

Math Lab 11 MS 1

Plotting and Interpreting Graphical Data for Mechanical Transducers

For best results, print this document front-to-back and place it in a three-ring binder.
Corresponding teacher and student pages will appear on each opening.

TEACHING PATH - MATH SKILLS LAB - CLASS M

RESOURCE MATERIALS

Student Text: Math Skills Lab

CLASS GOALS

1. Teach your students how to use manufacturer's specification bulletins to obtain information about transducers.
2. Teach your students how to solve problems that involve mechanical transducers (using manufacturer's specification bulletin information).

CLASS ACTIVITIES

1. Take five or ten minutes to go through the Student Exercises. Make sure your students understand the correct answers.
2. Complete as many activities as time permits. Students already should have read the discussion material and looked at the examples for each activity before coming to this class. (How much your students accomplish will depend on the math skills they already have.) Summarize the explanatory material for the activity: "Plotting and Interpreting Graphical Data for Mechanical Transducers." Then have students complete the Practice Exercises given at the end of the activity.
3. Supervise student progress. Help students obtain the correct answers.
4. Before the class ends, tell your students to read Lab 11M1, "Force Transducers."

Math Skills Laboratory



MATH ACTIVITY

Activity: Plotting and Interpreting Graphical Data for Mechanical Transducers

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you should be able to do the following:

- 1. Use manufacturer's specification bulletins to plot graphs of displacement versus output voltage for a specific type of mechanical transducer.*
 - 2. Use manufacturer's specification bulletins or a plotted graph to find displacement-versus-voltage data for a transducer.*
 - 3. Solve problems that involve transducer characteristics and operation.*
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LEARNING PATH

- 1. Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.*
 - 2. Study the examples.*
 - 3. Work the problems.*
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ACTIVITY

Plotting and Interpreting Graphical Data for Mechanical Transducers

MATERIALS

For this activity, you'll need graph paper and a calculator.

DISCUSSION

This lab involves plotting and interpreting data related to mechanical transducers.

One of the most useful mechanical transducers is the linear variable-differential transformer (LVDT). A series-240 LVDT transducer is used to measure physical properties that can be changed into a linear displacement. Typical uses include servo-position feedback, sensor for pressure transducers, strain measurement in structural members, automatic gaging and machine control.

NOTE: Help students understand what is graphed in Figure 3. They should see that armature position (about a null point) is plotted along the y-axis. (The null position for the armature corresponds to the zero along the y-axis.) They should see that the output voltage of the LVDT is plotted along the x-axis, from $-V_{\max}$ to $+V_{\max}$. At the null position, the output voltage is zero. At other positions it is either positive or negative, depending on whether the armature is above the null position or below the null position.

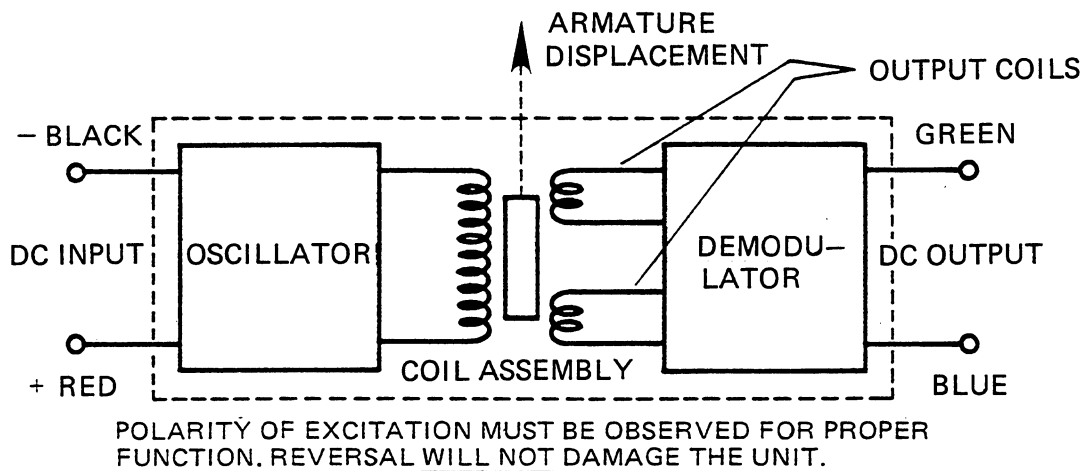


Fig. 1 Circuit diagram.

