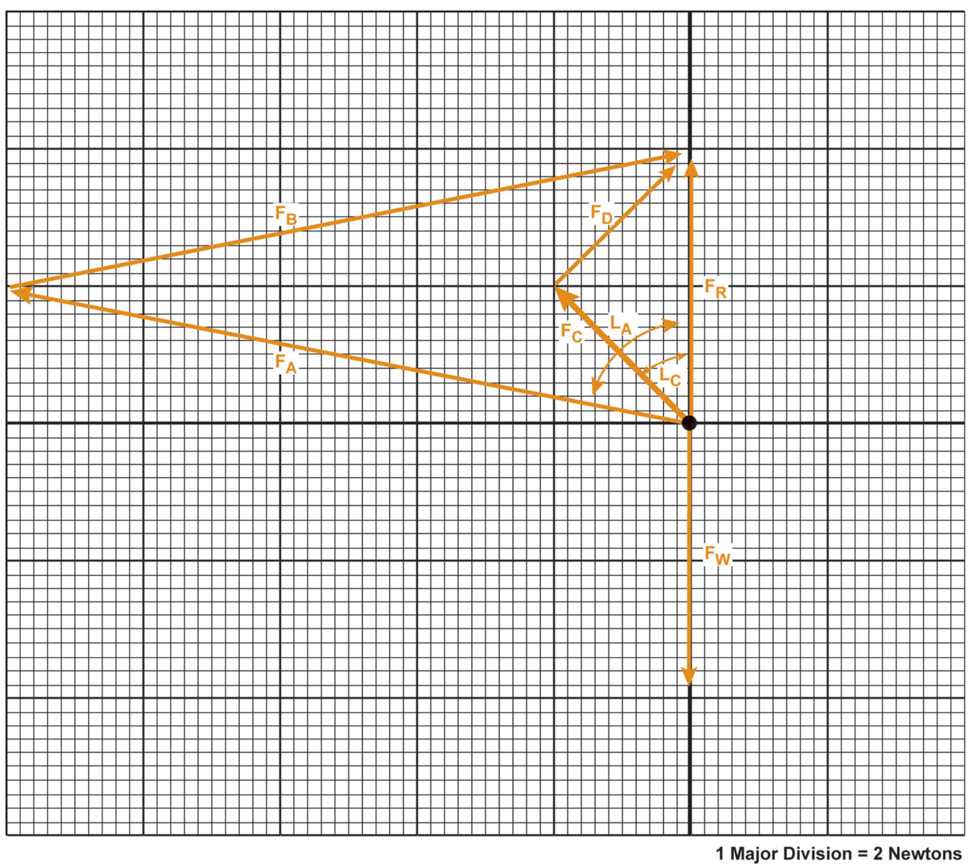

Figure 12
Graph for Question 5

Notes

Supplemental Experiment 2
Solving Vector Forces Using Trigonometric Functions

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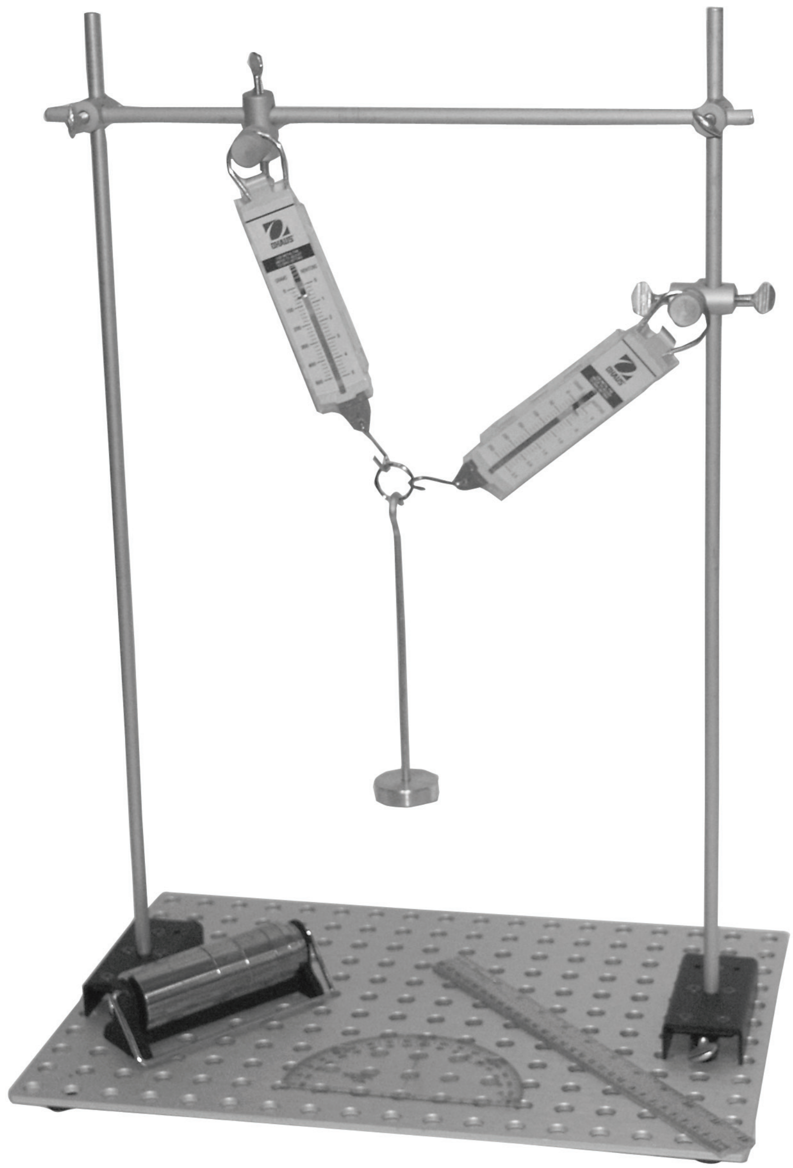
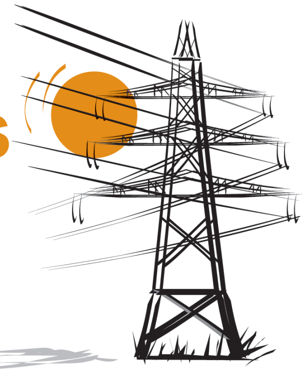


Figure 1
Setup for Supplemental Experiment 2

Supplemental
Experiment 2

Solving Vector Forces Using Trigonometric Functions



Experiment Objectives

- Calculate the theoretical values of distributed forces when a given mass is suspended from multiple points.
- Comprehend how forces work when the system is in equilibrium.
- Use mathematics, specifically, trigonometry, to calculate the vector forces.

Laboratory Proficiencies

- Use a protractor to measure degrees to the nearest degree.
- Use a scientific calculator or a trig table to find the trigonometric function values of a right triangle.

Discussion

In the Measuring Vector Forces experiment, we graphed data to calculate and find the resultant forces. In this experiment, we will be using trigonometry functions to show that we can arrive at the same result using mathematics only. Trigonometry functions are commonly used by architects, physicists, mathematicians, and engineers.

Three common functions in trigonometry are sine, cosine, and tangent. They can be found by using a scientific calculator or trig tables found in a textbook or online. These functions are used with a right triangle.

A right triangle is a triangle in which one angle is 90° and the three sides (base, height, and hypotenuse) satisfy the Pythagorean theorem ($Base^2 + Height^2 = Hypotenuse^2$). See Figure 1. For every triangle, inside angles total to 180°. Therefore, in a right triangle, the two non-right angles must total 90°. The hypotenuse is always the longest side of a right triangle and the hypotenuse is opposite the right angle.

