Math Lab 4 MS 1

Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

Solving Mechanical Resistance Problems

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

PREPARATORY MATH SKILLS NEEDED TO COMPLETE THIS LAB

There are Preparatory Math Skills Labs, located in a separate book entitled *PT Resource Manual* that contains concepts your students should have mastered before they begin this Math Skills Lab. These labs are coded and titled--

PMS13: "Checking Dimensions in Equations"
PMS14: "Converting Units From One Form to Another"
PMS15: "Multiplying and Dividing Fractions"
PMS16: "Multiplying and Dividing Units Written as Fractions"

Encourage students who need help in these areas to refer to these preparatory labs.

RESOURCE MATERIALS

Student Text: Math Skills Lab
PT Resource Manual

CLASS GOALS

1. Teach students how to rearrange symbols in resistance equations to isolate certain unknowns.
2. Teach students how to solve mechanical resistance problems.

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure that students understand the correct answers.
2. Complete as many activities as time permits. (How much is accomplished depends on the math skills that your students already have.)
   a. Summarize for the students the explanatory materials for "Activity 1: Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns." Then have students complete the Practice Exercises given at the end of Activity 1.
   b. Summarize the explanatory material for "Activity 2: Solving Mechanical Resistance Problems." Then have students complete the Practice Exercises given at the end of Activity 2.
3. Before the class ends, tell students to read Lab 4M1, "Investigating Friction," as homework.

Math Skills Laboratory

Lab 4 MS 1

MATH ACTIVITIES

Activity 1: Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

Activity 2: Solving Mechanical Resistance Problems

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Rearrange the equation for the coefficient of friction,*** $\mu = f/N$***. Isolate the frictional force (f) or the normal force (N) pressing the two surfaces together.***
2. ***Rearrange the equation for drag resistance,*** $R_D = F/v$***. Isolate the drag force (F) or the speed of the object (v) moving in a fluid.***
3. ***Restate the equation*** $\mu = f/N$ ***with proper subscripts for conditions of static and kinetic friction.***
4. ***Substitute appropriate numerical values and units in resistance equations. Solve the equations for a numerical value with the proper units.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems for Activities 1 and 2.***

ACTIVITY 1:

Rearranging Symbols in Resistance Equations to Isolate Certain Unknowns

MATERIALS

For this activity, you'll need pencil and paper.

A. Equations and formulas are used to express a relationship between several physical quantities. Equation 1 below shows the relationship between the coefficient of friction, a frictional force and the force pressing the two surfaces together.

$$\text{Coefficient of Friction} = \frac{\text{Frictional Force}}{\text{Normal Force}} \qquad \textit{\textbf{Equation 1}}$$

This relationship helps you find the value of one physical quantity if you know the value and units for the other quantities. It's easier to write this equation with symbols rather than words, as follows:

$$\mu = \frac{f}{N}$$

where: μ = coefficient of friction (a pure number—no units)
f = frictional force between the two surfaces (typically in pounds or newtons)
N = force pressing the two surfaces together (same units as f, typically in pounds or newtons)

B. Equation 1 describes ***friction.*** But friction is only one form of mechanical resistance. We also can describe ***drag resistance*** by a formula. Drag resistance is a form of resistance that occurs when a solid body passes through a fluid. The formula for finding drag resistance is given in Equation 2.

$$\text{Drag Resistance} = \frac{\text{Drag Force on Object}}{\text{Speed of Object}} \qquad \textit{\textbf{Equation 2}}$$

This relationship helps you find the value of one physical quantity if you know the value and units of the other two quantities. It's easier to write this equation with symbols rather than words, as follows:

$$R_D = \frac{F}{v}$$

where: R_D = drag resistance in units such as $\frac{\text{lb}}{\text{ft/sec}}$ or $\frac{\text{N}}{\text{m/sec}}$
F = drag force on an object due to movement through a fluid (pounds, newtons)
v = speed of the object through a fluid (feet/second, meters/second)