The LVDT transducer was first introduced in Problem 4 of Math Lab 7MS4, "Force Transformers." Figure 1 shows a circuit diagram of an LVDT. A displacement due to a force—either a push or a pull—causes the LVDT armature (movable core) to move away from the midpoint (null) position.

When located at the *null* point, the armature induces equal voltages (with opposite signs) across the two output coils. When voltage from the two coils is added, the resultant output voltage is zero. The result is the same as that for the two D-cells shown in Figure 2a.

For any other armature position, above or below the null point, the resultant voltage across the two output coils is not zero. It depends on the exact position of the armature coil. When the armature is at the top position, the voltages from the two coils add, giving twice the result. That's just like what happens in Figure 2b, when two cells add their voltages.

Look at the graph of output voltage versus armature displacement in Figure 3. It shows that when full-stroke displacement of the armature occurs (armature moves from the bottom of coil assembly in Figure 1 through the null point and on to the top of the coil assembly), there's a shift from maximum negative DC output voltage at the bottom, to zero voltage at the null point, and on to maximum positive DC output voltage at the top.

Use the electrical specifications bulletin (Table 1) given with the series-240 LVDT transducers to construct a graph of armature displacement versus output voltage. This graph becomes a data source for technicians who select, install or calibrate LVDT transducers.

From the series-240 LVDT transducers, choose model number 0244. See Table 1. All required specifications are listed in the column under the model number 0244.

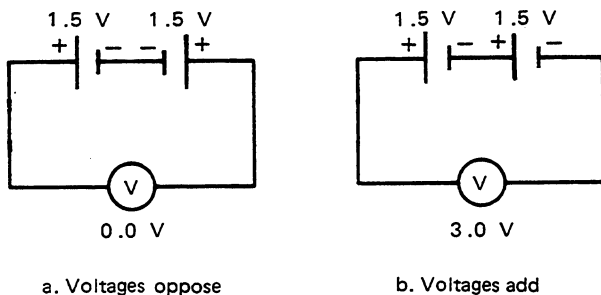


Fig. 2 Cell-orientation diagram.

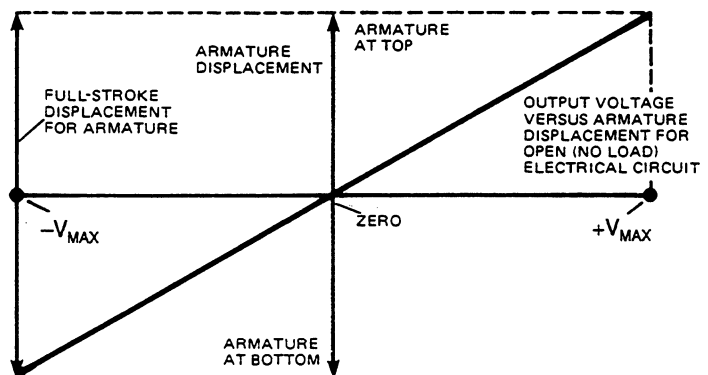


Fig. 3 Output voltage versus armature displacement for an LVDT.

NOTE: Table 1 is a portion of a manufacturer's specification sheet modified for use here. There are many other parameters that are beyond the scope of the text in manufacturer's specification sheets for design engineers, drafters and others. However, the technician will find them invaluable when choosing transducers or troubleshooting existing applications of transducers.

NOTE: The performance of Model 0244 LVDT, as graphed, shows that when the displacement of the armature varies from minus one inch below the null position to plus one inch above the null position, the output voltage varies from -4.3 volts to +4.3 volts. Quiz your students so that they can interpret the graph to approximate what the voltage output should be at +0.5 inch and -0.5 inch. They should read about +2.1 V or 2.2 V and -2.1 V or -2.2 V. Then, since curve is linear they should see that "exact" answer is $+4.3 \text{ V} \div 2$ (for 0.5 inch) and $-4.3 \text{ V} \div 2$ (for -0.5 inch).