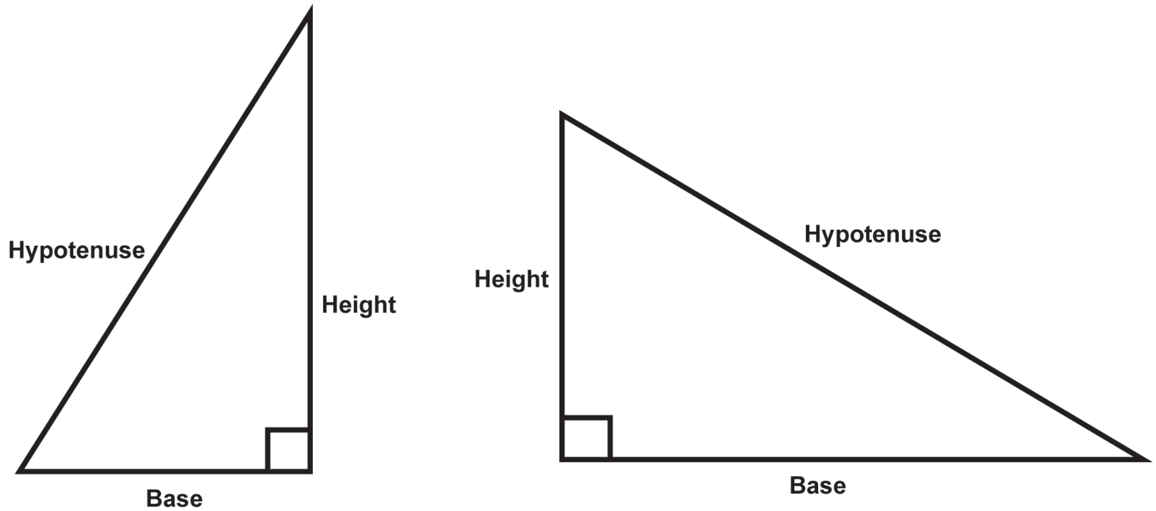


Figure 2
Examples of right triangles

Sine, Cosine, and Tangent Functions

The sin (sine) function finds the value of a side, hypotenuse, or angle as long as the triangle is a right triangle. The sine function is given by:

$$\sin(\text{angle}) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

The cos (cosine) function delivers a similar purpose as the sine function. However, the equation of cosine function would be:

$$\cos(\text{angle}) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Unlike sine and cosine, the tan (tangent) function does not require the hypotenuse. The equation for the tangent function looks like:

$$\tan(\text{angle}) = \frac{\text{opposite side}}{\text{adjacent side}}$$

Opposite and adjacent are relative to the chosen angle. Trig functions do not have units because they are in ratio. The phrase “Oscar Has A Hat On Always” can be used to memorize these three trig functions easily. Figure 3 shows the equations of the basic trigonometric functions along with an easy way to memorize them.

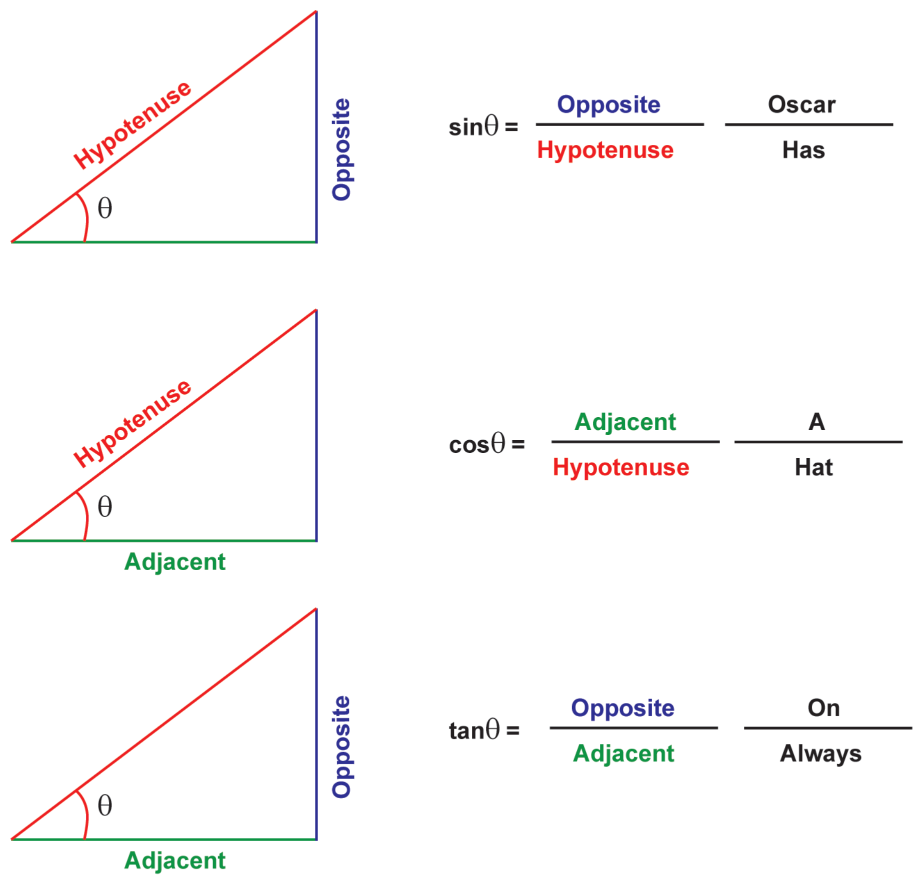
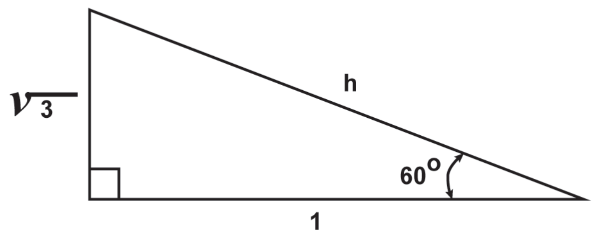


Figure 3
Easy memory trick for trig functions

We can use sine and cosine functions in this experiment to find theoretical values of vector forces. Figure 4 shows separate examples involving the Pythagorean theorem and the sine function giving the same result.



Using the Pythagorean theorem

$$\begin{aligned} h^2 &= (1)^2 + \left(\sqrt{3}\right)^2 \\ &= 1 + 3 \\ &= 4 \\ h &= 2 \end{aligned}$$

Using the sine function

$$\text{Sin(angle)} = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

$$\sin\left(60^\circ\right) = \frac{\sqrt{3}}{h}$$

$$h \sin\left(60^\circ\right) = \sqrt{3}$$

$$\begin{aligned} h &= \frac{\sqrt{3}}{\sin\left(60^\circ\right)} \\ &= \frac{1.732}{0.866} \\ &= 2 \end{aligned}$$

Figure 4
Pythagorean theorem *v.* sin function

Equipment and Materials Required

- Calculator (supports trig functions or trig tables)
- Metal O-Ring, 1”
- Protractor
- Ruler, 30 cm
- Small Slotted Weight Set
- Spring Scale, 2.5 Newton
- Spring Scale, 5 Newton
- Support Stand Set (following parts)
  - — Long Crossbar
  - — Mechanical Breadboard
  - — Rod Connectors, 4
  - — Scale Hangers, 2
  - — Support Rods with Base, 2

Procedure

Lab Setup

The lab setup is shown in Figure 1 at the beginning of the experiment. Refer to this figure and the detailed figures that follow when assembling the equipment.

❑ 1. Mount the two scale hangers. Position one of the scale hangers on the crossbar, 10 cm from the left rod connector. Position the other hanger on the right support rod, 9 cm below the crossbar.

❑ 2. Hang the 5 Newton scale from the scale hanger on the crossbar. Hang the 2.5 Newton scale from the scale hanger on the support rod. Fully extend the tongue of the 5 Newton scale. Use a pencil to draw a line down the middle of the tongue. Repeat for the 2.5 Newton scale.

❑ 3. The spring scales should read zero when hanging vertically. Adjust both scales by sliding the movable indicator plate up or down.

❑ 4. Attach the S-hook from each scale to the 1” O-Ring. Then suspend the 50-gram weight hanger from the ring.

❑ 5. Add 350 grams of mass to the weight hanger for a total hanging mass of 400 grams.

Observations and Data Collection

Enter your answers in the Student Journal.

❑ 1. Read the scales and center the forces in Data Table 1.

❑ 2. Use the protractor to measure the angle of each force with respect to the vertical axis. Position the protractor so that the center mark is aligned with the center of the O-ring. Align the baseline of the protractor along the shaft of the weight hanger.

Label the angle between the vertical axis and the 5 Newton scale as angle A. Label the angle between the vertical axis and the 2.5 Newton scale as angle B. See Figure 5 on the next page. Enter the angles in Data Table 1.

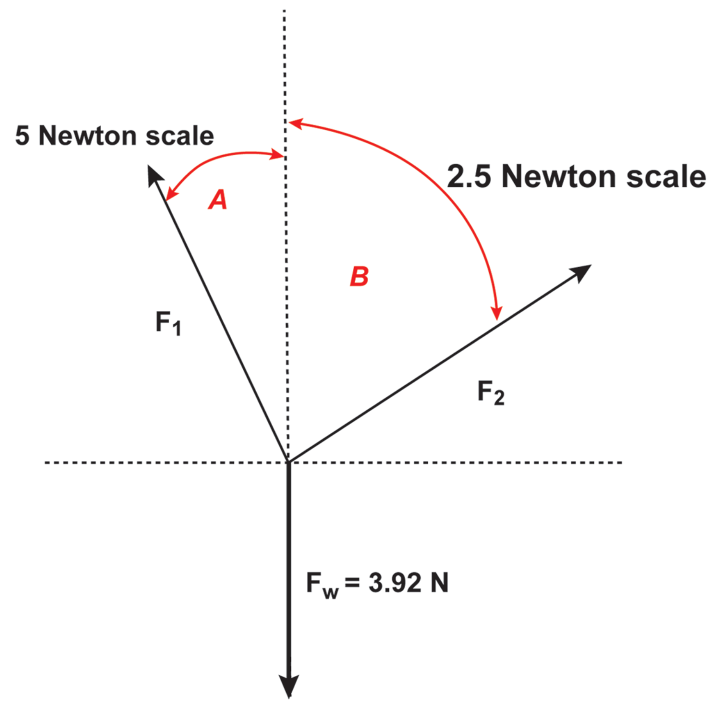


Figure 5
Identifying the measured angles

❑ 3. Subtract angle *A* from 90° and label this value as θ "theta." Subtract angle *B* from 90° and label this value as ϕ "phi." See Figure 6. In Figures 6 and 7 vector F_W has been left out in order to simplify the drawing. Enter the angle sin Data Table 1.