Drag resistance units are given above as force units divided by speed units. For example:

$$\frac{\text{lb}}{\text{ft/sec}} \text{ also can be written as } \frac{\text{lb}\cdot\text{sec}}{\text{ft}}$$

Here's how you obtain this alternate form:

$$\frac{\text{lb}}{\text{ft/sec}} = \frac{\text{lb/1}}{\text{ft/sec}} = \frac{\text{lb}}{1} \div \frac{\text{ft}}{\text{sec}}$$

Now ***invert*** ft/sec and change the division sign (÷) to a multiplication (×) sign. Then you get:

$$\frac{\text{lb}}{\text{ft/sec}} = \frac{\text{lb}}{1} \times \frac{\text{sec}}{\text{ft}} = \frac{\text{lb}\cdot\text{sec}}{\text{ft}}$$

C. When using Equation 1, the value of the coefficient of friction (μ) and the frictional force (f) depend on the type of friction involved. The types are noted by subscripts on the symbols, "f" and "μ." The ***subscripts*** are as follows:

s → static friction
k → kinetic friction
r → rolling friction

Table 1 lists the values for the coefficients of static, kinetic and rolling friction for several surfaces. You'll use Table 1 to solve problems in this math lab.

TABLE 1. APPROXIMATE COEFFICIENTS OF FRICTION (μ)

Material	μ_s	μ_k	μ_r
Wood on wood	0.7	0.4	—
Steel on steel	0.15	0.09	0.05
Metal on rubber	0.65	0.55	0.35
Wood on leather	0.5	0.4	—
Rubber on dry concrete	0.9	0.7	0.3
Rubber on wet concrete	0.7	0.57	0.19

Table 2 summarizes the units used with each physical quantity given in Equations 1 and 2. The table is meant to help you learn the units. Study Table 2.

TABLE 2. RESISTANCE UNITS FOR FRICTION AND DRAG RESISTANCE

		System of Units	
		English	SI
Equation 1: $\mu = \frac{f}{N}$	μ	—	—
	f	lb	N
	N*	lb	N
Equation 2: $R_D = \frac{F}{v}$	R_D	$\frac{\text{lb}}{\text{ft/sec}}$	$\frac{\text{N}}{\text{m/sec}}$
	F	lb	N
	v	$\frac{\text{ft}}{\text{sec}}$	$\frac{\text{m}}{\text{sec}}$

****Note:*** Normal force (N) equals the weight of one object pressing on the other object if the surface layer between the objects is horizontal.

ANSWERS TO REVIEW OF UNITS

a. pounds
b. newtons
c. m/sec
d. drag resistance
e. is a pure number

ANSWERS TO PRACTICE EXERCISES FOR ACTIVITY 1

Problem 1: $\mu = f/N$. Find "f" (worked out in text).
$f = \mu \times N$.

Problem 2: $\mu = f/N$; find "N."
First, multiply each side by "N." Cancel "N's" on the right side.

$$\mu \times N = \frac{f}{\cancel{N}} \times \cancel{N}$$

So $\mu \times N = f$. Now divide both sides by "μ." Cancel the "μ's."

$$\frac{\cancel{\mu} \times N}{\cancel{\mu}} = \frac{f}{\mu}$$

The final answer is: $N = \dfrac{f}{\mu}$.

Problem 3:

$$R_D = \frac{F}{v}; \text{ find "F."}$$

$$R_D \times v = \frac{F}{\cancel{v}} \times \cancel{v}$$ (Multiply both sides by "v." Cancel "v" units on the right side.)

$R_D \times v = F$, or

rearrange equation with "F" on the left side.

$F = R_D \times v$.

LET'S REVIEW UNITS!