TABLE 1. ELECTRICAL SPECIFICATIONS FOR SERIES-240 LVDT

Model Number	0240	0241	0242	0243	0244	0245	0246
Range, working (inches)	± 0.050	± 0.100	± 0.250	± 0.500	± 1.00	± 2.00	± 3.00
Maximum usable Input, volts DC	± 0.075	± 0.150	± 0.375	± 0.750	± 1.50	± 2.75	± 4.00
Nominal FS Output $\pm V$ DC (open circuit)	6.0 minimum to 30 maximum						
@ 6-V input	1.2	2.1	1.6	3.0	4.3	4.0	3.1
@ 15-V input	3.0	5.4	4.2	7.5	10.8	10.0	7.8
@ 24-V input	5.0	9.0	7.0	12.5	18.0	16.0	13.0
@ 30-V input	5.9	10.7	8.3	14.8	21.4	20.0	15.4

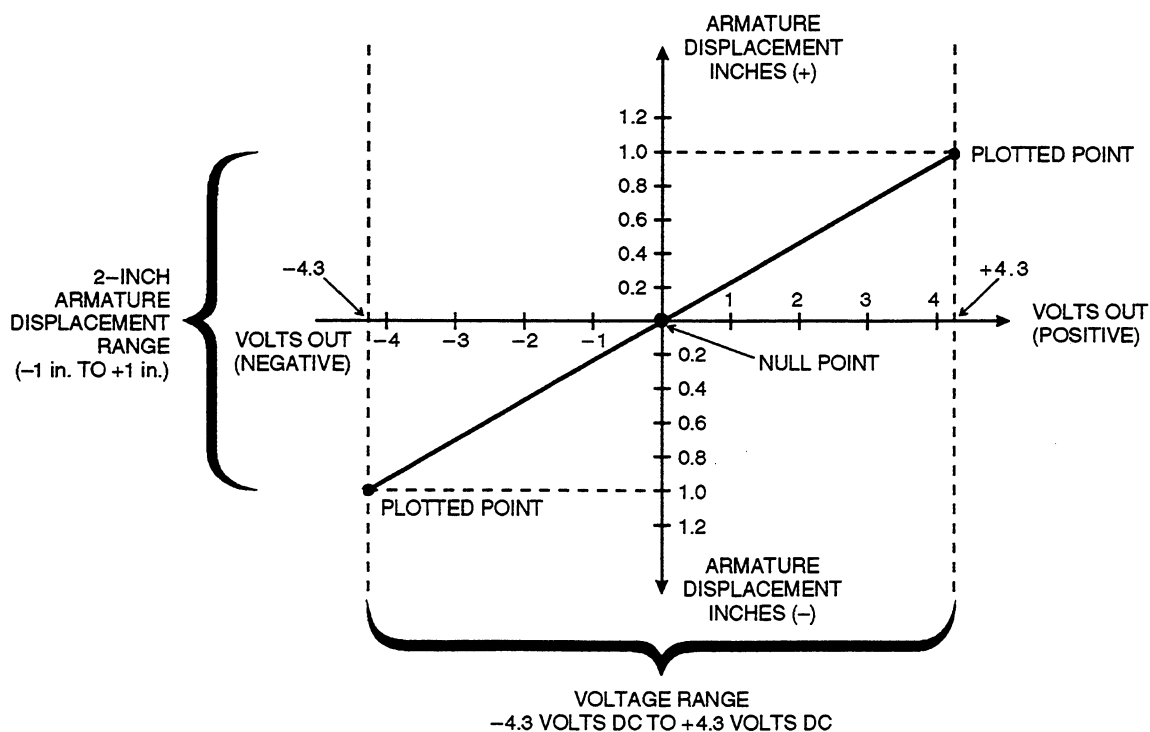
Example 1: Graph of Armature Displacement Versus Output Voltage

Given: Electrical specifications table for Model-0244 LVDT transducer with 6-V DC input.

Construct: A graph of armature displacement versus output voltage that will give the displacement and output voltage over the working range of the LVDT.

- Solution:
- From the electrical specifications, you'll find that the 0244 LVDT has—
 - a working range (displacement) of 1.00 inch on either side of the null point
 - at 6-V DC input, the LVDT has an output voltage range of 4.3-V DC on either side of the null point. (**Note:** "FS" means "full scale.")
 - With this data, you can graph the displacement versus output voltage over the ranges specified. Since the LVDT is a **linear** transducer, the plotted data produces a straight line, as shown in the following graph.