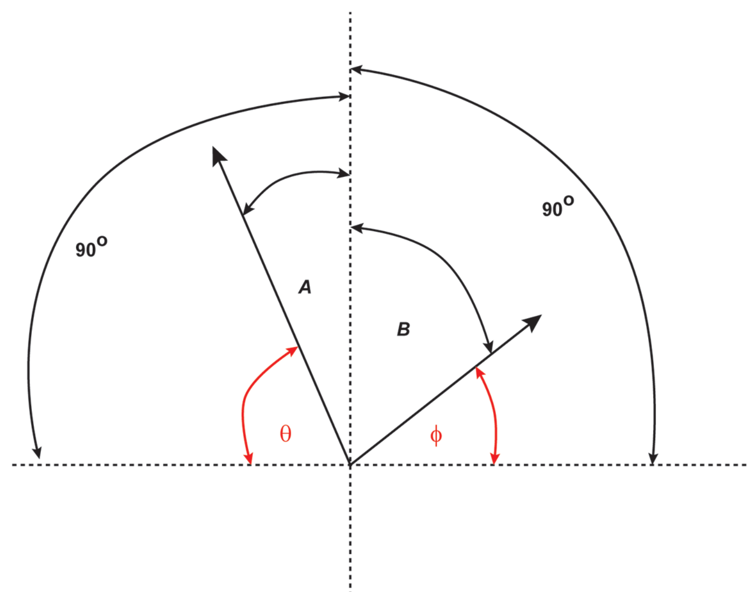


Figure 6
Identifying angles θ and ϕ

Horizontal Balance of Forces

❑ 1. Figure 7 shows the horizontal forces that are derived from the two force vectors.

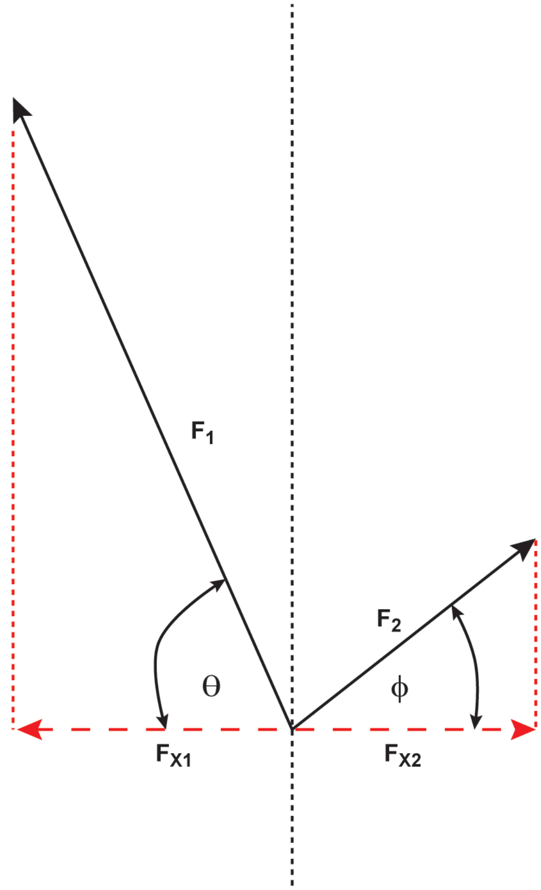


Figure 7
The horizontal forces that are derived from the force vectors

❑ 2. Although in the experiment the horizontal forces cannot be measured directly, they can be calculated mathematically. Enter the values in Data Table 2.

$$\cos(\theta \text{ or } \phi) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{F_{X1}}{F_1} \qquad \cos\phi = \frac{F_{X2}}{F_2}$$

$$F_{X1} = F_1\cos\theta \qquad F_{X2} = F_2\cos\phi$$

❑ 3. Since the system is in equilibrium, meaning there is no horizontal movement, the horizontal forces are equal but opposite in direction. Are the forces equal?

Vertical Balance of Forces

❑ 1. The vertical component of F_1 is shown in Figure 8.The vertical component of F_1 can be written as:

$$\mathbf{F_{Y1} = F_1 \sin\theta}$$

Enter F_{Y1} in Data Table 3.

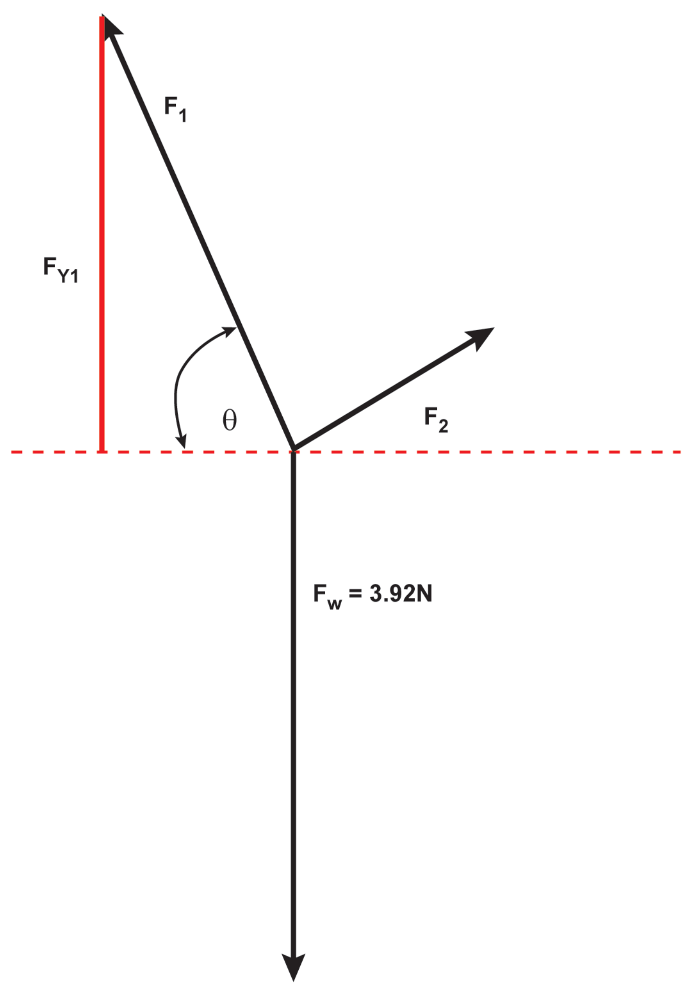


Figure 8
The vertical component of F_1

- ❑ 2. Figure 9 shows the vertical component of F_2. The vertical component of F_2 can be written as:

$$\mathbf{F_{Y2} = F_2 \sin\phi}$$

Enter F_{Y2} in Data Table 3.

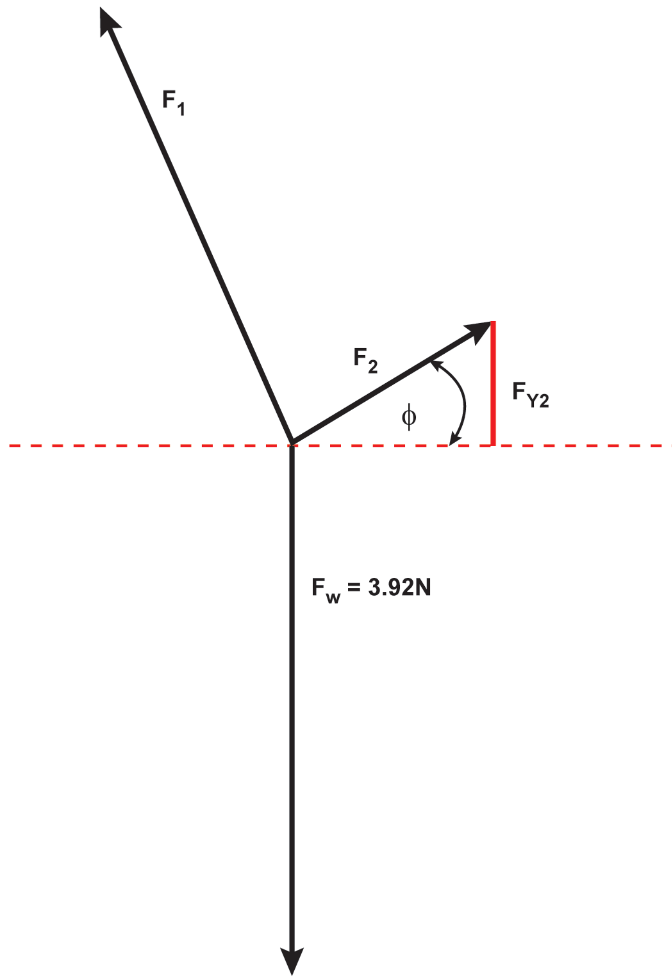


Figure 9
The vertical component of F_2.