Before beginning the Practice Exercises, answer the following questions on units for mechanical resistance. Complete the sentences with the correct word or words.

a. A frictional force in the English system of units is usually measured in _____ (pounds; newtons).

b. A frictional force in SI is usually measured in _____ (pounds; newtons).

c. The unit _____ (mi/hr; ft/sec; in./min; m/sec) is NOT a rate in the English system of units.

d. The units $\frac{\text{lb}}{\text{ft/sec}}$ or $\frac{\text{N}}{\text{m/sec}}$ are correct when describing a ______ (drag force; drag resistance).

e. The coefficient of friction (μ) is the ratio of a force divided by a force. Therefore, the coefficient of friction (μ) _____ (is a pure number; has units of $\frac{\text{lb}}{\text{ft/sec}}$ or $\frac{\text{N}}{\text{m/sec}}$).

PRACTICE EXERCISES FOR ACTIVITY 1

The solution to Problem 1 is given for you. Complete the solutions for Problems 2, 3, and 4.

Problem 1: Given: $\mu = {}^{f}\!/\!_{N}$ (Equation 1)

Find: f

Solution: (Rearrange the equation. Isolate f.)

Step 1: Isolate f by rearranging Equation 1. First write down Equation 1:

$$\mu = \frac{f}{N}$$

Step 2: Multiply both sides by N.

$$\mu \times N = \frac{f}{\cancel{N}} \times \cancel{N} \qquad \text{(Cancel N on the right side.)}$$

Step 3: Rewrite the equation without the canceled Ns.

$$\mu \times N = f \qquad \text{(The f is isolated.)}$$

Step 4: Reverse the order of the equation. Put f on the left side.

$$f = \mu \times N$$

The problem has been solved. The equation, $\mu = {}^{f}\!/\!_{N}$, has been rearranged. The variable f has been isolated.

Problem 2: Given: $\mu = {}^{f}\!/\!_{N}$ (Equation 1)

Find: N

Solution: (Rearrange the equation. Isolate N.)

Problem 3: Given: $R_D = {}^{F}\!/\!_{v}$ (Equation 2)

Find: F

Solution: (Rearrange the equation. Isolate F.)

Problem 4:

$$\text{Drag Res} = \frac{F}{v}\text{; find "v."}$$

From Problem 3 – –

$$F = R_D \times v$$

$$\frac{F}{R_D} = \frac{\cancel{R_D}}{\cancel{R_D}} \times v$$ (Divide both sides by R_D. Cancel like terms.)

$$\frac{F}{R_D} = v\text{, or } v = \frac{F}{R_D}.$$

SOLUTION TO STUDENT CHALLENGE PROBLEM

Problem 5: Solutions are given for the English units. Solutions are similar for SI units.

a. $f = \mu \times N$

$lb = (-) \times lb$

$lb = lb$. (Check!)

b. $N = \dfrac{f}{\mu}$

$lb = \dfrac{lb}{(-)}$

$lb = lb$. (Check!)

c. $F = R_D \times v$

$lb = \dfrac{lb}{ft/sec} \times ft/sec$

$lb = lb$. (Check!)

d. $v = \dfrac{F}{R_D}$

$ft/sec = \dfrac{lb}{\dfrac{lb}{ft/sec}}$

$ft/sec = ft/sec$. (Check!)

(See page T-23c for Problems 6, 7 and 8.)

SOLUTIONS TO PRACTICE EXERCISES FOR ACTIVITY 2

Problem 6: Use $\mu_s = \dfrac{f_s}{N}$. (Equation 1) Solve for "f_s."

The horizontal force is equal and opposite to static friction. Therefore, a force just greater than f_s will cause the box to slide.

$f_s = \mu_s N$ where: μ_s for wood-on-wood = 0.7
N = weight = 50 lb

$f_s = 0.7 \times 50$ lb
$f_s = 35$ lb.

Problem 7: Use $\mu_k = f_k/N$. (Equation 1) Solve for "f_k."

a. Once moving, the force required to keep the box moving is less than the force required to start it moving. Static friction is always greater than kinetic friction.