GRAPH OF DISPLACEMENT VERSUS OUTPUT VOLTAGE FOR THE MODEL-0244 LVDT WITH 6-V DC INPUT.



NOTE: Review the earlier discussions of how an LVDT works if your students have trouble with the positive and negative outputs of the graph.

NOTE: The constant (C) used with the LVDT here makes it easy to find other values of voltage versus displacement as long as either output voltage or displacement is known. This is shown in Example 2.

You can show students how C is used. Solve the following problems for them:

- Find output voltage for an armature displacement of 0.2 inches.

$$V_{out} = C \times (\text{armature displacement})$$

$$V_{out} = \frac{4.3 \text{ V}}{1 \text{ in}} \times 0.2 \text{ in}$$

$$V_{out} = \left(\frac{4.3 \times 0.2}{1} \right) \left(\frac{\cancel{\text{V}} \cdot \cancel{\text{in}}}{\cancel{\text{in}}} \right) = 0.86 \text{ V}$$

$$V_{out} = +0.86 \text{ V}$$

- Find output voltage for an armature displacement of -0.2 inches.

$$V_{out} = C \times (\text{armature displacement})$$

$$V_{out} = \frac{4.3 \text{ V}}{1 \text{ in}} \times -0.2 \text{ in}$$

$$V_{out} = \left[\frac{4.3 \times (-0.2)}{1} \right] \left(\frac{\cancel{\text{V}} \cdot \cancel{\text{in}}}{\cancel{\text{in}}} \right) = -0.86 \text{ V}$$

$$V_{out} = -0.86 \text{ V}$$

With this graph, a technician can choose any value of positive or negative armature displacement and read the corresponding output voltage. Let's use this graph to find some output voltages for given displacements.

Given a displacement of the armature—output voltage is:

- | | |
|--|---------|
| a. depressed -0.6 inch from null point | -2.6 V |
| b. raised +0.6 inch from null point | +2.6 V |
| c. depressed -0.4 inch from null point | -1.75 V |
| d. raised +0.9 inch from null point | +3.9 V |

Now use the graph to find armature displacements that correspond to a certain output voltage.

For the following voltage output—armature displacement must be:

- | | |
|-----------|-------------------------------------|
| e. +3.5 V | raised +0.8 inch from null point |
| f. -2.6 V | depressed -0.6 inch from null point |
| g. +3.0 V | raised +0.7 inch from null point |
| h. -4.3 V | depressed -1.0 inch from null point |

If you know the output voltage for a given displacement, you can write that amount as a ratio or as a proportionality constant (C), where $C = \text{voltage/displacement}$.

Since the LVDT is a linear device, this constant will be valid over the entire range of operation of the transducer. You can calculate "C" from values in Table 1. You know that voltage output is 4.3 V when the armature moves 1 inch from the null position. Thus,

$$C = \frac{V}{d}$$

where: V = output voltage = 4.3 V

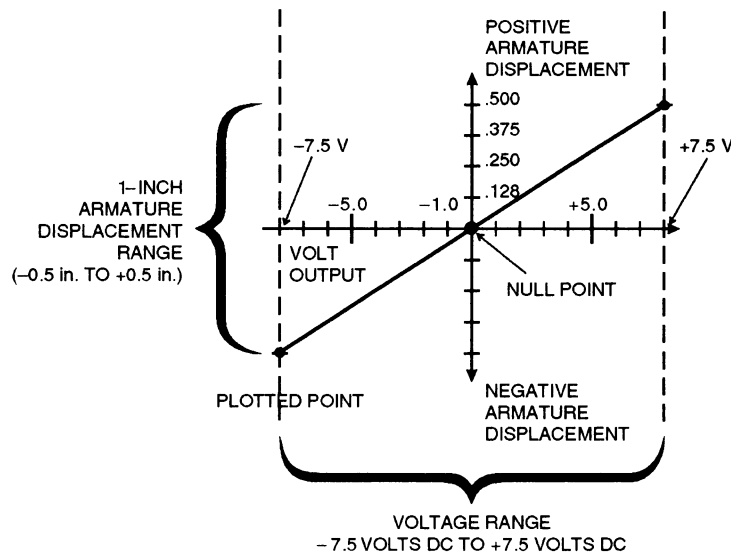
d = displacement from null point (\pm)

$$C = \frac{4.3 \text{ V}}{1 \text{ in}} = 4.3 \text{ volts/inch}$$

Once "C" for the particular LVDT is found, you can find other voltages or displacements as long as either output voltage or displacement is known.