- ❑ 3. Since the vertical components are acting in the same direction, they can be added as shown in Figure 10. The summation can be written as:

$$\mathbf{F_R = F_{Y1} + F_{Y2}}$$

Enter the value in Data Table 4.

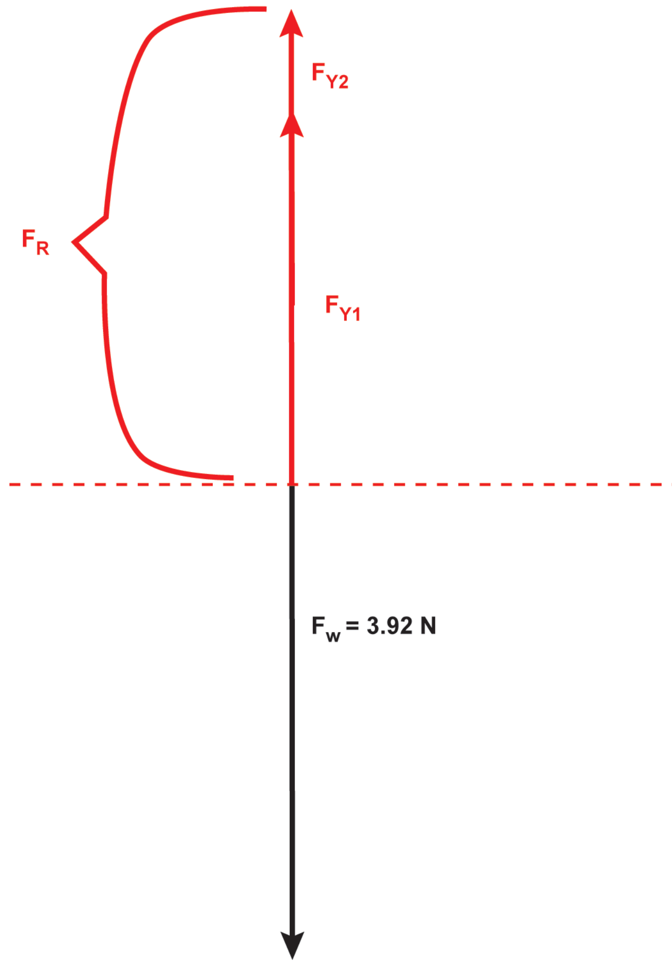


Figure 10
The addition of vectors

- ❑ 4. Convert the mass of the hanging mass to kilograms. Enter the value in Data Table 4.

$$1 \text{ g} = \frac{1 \text{ kg}}{1000 \text{ g}}$$

- ❑ 5. Use the following equation to convert the mass of the hanging weight to units of force in Newtons.

$$F_w = \text{Mass}(\text{kg}) \times \frac{9.81 \text{ N}}{\text{kg}}$$

Enter your answer in Data Table 4.

- ❑ 6. Is F_w equal to F_R? Enter your answer in the Student Journal.

Questions and Interpretations

1. Show mathematically how F_{Y1} is the same as $F_1 \sin\theta$, and F_{Y2} is the same as $F_2 \sin\phi$. Refer to the equations in step 6.

2. If θ "theta" was to equal 47° and ϕ "phi" was to equal 12°, what would the new F_1 and F_2 be? Assume $F_R = 3.92$ N.

3. If we changed the height of 2.5 Newton scale from 9 cm below the crossbar to 3 cm below the crossbar, how would that affect the vector force of the 5 Newton scale (F_1)? Will it increase, decrease, or stay the same? Refer to Figure 5.

4. Where should you position the 2.5 Newton scale in order for the 5 Newton scale to support the whole weight (vertically in line with the weight)?

Notes

Supplemental Experiment 3
Projectile Motion

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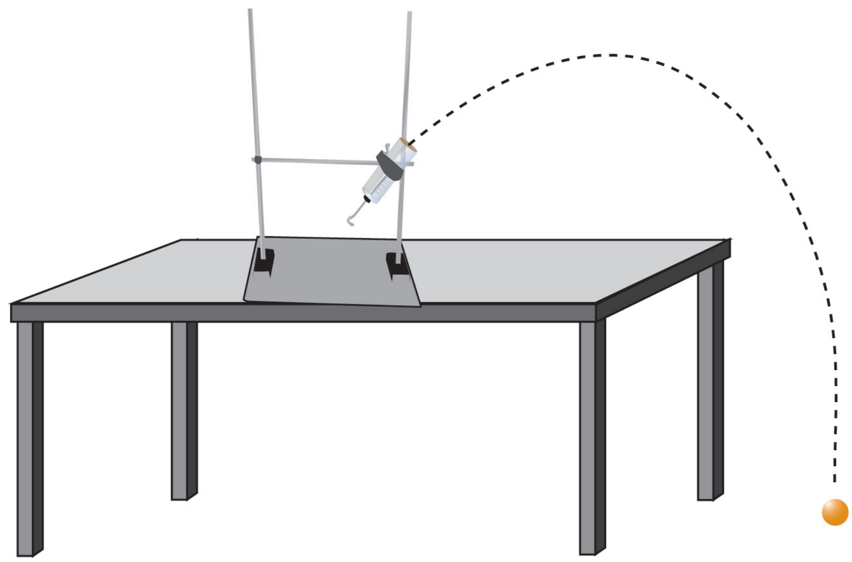


Figure 1
Setup for Supplemental Experiment 3

Supplemental
Experiment 3

Projectile Motion

Experiment Objectives

- Explain how gravity works in projectile motion.
- Apply the concept of vector components to solve projectile motion problems.
- Understand how air resistance affects projectile motion.

Laboratory Proficiencies

- Set up and perform an experiment that will demonstrate projectile motion.
- Measure the weight of the projectile using a triple beam balance.

Discussion

A projectile is any object that is projected by something and continues in motion by its own **inertia**. Projectiles follow curved paths that may seem complicated at first, but are surprisingly simple when the horizontal and vertical components of motion are looked at separately.

The horizontal component of projectile motion is the same as a ball rolling at a constant rate horizontally on a table without friction. The ball covers equal distances in equal intervals of time by its own inertia. It rolls without accelerating because there are no components of force acting in its direction of motion.

Inertia A property of an object by which it resists change in motion.

The vertical component of projectile motion is the applied force acting as it goes up and is the same as a free falling object. Like a free falling object, projectile motion accelerates downward towards the direction of earth's gravity. Gravity increases the speed in the vertical direction which causes greater distances to be covered in equal time intervals. When the gravity vector is in the same direction as the vertical component of the velocity, the velocity increases; however, if the gravity vector is against the velocity vector, then the velocity decreases. The horizontal and vertical components of projectile motion are completely independent of each other. The curved motion is produced by the combination of the two components.

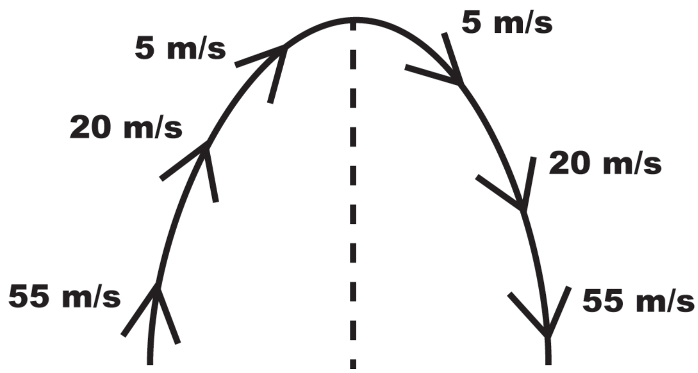
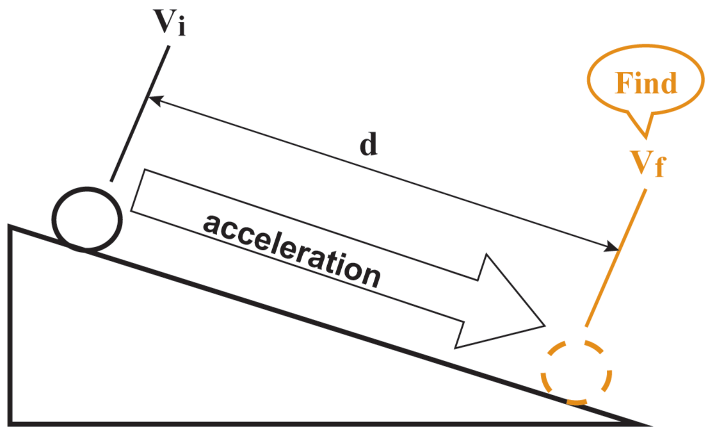


Figure 2
Without air resistance the speed gained while going down equals the speed lost while going up.