$f_k = \mu_k N$ where: μ_k for wood-on-wood = 0.4
N = weight = 50 lb

$f_k = 0.4 \times 50$ lb
$f_k = 20$ lb.

b. As explained above, $f_s > f_k$. This is verified by the calculations, since $f_s = 35$ lb and $f_k = 20$ lb.

This is reasonable. It is known to be true from our everyday experiences. We find it takes more force to start a load moving than to keep it moving once started.

Problem 8: Since metal is moving between "rolling wheels," use $f_r = \mu_r N$ at either surface. Solve for f_r, then double to find the total pulling force.

$f_r = \mu_r N$, where: μ_r for rubber on metal = 0.35
N = normal force between roller and sheet = 50 lb

$f_r = 0.35 \times 50$ lb = 17.5 lb

Total force = 2 × 17.5 lb = 35 lb

This is the "feeding" force provided by friction between the rollers and metal surface, when the rollers are turned by motors to "feed" the metal sheets toward the stamping machine.

Problem 4: Given: $R_D = F/v$
Find: v
Solution: (Rearrange the equation. Isolate v.)

Student Challenge

Problem 5: Given: Problems 1 through 4 and your solution to each problem.
Find: Whether or not the equation is stated correctly by substituting English or SI units for the symbols.
Solution: (***Hint:*** If the equation is stated correctly, the units on the left side of the equation will be the same as the units on the right side.)

ACTIVITY 2:

Solving Mechanical Resistance Problems

MATERIALS

For this activity, you'll need a hand calculator.

Mechanical resistance formulas for this activity are given by Equations 1 and 2 in Activity 1. You'll need to choose the correct equation for the problem you want to solve.

What if it's not in the correct form? First, rearrange the equation to isolate the symbol that represents the physical quantity you want. Then solve the rearranged equation for the unknown value. Your final answer should include a correct numerical answer with the proper units. Use Table 1 of values for μ. Use Table 2 to help check your units in the answer.

PRACTICE EXERCISES FOR ACTIVITY 2

Problem 6: Given: A wooden box weighing 50 lb is placed on a flat wooden table.
Find: The horizontal force required to overcome static friction and just cause the box to slide.
Solution: (***Hint:*** Use the equation, $\mu_s = \frac{f_s}{N}$.)

Problem 7: Given: The same conditions stated in Problem 6.
Find:
a. The horizontal force required to keep the box moving once it starts to move.
b. Whether or not this force is greater than the force needed to start the box moving.

Solution: (***Hint:*** This problem involves kinetic [moving] friction.)

Problem 8: Given: Rubber-faced rollers "feed" metal sheets into a machine that stamps out clock parts. The "pinched" rollers press on the sheet, from above and below, with a force of 50 lb. at each surface. Friction between the rollers and sheet move the sheet towards the machine.
Find: The total force needed to pull the sheet through the "pinched" rollers—initially at rest—and cause them to turn. (This "pulling" force should be the same as the "driving" force provided by the rollers when **motors** turn the rollers to "feed" the sheet through.)
Solution: Will $f_r = \mu_r N$ work here? What is "N"?

SOLUTIONS TO PRACTICE EXERCISES, Continued

Problem 9: Use $R_D = F/v$ (**Equation 2**)

$$R_D = \frac{F}{v}$$

where: F = drag force = 200 N

v = speed = 12 m/sec

$$R_D = \frac{200\ N}{12\ m/sec}$$

$$R_D = 16.67\ \frac{N}{m/sec}$$

$$R_D = 16.67\ \frac{N \cdot sec}{m}$$

If students have difficulty with the units, review Math Skills Lab Activity PMS16 with them.

SOLUTIONS TO STUDENT CHALLENGE PROBLEMS

Problem 10: a. Use $\mu_s = f_s/N$. (**Equation 1**)

The truck is stopped. So it must be moved by a force slightly larger than the static frictional force (f_s), which is 2250 lb. Solve for "μ_s."

$f_s = \mu_s N$ Solve for "μ_s" by dividing both sides by "N" to isolate "μ_s."

$$\frac{f_s}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}}$$

(Cancel like terms on the right side. Rearrange the equation.)

$$\mu_s = \frac{f_s}{N}$$

where: f_s = 2250 lb

N = truck weight = 2500 lb

$$\mu_s = \frac{2250\ lb}{2500\ lb}$$

$\mu_s = 0.9$. This is the same value of μ_s for rubber on dry concrete, as given in Table 1.

(Solutions for Parts b and c are on page T-24c)

(See page T-24c for Problems 9, 10 and 11.)

b. Use $\mu_s = f_s/N$. **(Equation 1)**
The conditions are the same as part "a" of this problem, except that the surface is now wet concrete rather than dry concrete. Solve for the truck weight. The weight is the same as the normal force (N) on a flat surface.

$f_s = \mu_s N$ (Divide both sides by "μ_s" to isolate "N.")

$\frac{f_s}{\mu_s} = \frac{\mu_s N}{\mu_s}$ (Cancel like terms on the right. Rearrange the equation with "N" on the left side.)

$N = \frac{f_s}{\mu_s}$ where: $f_s = 2250$ lb
$\mu_s = 0.7$

$N = \frac{2250 \text{ lb}}{0.7}$

$N = 3214.3$ lb.

c. Use $\mu_k = f_k/N$. **(Equation 1)** Solve for "f_k."
A force greater than the kinetic friction will keep the truck moving.

$f_k = \mu_k N$ where: μ_k rubber on wet concrete = 0.57
N = 3214.3 lb (from part b.)

$f_k = 0.57 \times 3214.3$ lb

$f_k = 1832.1$ lb.

Note: Any force greater than this will cause the truck to accelerate as it slides.

Problem 11: At low speeds and low turbulence, drag force versus speed is a straight line, when graphed as shown in the student text.

For the first case:

$$R_D = \frac{F}{V} = \frac{200 \text{ N}}{12 \text{ m/sec}} = 16.67 \frac{\text{N}\cdot\text{sec}}{\text{m}}.$$

For the second case:

$$R_D = \frac{F}{V} = \frac{400 \text{ N}}{24 \text{ m/sec}} = 16.67 \frac{\text{N}\cdot\text{sec}}{\text{m}}.$$

Since drag resistance remains constant for a given material of given shape, doubling the speed requires doubling the drag force to 400 N.

Problem 9: Given: A drag force of 200 N is experienced by a hydrofoil boat moving through the water at a speed of 12 m/sec. (A hydrofoil is a wing-like structure made to lift the boat hull out of the water at high speeds.)

Find: The drag resistance of the hydrofoil.

Solution: (***Hint:*** $R_D = \frac{F}{v}$.)

Student Challenge

Problem 10: Given: A horizontal force of 2250 lb will cause a stationary 2500-lb truck to slide (with wheels locked) when the truck is on a dry concrete surface. The truck tires are made of rubber. On a wet concrete surface, the same force will cause a more heavily loaded truck to slide.

Find:

a. The coefficient of sliding friction for the truck on dry concrete. Does it match the value given in Table 1?
b. How heavy can the truck be and still just start to slide when 2250 lb of horizontal force is applied if the truck is on a ***wet concrete*** surface?
c. What force will keep the truck moving, once it starts to slide on a wet concrete surface?

Solution: (***Hint:*** Static friction is always greater than kinetic or sliding friction. If the applied force just starts the truck sliding, a ***smaller*** applied force will keep it sliding.)

Problem 11: Given: The same hydrofoil described in Problem 9.

Find: The drag force when the speed is doubled to 24 m/sec. (***Note:*** This is still a low speed. It produces very little turbulence for the hydrofoil.)

Solution: (***Hint:*** Use the fact that the graph of drag force versus speed is a ***straight line or linear relationship*** at low speeds.)