ANSWERS TO STUDENT EXERCISES

- Problem 1:** a. Graph of displacement versus output voltage for the model 0243 LVDT with 15-V DC input. The graph should show a full stroke armature displacement from -0.5 in to +0.5 in and a corresponding voltage change from -7.5 V DC to +7.5 V DC.



b. $C = \frac{V}{d}$

where: V = voltage when armature is at top of stroke

$$V = 7.5 \text{ V}$$

d = maximum distance of armature above null position.

$$d = 0.5 \text{ in}$$

$$C = \frac{7.5 \text{ V}}{0.5 \text{ in}} = \frac{7.5}{0.5} \cdot \frac{\text{V}}{\text{in}} = 15 \frac{\text{V}}{\text{in}}$$

$C = 15 \text{ V/in}$ (for the model 0243 LVDT).

Answers to Student Exercise #2 follows on page T-21c.

Problem 2: a. At the +0.25-inch point on the vertical axis (armature displacement), draw a horizontal line parallel to the horizontal axis (voltage output). From the point where this line intersects the graph line, drop a vertical line to the horizontal axis. The output voltage can be read directly at the point where the vertical line crosses the horizontal axis. It is +3.75 V DC output voltage.

b. $C = \frac{V}{d}$ Solve for "V." As shown in Example 2, we have:

$$V = Cd$$

where: $C = 15 \text{ V/in}$ (from part "b," Problem 1)
 $d = 0.25 \text{ in}$

$$V = Cd = 15 \frac{\text{V}}{\text{in}} \times 0.25 \text{ in}$$

$$V = (15 \times 0.25) \left(\frac{\text{V}}{\text{in}} \cdot \text{in} \right)$$

$$V = +3.75 \text{ V. (The answer is the same).}$$

Example 2: Using a Proportionality Constant to Find Displacement or Voltage from an LVDT Transducer

Given: Proportionality constant—

$$C = \frac{V}{d} \quad \text{where: } C = 4.3 \text{ V/in for an LVDT transducer (Model 0244, at 6-V input)}$$

Find: Output voltage when LVDT armature is depressed -0.6 inch from null point.

Solution: $C = V/d$. Solve for "V" by multiplying both sides by "d."

$$(d) \times C = \frac{V}{d} \times (d) \quad (\text{Cancel like terms. Rearrange equation.})$$

$$V = C \times d = 4.3 \text{ V/in} \times (-0.6 \text{ in})$$

$$V = -(4.3 \times 0.6) \frac{\text{V} \cdot \cancel{\text{in}}}{\cancel{\text{in}}}$$

$$V = -2.58 \text{ V} \approx 2.6 \text{ V.}$$

Note: This is the same value of voltage output as the "approximate" reading from the graph for a displacement of -0.6 inch.

PRACTICE EXERCISES

Complete the following exercises.

Problem 1: Given: Table 1, Electrical Specifications for Series-240 LVDT Transducers.

- Find:
- For Model-0243 with 15-V DC input, plot a graph of armature displacement versus output voltage. (It should be similar to the graph plotted for Model-0244.)
 - Find the proportionality constant (C) for the LVDT where $C = V/d$. (Look back at Example 2.)

Solution:

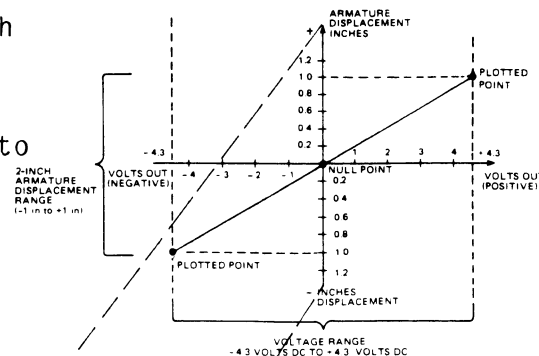
Problem 2: Given: The graph and proportionality constant found in Problem 1 for LVDT transducer Model-0243.