The horizontal velocity vector is consistent throughout the motion, only the vertical component changes. At the maximum height, the vertical component reduces to zero. For ideal projectile motions, the path of an object draws a perfect parabola, but in the presence of air resistance, the projectile motion falls short of a parabola. Figure 2 shows the curved path with no air resistance. The velocity lost while going up is equivalent to velocity gained while coming down. This also means that the time up equals the time down.

Basic Formulas of Kinematics

The basic formulas of **kinematics** are given below. If you know the initial velocity, acceleration, and distance, use the formula in Figure 2 to find the final velocity.



$$v_f^2 = v_i^2 + 2ad$$

Where

v_f = Final velocity in meters/second

v_i = Initial velocity in meters/second

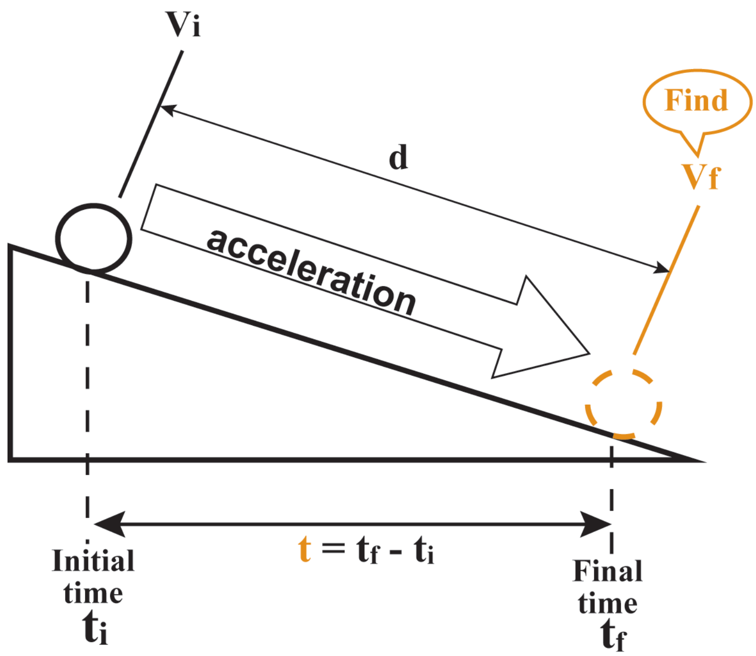
a = Acceleration in meters/second^2

d = Displacement in meters

Figure 3
Finding the final velocity, given the initial velocity, acceleration, and displacement

Kinematics The branch of mechanics that describes the motion of objects without considering the forces that create it.

See Figure 4 to find the final velocity if the initial velocity, acceleration, and time are known.



$$v_f = v_i + at$$

Where

v_f = Final velocity in meters/second

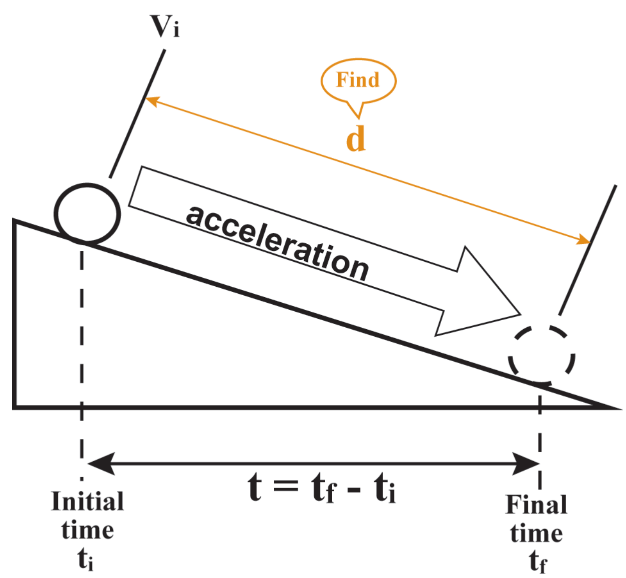
v_i = Initial velocity in meters/second

a = Acceleration in meters/second^2

t = Time in seconds

Figure 4
Finding the final velocity, given the initial velocity, acceleration, and the time in which the acceleration is acting

See Figure 5 to the find the displacement, given the initial velocity, acceleration, and the time of the displacement.



$$d = v_i\, t + 0.5\, at^2$$

Where

d = Displacement in meters

v_i = Initial velocity in meters/second

a = Acceleration in meters/second^2

t = Time in seconds

Figure 5
Finding the displacement, given the initial velocity, acceleration, and the time the acceleration is acting

Equipment and Materials Required

- Balls (2)
- Long Crossbar
- Mechanical Breadboard
- Meter Stick
- Projectile Apparatus
- Protractor
- Rod Connectors (3)
- Stopwatch
- Supports Rods (2)
- Triple Beam Balance

Procedure
Part 1

The students should work in groups of three.

- ❑ 1. Have one person be in charge of rolling a ball off a table. See Figure 6.

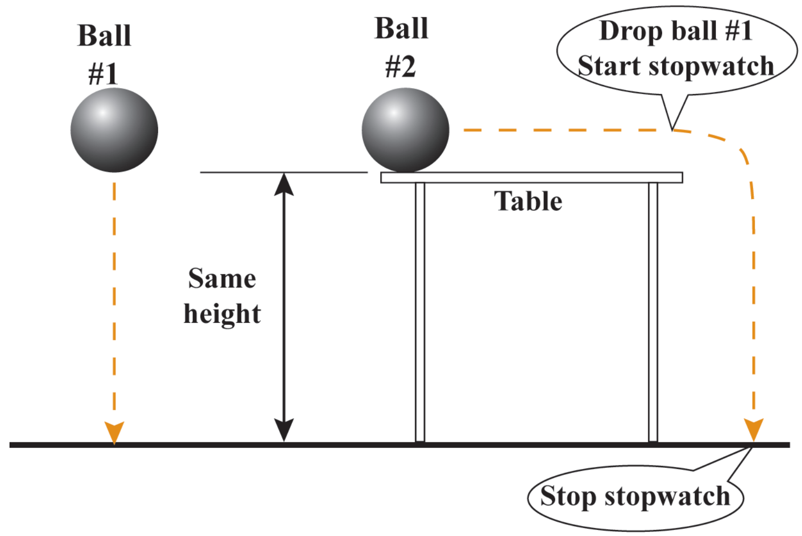


Figure 6
A ball being dropped from a hand verses a ball rolling off a table at the same height

- ❑ 2. Have a second person be in charge of dropping the other ball from the same height as the table. This person should drop the ball as ball #2 reaches the edge of the table.

- ❑ 3. The last person will be responsible for starting and stopping the stopwatch. This person should start the stopwatch as both balls start to fall to the ground and stop it once they both hit the ground.

- ❑ 4. Roll the ball at different speeds and note the time for each try. Record these answers in Data Table 1 of your Student Journal.

- ❑ 5. Did the speed of the rolling ball matter? If so, how? Enter you answer in your Student Journal.

Procedure
Part 2

❑ 1. Have one person throw a ball straight up and catch the ball where it was released. Refer to Figure 7. Try this a few times before proceeding to the next step.

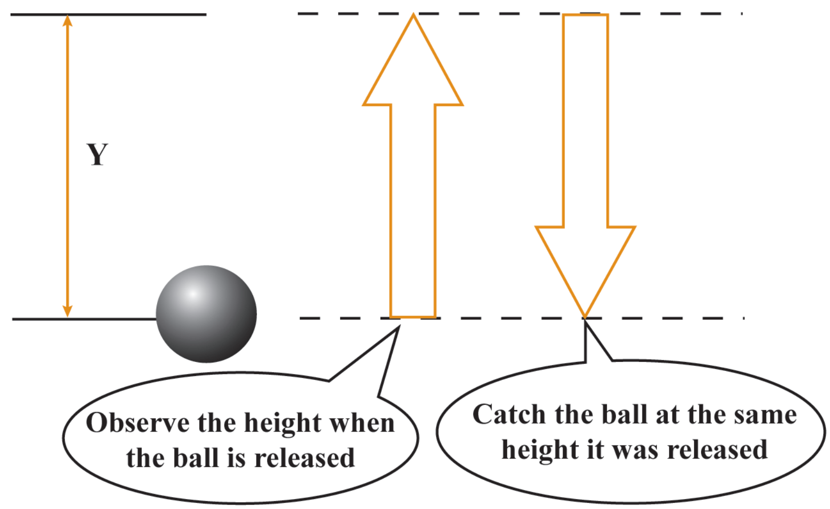


Figure 7
Throwing a ball upwards and catching it at the same spot

❑ 2. Have a second person start the stopwatch as the ball is released, and stop the stopwatch as soon as the ball is caught. See Figure 8. Enter the time in Data Table 2.