- Find:
- The output voltage for 0.25-in positive displacement, using the graph.
 - The output voltage for 0.25-in positive displacement, using the proportionality constant (C) and the equation, $C = V/d$.

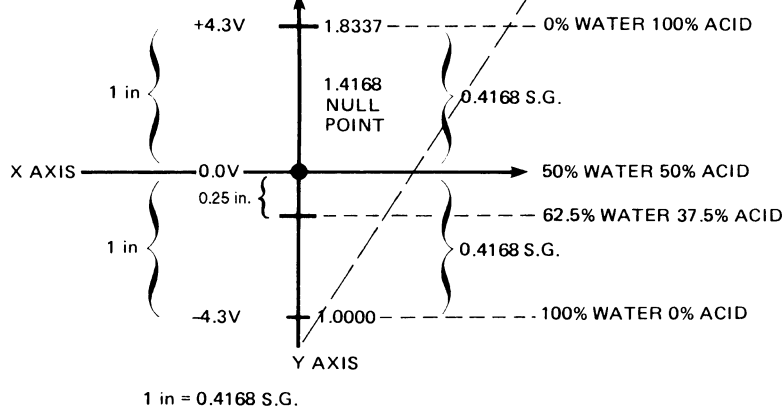
Solution:

Problem 3: From Example 1 of the Math Skills Lab, you know the following about the model 0244 LVDT.

- Displacement goes from -1.00 inch through the null point to +1.00 inch.
- Output voltage goes from -4.3 V to zero to +4.3 V as armature goes from -1.00 inch to +1.00 inch.
- There is a linear relationship between output voltage and displacement for the LVDT.



- a. Assuming the "y" axis (displacement) is the hydrometer rise or fall from the null point, a line can be constructed so that there's a difference of 0.4168 in specific gravity (SG) from the null point to the SG of pure acid (1.8337 - 1.4168) and an equal difference of 0.4168 from the null point to SG of pure water (1.4168 - 1.0000).



At 0.25 inch below the null point there would be more than 50% water and less than 50% acid.

The relation is linear, so:

$$\frac{0.25 \text{ in}}{1 \text{ in}} = \frac{x}{50\%} \text{ or } x = \frac{0.25 \text{ in} \times 50\%}{1 \text{ in}} = 12.5\%$$

Add 12.5% to 50% = 62.5% water

Subtract 12.5% from 50% = 37.5% acid

The ratio is 62.5% water, 37.5% acid

The answers to Problems 3b, 3c, and Problem 4 are on page T-22c.

b. $C = \frac{V}{d} = \frac{4.3 \text{ V}}{1 \text{ in}} = 4.3 \text{ V/in}$

$$C = \frac{V}{\Delta SG} = \frac{4.3 \text{ V}}{0.4168 \text{ SG}}, \text{ where } \Delta SG = 1.4168 - 1.0000$$

$C = 10.3 \text{ V per unit of specific gravity}$

c. $C = \frac{V}{\Delta SG}$ (Solve for "V.")

$V = C \times \Delta SG$ where: $C = 10.3 \text{ V/unit of SG}$
 $\Delta SG = \text{units of SG from null point}$

Since SG reading is 1.2600, it falls **below** the null point (1.4168) by an amount $1.2600 - 1.4168 = -0.1568$.

(So $\Delta SG = -0.1568$.) Then $V = \frac{10.3 \text{ V}}{\Delta SG} \times (-0.1568 \text{ SG})$

Then $V = C \times \Delta SG = \frac{10.3 \text{ V}}{\text{units of SG}} \times -0.1568 \text{ units of SG} = [10.3 \times (-0.1568)] \text{ V}$