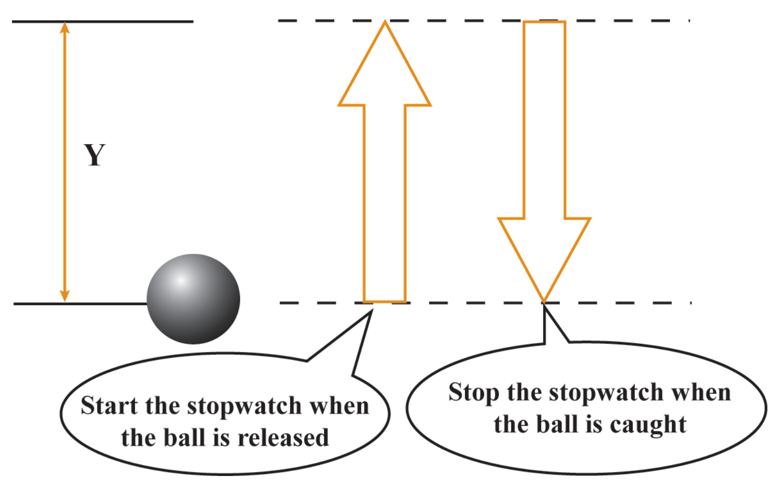


Figure 8
Starting and stopping the stopwatch

❑ 3. To find the height of the ball this problem needs to be divided into two parts. First, the ball is thrown up, and second the ball comes down. In the first or going up part we do not know what the initial velocity is; however, the second or coming down part has more information. See Figure 9.

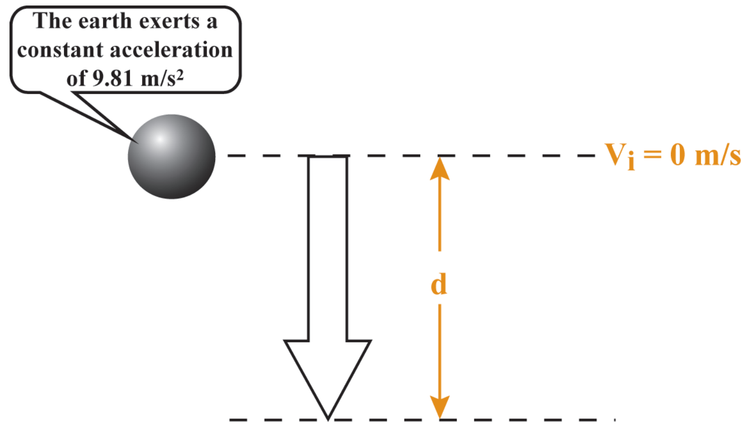


Figure 9
The known information about the ball toss

❑ 4. Of the formulas we have seen, the following formula uses initial velocity and acceleration to find the distance. Enter "d" in Data Table 2.

$$\mathbf{d = v_i\, t + 0.5\, at^2}$$

❑ 5. To find the final velocity we have a choice of two formulas:

$$\mathbf{v_f = v_i + at \quad \text{or} \quad v_f^2 = v_i^2 + 2ad}$$

Use one of these formulas to find the velocity just before the ball is caught. Enter your answer in Data Table 2.

What is the velocity of ball when it is released from the hand? Enter you answer in Data Table 2.

Procedure Part 3

- ❑ 1. Set up the lab equipment as shown in Figure 10. Position the black bases along the second column of holes as Figure 10 shows.

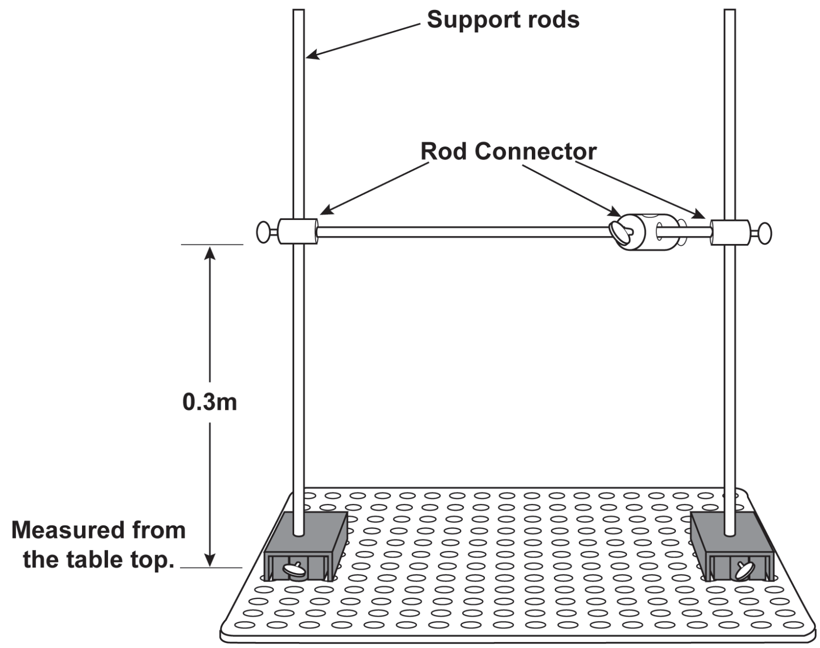


Figure 10
Setup for Part 3

- ❑ 2. Mount the rod connector 30 cm from the table, not the breadboard.
- ❑ 3. Insert the projectile apparatus into the middle rod connector.
- ❑ 4. Adjust the projectile apparatus for a 70° angle to the long crossbar. See Figure 11.

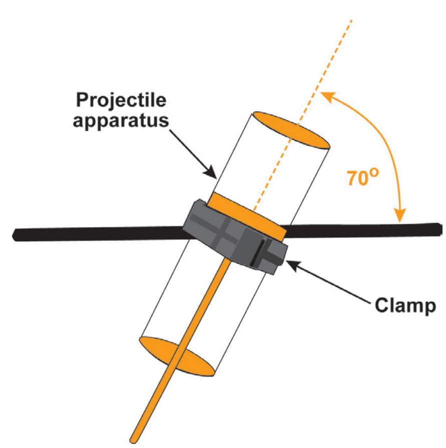


Figure 11
Setting the projection angle

❑ 5. Clamp the projectile apparatus at the 60 mm mark of the scale as shown in Figure 12.

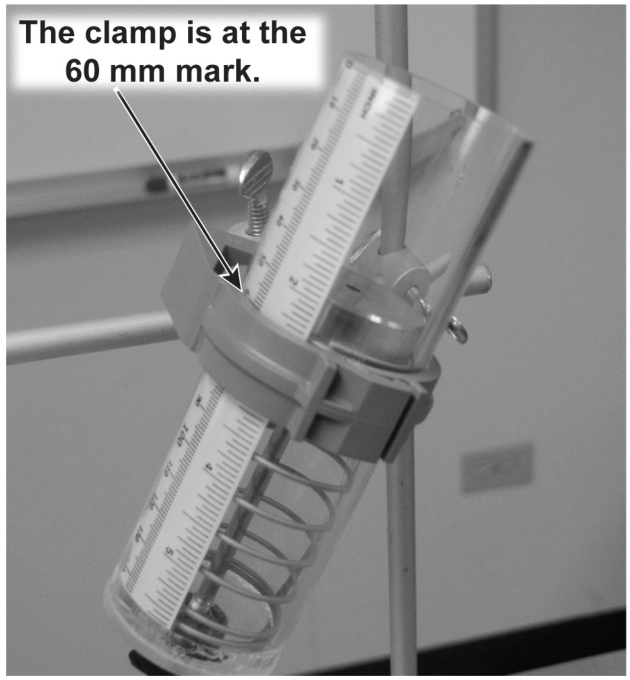


Figure 12
The projectile apparatus clamped at 60 mm

❑ 6. Adjust the rubber stopper until the top of the spring is positioned at 50 mm. See Figure 13.

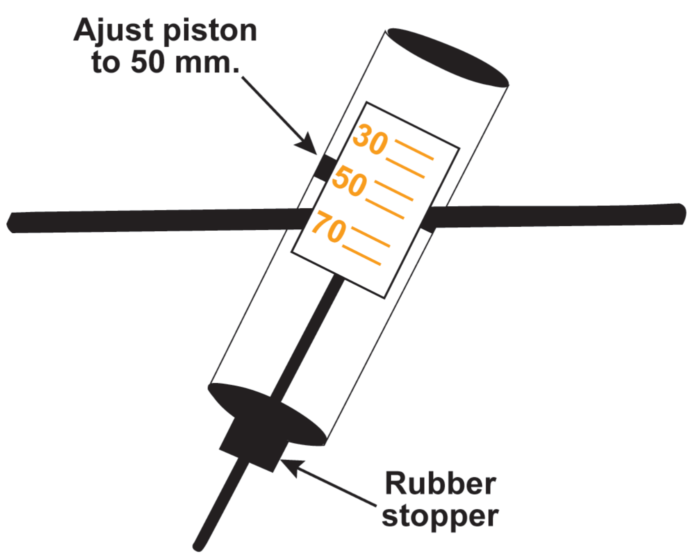


Figure 13
Adjusting the rubber stopper

❑ 7. Measure mass of the ball in grams using a triple beam balance. Record the mass in Data Table 3.

❑ 8. Measure the height of the table. Record the mass in Data Table 3.

❑ 9. Note the position of the piston. Record this value as the initial piston position ℓ_i in Data Table 3. (The value should be near 50.)

❑ 10. Hold the projectile apparatus with one hand and pull the piston all the way into the cylinder. Record the value as the final piston position (ℓ_f) in Data Table 3.

❑ 11. Orientate the projectile apparatus so the ball will land on the floor.

❑ 12. Assign each student to a task.

A. One student to insert the ball into the projectile apparatus.

B. One student to measure the time the ball is in flight and to estimate the maximum height the ball is above the table.

C. One student to observe the spot where the ball lands and to mark that spot on the floor. The horizontal displacement is measured as shown in Figure 14.

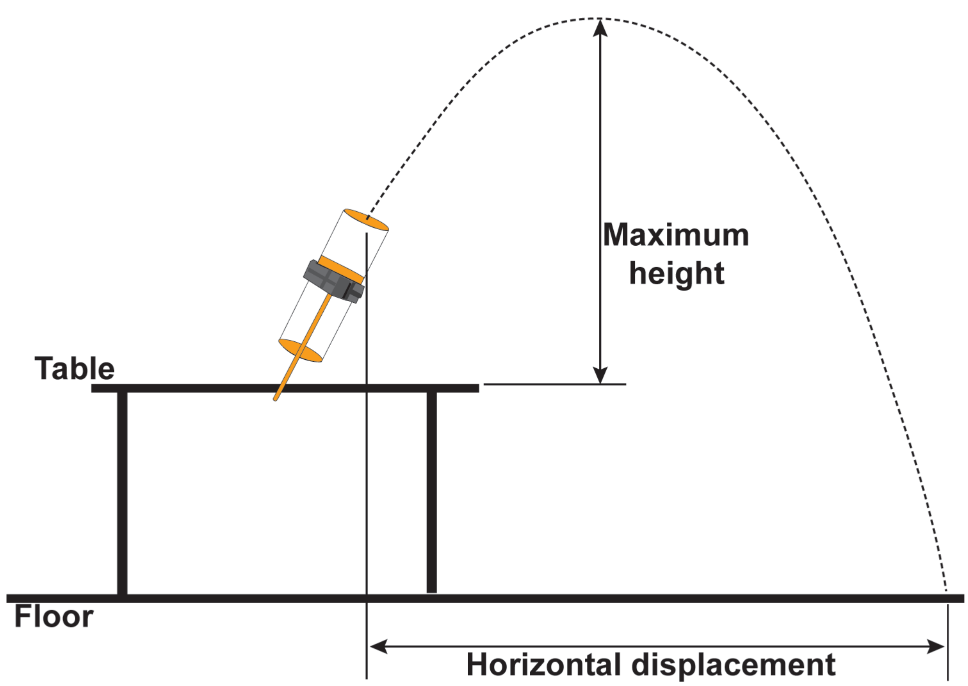


Figure 14
Measuring the horizontal displacement and estimating the maximum height

❑ 13. Place the ball in the projectile apparatus. Hold the projectile apparatus and pull the spring all the way back. With the other students ready for action, release the spring.

❑ 14. Enter the time of flight, the horizontal displacement, and the estimated height above the tablein Data Table 4 .

❑ 15. Repeat steps 12 through 13 two more times.

Observations and Calculations

Enter your answers in the Student Journal.

Finding the Ejection Velocity

❑ 1. From Data Table 4, find the average flight time, the average horizontal displacement, and the average estimated height above the table. Enter your answers in Data Table 4.

❑ 2. In the projectile motion experiment the potential energy of the stretched spring is converted into the kinetic energy given to the ball. This conversion can be stated as:

$$\mathbf{KE_{ball} = PE_{spring}}$$

❑ 3. The equation for the Potential Energy of the spring is:

$$\mathbf{PE_{spring} = {}^1/_2\, k(\Delta\ell)^2}$$

Where

k = spring constant

$\Delta\ell$ = Initial velocity in meters/second

❑ 4. The kinetic energy developed by the spring is imparted not only to the ball but also to the piston and shaft.

$$\mathbf{m_{piston\ \&\ shaft} = 56.9\ gm}$$

Enter the total mass in Data Table 5.

$$\mathbf{m_{total} = m_{ball} + 56.9\ gm}$$

❑ 5. Before we can solve the previous equation we need to convert the total mass of the moving mechanism from grams to kilograms. Enter your answer in Data Table 5.

$$\mathbf{1\ gram = {}^{1}/_{1000}\ kilogram}$$

❑ 6. Likewise we need to find the distance the piston moves. Enter your answer in Data Table 5.

$$\Delta\ell = \ell_{\mathbf{f}} - \ell_{\mathbf{i}}$$

❑ 7. Convert $\Delta\ell$ from millimeters to meters. Enter your answer in Data Table 5.

❑ 8. The value of the spring constant is:

$$\mathbf{k = 170\ N/m}$$

❑ 9. If you have a scientific calculator you can now solve this equation for v. To remind us that the mass is the total mass, the equation has been changed to reflect the total mass.

$$\mathbf{{}^{1}/_{2}\ m_{total}v^{2} = {}^{1}/_{2}\ k(\Delta\ell)^{2}}$$

If you do not have a scientific calculator you can use this form of the above equation:

$$v = \sqrt{\frac{k\left(\Delta\ell\right)^{2}}{m_{total}}}$$

Enter the ejection velocity in Data Table 5.

Determining the Final Velocity

❑ 1. To work with the projectile path we need to divide the path into sections that identify the important points. See Figure 15.

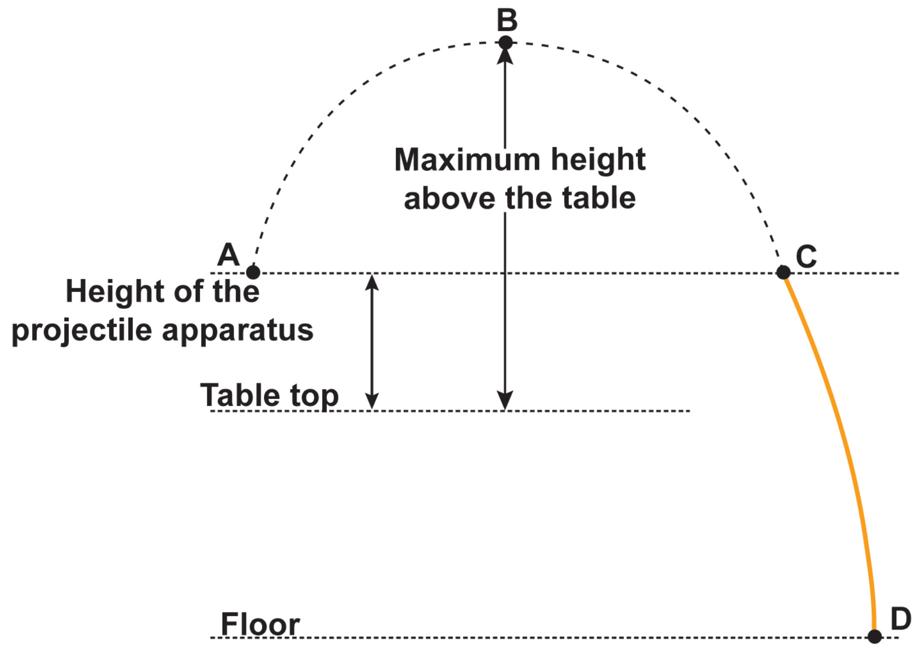


Figure 15
The important points of the projectile path

❑ 2. To find the final velocity of ball prior to hitting the floor at point D, we will work with the projectile path from points C to D. The formula that will gives us the final velocity from points C to D is:

$$v_f^2 = v_i^2 + 2ad$$

Before we can use this equation we need to rework the equation and prepare the data as outlined in the following steps.

❑ 3. At point C the projectile has gained all the velocity it had at point A if we neglect friction. Also the initial projectile velocity is the ejection velocity from the spring.

$$v_{i\ (\text{projectile velocity})} = v_{(\text{ejection velocity})}$$

Enter the initial projectile velocity in Data Table 6.

❑ 4. At point C the angle of the velocity will be the same as the angle at the projectile apparatus. Because of the angle of the projectile velocity at point C, the vertical component is the one to use in the equation. See Figure 16.

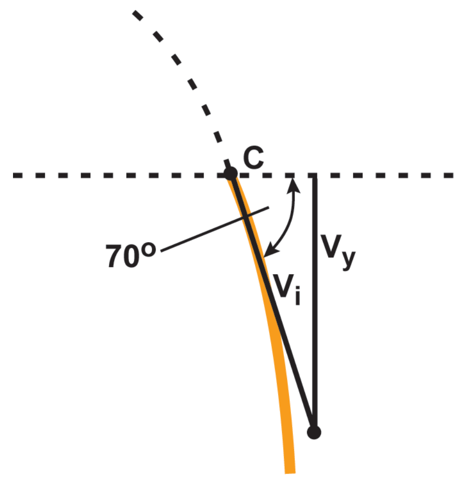


Figure 16
The angle of the velocity determines the vertical component at Point C

❑ 5. Applying trigonometry to point C, the vertical component of vi can be found as follows:

$$\frac{v_y}{v_i} = \sin(70°)$$

$$v_y = v_i \sin(70°)$$

Enter the value of v_Y in Data Table 6 in your Student Journal.

❑ 6. Convert the table height to meters. Enter the table height in Data Table 6.

❑ 7. Convert the height of the projectile apparatus above the table to meters. Enter in Data Table 6.

❑ 8. Find the height between points C and D. Enter it in DataTable 6.

- [ ] 9. To find the vertical component of the final velocity at point D, we can rewrite the equation in terms of the vertical components:

$$(v_{yfinal})^2 = v_y^2 + 2ay$$

Where

v_{yfinal} = vertical velocity at point D

y = the distance between Points C and D

v_i = v_y (the vertical component of v_i at point C)

a = 9.81 m/s2 (the acceleration due to gravity)

Enter the value of v_{yfinal} in Data Table 6.

- [ ] 10. To find the final projectile velocity at point D, we need to do vector addition of the vertical and horizontal components. Without friction the horizontal component remains the same through all the horizontal positions.

$$\frac{v_x}{v_i} = \cos(70°)$$
$$v_x = v_i \cos(70°)$$

Enter the value of v_x in Data Table 6.

- [ ] 11. Use the Pythagorean theorem to find V_f (right before the ball hits the ground). Record this value in Data Table 6.

$$v_f^2 = v_x^2 + v_{yfinal}^2$$
$$v_f = \sqrt{v_x^2 + v_{yfinal}^2}$$

Determine the Maximum Height the Ball Reaches

- [ ] 1. So far we only know the total flight time. However, we can calculate the time it took for the ball to fall to the ground from point C to point D. See Figure 17. Use the vertical components of the velocities. Enter the value of t(C to D) in Data Table 7 of your Student Journal.

$$v_y + at_{(C\ to\ D)} = v_{yfinal}^2$$
$$at_{(C\ to\ D)} = v_{yfinal}^2 - v_y$$
$$t_{(C\ to\ D)} = \frac{v_{yfinal} - v_y}{a}$$

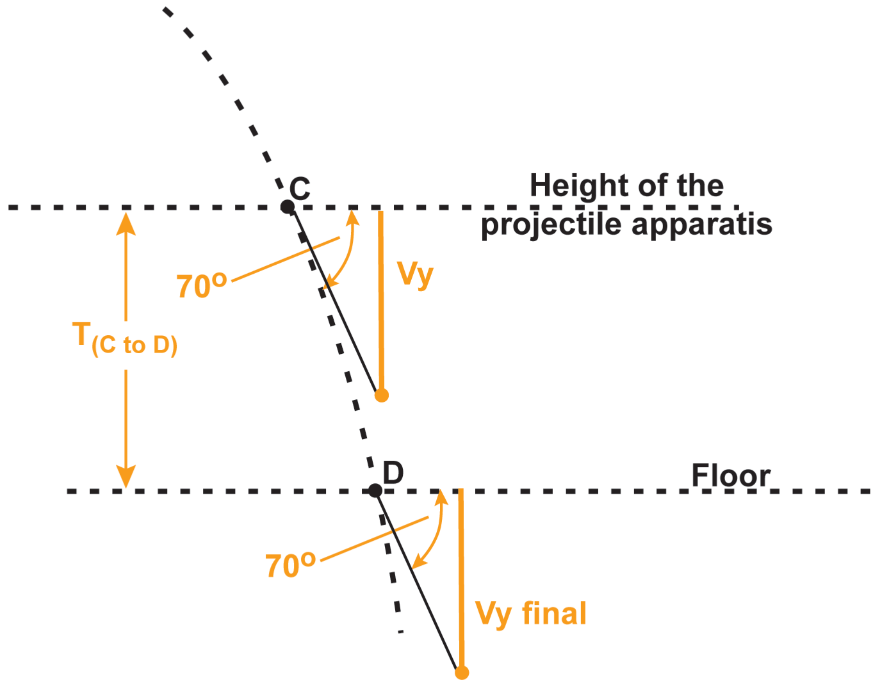


Figure 17
Finding $t_{(C\ to\ D)}$

❑ 2. Subtract $t_{(C\ to\ D)}$ from the stopwatch time. The result is $t_{(A\ to\ C)}$, the time for the ball to travel from point A to point C. Enter $t_{(A\ to\ C)}$ in Data Table 7.

$$t_{(A\ to\ C)} = t_{stopwatch} - t_{(C\ to\ D)}$$

❑ 3. Figure 18 shows the data needed to find the maximum height that the ball reaches. The time $t_{(A\ to\ B)}$ is one half of the time from point A to point C. Enter $t_{(A\ to\ B)}$ in Data Table 7.

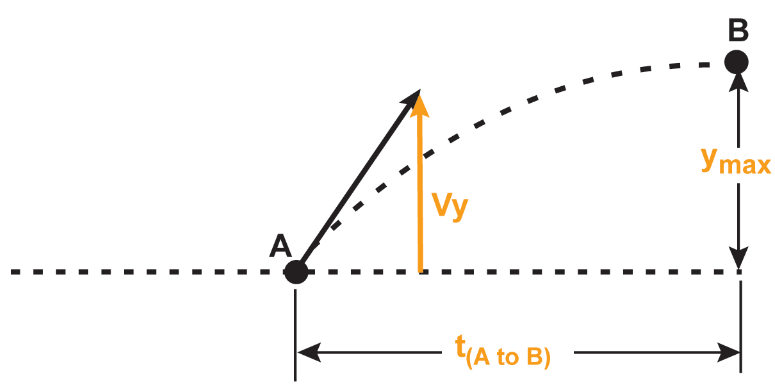


Figure 18
Finding the maximum height

- ❑ 4. Previously we found the vertical component of the ejection velocity. Enter the vertical component v_y in Data Table 7.

- ❑ 5. This is an application for the following formula.

$$d = v_i t + 0.5 at^2$$

We can rewrite the equation as:

$$y_{max} = v_y t + 0.5 at_{(A \text{ to } B)}{}^2$$

Enter the height y_{max} in Data Table 7.

- ❑ 6. Determine the total height above the table. Enter the value in Data Table 7.

- ❑ 7. Enter the estimated total height above the table in Data Table 7.

- ❑ 8. In your Student Journal list some of the factors that would account for a large difference between the calculated height and the estimated height.

Calculating the Horizontal Displacement

- ❑ 1. Enter the horizontal velocity in Data Table 8 of the Student Journal.

- ❑ 2. Enter the average flight time in Data Table 8.

- ❑ 3. Calculate the displacement based on the measured flight time. Enter your answer in Data Table 8.

$$\textbf{Horizontal displacement} = v_x \times \textbf{flight time}$$

- ❑ 4. Enter the measured horizontal displacement in meters in Data Table 8.

- ❑ 5. Compare the measured horizontal displacement to the calculated horizontal displacement. Enter your answer in the Student Journal.

Questions and Interpretations

1. A basketball player throws a ball from the same distance to the basket with the same projection velocity. For angles other than 45º there are two possible angles. Will the low angle shot take more or less time than a high angle shot? ____
____
____
____

2. You are playing deep center field and you need to make a throw to home plate. However, you have very little strength. At want angle do you throw the ball to get the most distance? Neglect air resistance? ______________________
____
____
____
____

3. Neglecting air resistance, if somebody dropped a sheet of paper and a bowling ball from a very tall building which would hit the ground first? ______
____
____
____
____

Notes