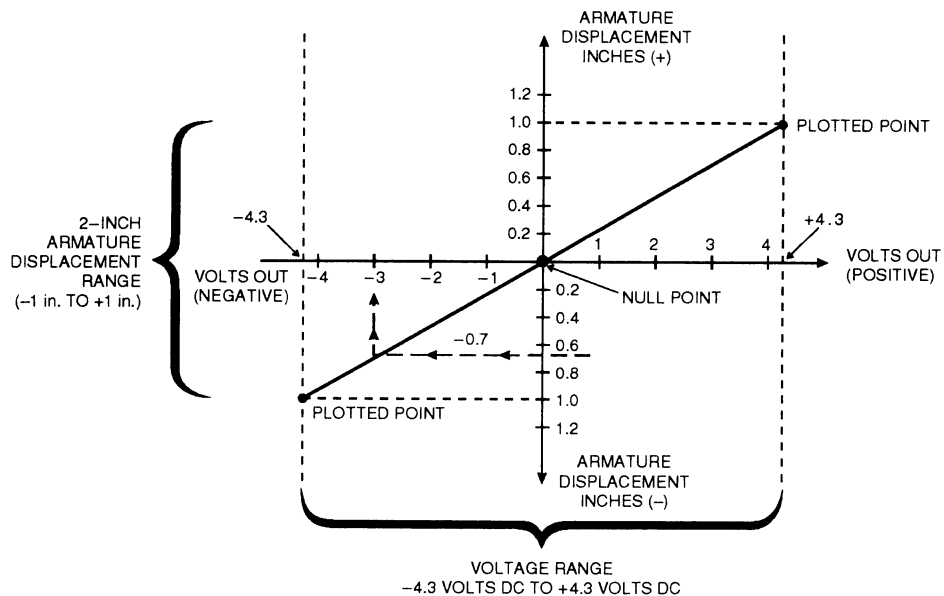
$V = -1.61 \text{ V}$ (Voltage output is -1.61 volts.)

Problem 4:

Since the floor deforms downward, the output voltage will be negative.

From the graph (-0.70 in displacement), a horizontal line intersects the graph line at -3 volts.

Therefore, when the load exceeds 2400 lb , a negative 3-volt output voltage is available to trigger the safety buzzer.



Problem 3: Given: A Model-0244 LVDT transducer, input voltage 6-V DC, similar to the one in Example 1 is used in a sulfuric acid-water mixing vat. This transducer gives data to a microprocessor that controls mixing valves.

In this case, voltage output is related to specific gravity of the liquid. Pure water has a specific gravity of 1.0000. Pure sulfuric acid has a specific gravity of 1.8337.

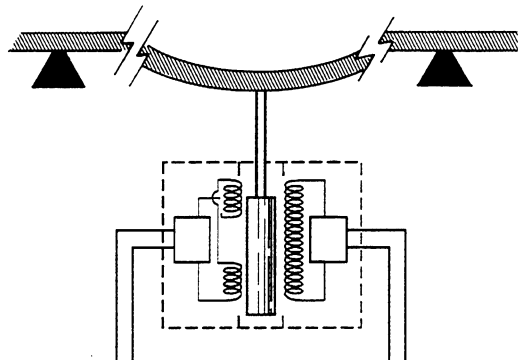
A hydrometer that measures specific gravity between 1.0000 and 1.8337 sinks in pure water until the 1.000 mark is at the water level. It rises gradually as the ratio of acid to water increases. The hydrometer floats in 100% sulfuric acid with the 1.8337 mark at the sulfuric acid level.

Let the percent mixture vary from pure water (0%) to pure sulfuric acid (100%). At a 50% mixture, the specific gravity is 1.4168. (This is the null point.)

- Find:
- What ratio of acid to water is present when the hydrometer bulb is 0.25 inch **below** the null point.
 - What's the proportionality constant if specific gravity is substituted for displacement in the equation, $C = V/d = V/\text{sp gr}$?
 - What voltage output corresponds to a specific gravity of 1.2600?

Solution:

Problem 4: Given: The graph in Example 1 for the Model-0244 LVDT transducer. An elevator-load buzzer is activated by an LVDT transducer when the elevator floor is deformed by an overload as shown in the sketch. The technician knows that the floor deforms 0.70 inch beyond the null position when a 2400-lb load is applied. (At the **null** position, the output voltage is zero.)



- Find: The voltage output available to trigger the safety buzzer when the load exceeds 2400 lb. Assume that the input voltage is 6-V DC.

Solution: