Math Skills Laboratory

Lab 10 MS 1

MATH ACTIVITY

Activity: ***Solving Energy-Conversion Problems For Mechanical Energy Convertors***

MATH SKILLS LAB OBJECTIVES

When you complete these activities, you should be able to do the following:

1. ***Solve energy-conversion problems for mechanical energy convertors.***
2. ***Substitute correct numerical values and units in energy-conversion equations. Solve the equations for an unknown numerical value with the proper units.***
3. ***Find the efficiency of mechanical energy convertors.***

LEARNING PATH

1. ***Read the Math Skills Lab. Give particular attention to the Math Skills Lab Objectives.***
2. ***Study the examples.***
3. ***Work the problems.***

ACTIVITY

Solving Energy-Conversion Problems For Mechanical Energy Convertors

MATERIALS

For this activity, you'll need a calculator.

In this Math Skills Lab, you'll review the important formulas for work and energy. You'll review the units used to measure work, energy and power. You'll solve problems that involve mechanical energy convertors.

LET'S REVIEW FORMULAS AND UNITS!

Examine Tables 1, 2 and 3. Table 1 lists the important formulas for work and energy that you've already studied. Table 2 summarizes common units that you'll use in energy-conversion problems. Table 3 lists some conversions between units.

You'll find these conversions helpful when you solve the problems in this activity.

TABLE 1. REVIEW OF BASIC FORMULAS FOR WORK AND ENERGY

Energy System	Formulas	Definition of Symbols
MECHANICAL		
Linear Work	$W = F \times D$	F = applied force D = distance moved
Rotational Work	$W = T \times \theta$	T = torque applied θ = angular distance rotated
Gravitational Potential Energy	$E_p = w \times h$	w = weight h = height raised above reference level
Elastic Potential Energy	$E_p = \frac{1}{2} kd^2$	k = force or spring constant d = distance spring is stretched or compressed
Linear Kinetic Energy	$E_k = \frac{1}{2} mv^2$	m = mass v = speed
Rotational Kinetic Energy	$E_k = \frac{1}{2} I\omega^2$	I = moment of inertia ω = angular speed
FLUID		
Fluid Work	$W = p \times (\Delta V)$; $W = (\Delta p) \times V$	$p; \Delta p$ = pressure or pressure difference $\Delta V; V$ = volume of fluid moved
Fluid Kinetic Energy	$E_k = \frac{1}{2} (\rho V)v^2$	ρ = mass density V = volume v = speed
ELECTRICAL		
Electrical Work	$W = q \times \Delta V$	q = electrical charge moved ΔV = voltage difference
Electrical Energy	$E_{elec} = P_{elec} \times t$	P_{elec} = electrical power used t = time power is used
THERMAL		
Thermal Energy	$H = mc\Delta T$	m = mass c = specific heat ΔT = temperature difference

TABLE 2. SUMMARY OF COMMON UNITS USED IN ENERGY CONVERSION CALCULATIONS

Quantity	Symbol	English Unit	SI Unit
Mass	m	slug or (lb·sec^2/ft)	kg
Weight	w	lb	N
Force	F	lb	N
Work	W	ft·lb	N·m or J
Kinetic Energy	E_k	ft·lb	N·m or J
Potential Energy	E_p	ft·lb	N·m or J
Power	P	(ft·lb)/sec; hp	(N·m)/sec or watt
Spring Constant	k	lb/ft; oz/in.	N/m; kgf/m *
Angular Displacement	θ	rad	rad
Angular Speed	ω	rad/sec	rad/sec
Linear Speed	v	ft/sec	m/sec
Mass Density	ρ	slug/ft^3; lbm/ft^3 **	kg/m^3
Moment of Inertia	I	slug·ft^2	kg/m^2
Pressure or Pressure Difference	p; Δp	lb/ft^2; oz/in^2	N/m^2
Electrical Charge	q	—	coulomb (C)
Torque	T	lb·ft;lb·in.	N·m
Fluid Volume	V; ΔV	ft^3	m^3
Electrical Current	I	—	ampere (A)
Electrical Voltage	V; ΔV	—	volt (V)
Electrical Resistance	R	—	ohm (Ω)
Heat Energy	H	Btu	cal
Temperature	T	°F	°C
Temperature Difference	ΔT	F°	C°
Specific Heat	c	$\frac{\text{Btu}}{\text{lb·F°}}$	$\frac{\text{kcal}}{\text{kg·C°}}$

* kgf ⇒ kilogram force
** lbm ⇒ pound mass

TABLE 3. USEFUL CONVERSIONS FOR UNITS

1 slug	=	32 pound mass (32 lbm)
1 kilogram force (kgf)	=	9.8 newtons
1 newton·meter	=	1 joule
1 volt·coulomb	=	1 joule
1 ampere·second	=	1 coulomb
1 radian	=	57.3°
1 horsepower	=	550 ft·lb/sec
1 horsepower	=	746 watts
1 watt	=	1 newton·meter per second
1 watt	=	1 joule per second
1 watt	=	1 volt·ampere
1 watt·sec	=	1 joule = 0.7376 ft·lb
1 kilowatt·hour (kWh)	=	3.6×10^6 joules
1 calorie	=	3.086 foot·pounds
1 calorie	=	4.184 joules
1 Btu	=	778 foot·pounds
1 Btu	=	1055 joules

Now that you've reviewed Tables 1, 2 and 3, use them to help you answer the following questions.

1. The formula for rotational work in mechanical systems is: ____.
2. The formula for linear kinetic energy is: ____.
3. The formula for rotational kinetic energy is: ____.
4. The formula for fluid kinetic energy is: ____.
5. The formula for electrical work is: ____.
6. The formula for electrical energy used, given the electrical power and time of operation, is: ____.
7. The heat energy lost by an object of mass m and specific heat c, while cooling through a temperature difference of ΔT, is given by the formula: ____.
8. A mechanical energy convertor changes 1000 ft·lb of input mechanical energy to 1 Btu of output heat energy. One Btu of energy equals ____ ft·lb of energy.
9. The percent efficiency of the mechanical energy convertor in Question 8 is: ____.
10. A mechanical energy convertor changes 1000 joules of input mechanical energy to 200 calories of output heat energy. Two hundred calories of energy equals ____ joules of energy.
11. The percent efficiency of the mechanical energy convertor in Question 10 is: _____.
12. A mechanical energy convertor changes 1 horsepower of input mechanical power into 700 watts of output electrical power. One horsepower equals ____ watts of power.
13. The percent efficiency of the mechanical energy convertor in Question 12 is: ____.
14. A water pump (mechanical-to-fluid energy convertor) delivers 10 m^3 of water (mass density of water $\rho = 1000\ kg/m^3$) at a speed of 15 m/sec each second it operates. The kinetic energy of the water delivered in a second is: ____ $kg{\cdot}m^2/sec^2$, or _____ N·m.
15. The power delivered by the pump in Question 14 equals ____ N·m/sec, or ____ J/sec, or _____ watts.

LET'S STUDY ENERGY-CONVERSION PROBLEMS THAT INVOLVE MECHANICAL ENERGY CONVERTORS.

Study the two following examples of energy-conversion problems. Then solve the exercises that follow.

Example 1: *Efficiency of a Windmill Generator (Mechanical-to-Electrical Energy Conversion)*

Given: The blades of a windmill transfer 5.8×10^4 joules of rotational mechanical energy each second to the spinning windmill shaft. The shaft turns an electrical generator. It produces 20 kilojoules of electrical energy each second.

Find: The efficiency of the windmill generator.

Solution: Use the energy IN and the energy OUT to find the efficiency of this mechanical-to-electrical energy convertor.

$$\text{Eff}(\%) = \frac{E_{OUT}}{E_{IN}} \times 100\%$$

where: $E_{OUT} = 20\ kJ = 20{,}000\ J = 2 \times 10^4\ J$ (generator energy output)

$E_{IN} = 5.8 \times 10^4\ J$ (shaft energy input from windmill)

Thus,

$$\text{Eff(\%)} = \frac{2 \times 10^4 \cancel{J}}{5.8 \times 10^4 \cancel{J}} \times 100\% \qquad \text{(Cancel units.)}$$

$$\text{Eff(\%)} = \frac{2}{5.8} \times 10^{4-4} \times 100\%$$

$$\text{Eff(\%)} = 0.3448 \times 10^0 \times 100\% \qquad \text{(Remember, } 10^0 = 1.\text{)}$$

$$\text{Eff(\%)} = 34.5\% \text{ (rounded)}$$

Note: The windmill generator is a power convertor as well as an energy convertor. So this problem can also be solved as a "power" problem. Here's the way you do it.

Remember that $P = E/t$. We know the energy IN during a time t = 1 second and the energy OUT during a time t = 1 second from the *Given* information. So we can calculate the input power P_{IN} and the output power P_{OUT} and determine the efficiency with the formula

$$\text{Eff(\%)} = \frac{P_{OUT}}{P_{IN}} \times 100\%$$

First, find P_{OUT} for a time t = 1 second. (Remember, 1 J/sec = 1 watt.)

$$P_{OUT} = \frac{E_{OUT}}{t} = \frac{2 \times 10^4 \text{ J}}{1 \text{ sec}} = 2 \times 10^4 \text{ J/sec} = 2 \times 10^4 \text{ watts}$$

Similarly, find P_{IN} for a time t = 1 second.

$$P_{IN} = \frac{E_{IN}}{t} = \frac{5.8 \times 10^4 \text{ J}}{1 \text{ sec}} = 5.8 \times 10^4 \text{ J/sec} = 5.8 \times 10^4 \text{ watts}$$

Then, calculate the percent efficiency.

$$\text{Eff(\%)} = \frac{2 \times 10^4 \cancel{\text{watts}}}{5.8 \times 10^4 \cancel{\text{watts}}} \times 100\% \qquad \text{(Cancel units.)}$$

$$\text{Eff(\%)} = \frac{2}{5.8} \times 10^{4-4} \times 100\%$$

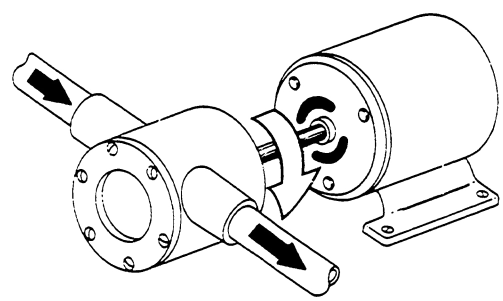
$$\text{Eff(\%)} = 0.3448 \times 10^0 \times 100\% \qquad \text{(Remember, } 10^0 = 1.\text{)}$$

$$\text{Eff(\%)} = 34.5\% \text{ (rounded)}$$

The answer is the same!

Example 2: *Efficiency of a Water Pump (Mechanical-to-Fluid Energy Conversion)*

Given: A ¼-hp water pump is rated as 75% efficient. The motor shaft that turns the pump rotor delivers 137.5 ft·lb of mechanical energy to the pump each second.



Find: The energy output of the water pump available to do work in moving water.

Solution: Use the formula for the efficiency of this mechanical-to-fluid energy convertor to find E_{OUT}.

$$\text{Eff(\%)} = \frac{E_{OUT}}{E_{IN}} \times 100\%$$

where: Eff(%) = 75%

E_{OUT} = energy output of the pump (to be determined)

E_{IN} = 137.5 ft·lb, the energy input from the motor shaft

Rearrange the equation to isolate E_{OUT}. This gives

$$E_{OUT} = \frac{\text{Eff}(\%) \times E_{IN}}{100\%}$$

Substitute given values for Eff(%) and E_{IN}.

$$E_{OUT} = \frac{75\% \times 137.5 \text{ ft·lb}}{100\%}$$

$$E_{OUT} = \left(\frac{75 \times 137.5}{100}\right) \left(\frac{\cancel{\%} \times \text{ft·lb}}{\cancel{\%}}\right) \qquad \text{(Cancel \% symbol.)}$$

$$E_{OUT} = 103.1 \text{ ft·lb}$$

Thus, the pump is able to provide 103.1 ft·lb of *energy* to move water.

Note: As in Example 1, the water pump is also a power convertor. Time is "included" since the motor shaft is rated in horsepower. Since 1 hp = 550 ft·lb/sec, P_{IN} = ¼ hp = 137.5 ft·lb/sec. And so, since

$$\text{Eff}(\%) = \frac{P_{OUT}}{P_{IN}} \times 100\%$$

we can solve for P_{OUT} to obtain

$$P_{OUT} = \frac{\text{Eff}(\%) \times P_{IN}}{100\%} \qquad \text{where: Eff}(\%) = 75\%, \quad P_{IN} = 137.5 \text{ ft·lb/sec}$$

Substitute these values.

$$P_{OUT} = \frac{75\% \times 137.5 \text{ ft·lb/sec}}{100\%}$$

$$P_{OUT} = \left(\frac{75 \times 137.5}{100}\right) \left(\frac{\cancel{\%} \times \text{ft·lb/sec}}{\cancel{\%}}\right) \qquad \text{(Cancel \% symbol.)}$$

$$P_{OUT} = 103.1 \text{ ft·lb/sec}$$

Thus, the pump is able to provide 103.1 ft·lb/sec of *power* to move water.

PRACTICE EXERCISES

Problem 1: Given: The piston in a piston-type water pump receives 600 ft·lb of linear mechanical energy each second to move three gallons of water to a higher position. The pump puts 579 ft·lb of energy each second into moving the water to its higher level.

Find: The pump (energy-convertor) efficiency.

Solution:

Problem 2: Given: A vane-type pump, driven by an electric motor, converts 705,000 N·m of input mechanical energy (rotational energy of the motor shaft) into fluid energy. The fluid energy delivered by the pump at its output is just enough to lift 3000 liters (3 cubic meters) of water to a height of 18 meters. The mass density of water is $\rho = 1 \text{ gm/cm}^3$ or 1000 kg/m^3.

Find:
a. The output energy of the pump.
b. The pump efficiency.

Solution: (***Hint:*** The value for E_{OUT} at the pump equals the kinetic energy of E_k of the water being pumped out. But E_k is equal to the potential energy E_p of the water that's raised a height of 18 meters. Therefore, $E_{OUT} = E_k = E_p$. To find E_{OUT}, find Ep. Remember that $E_p = [\rho V]gh$, where ρ is the mass density, V is the volume, g is 9.8 m/sec², and h is the height.)

Problem 3: Given: A motor-control device known as a *tachometer generator* has an output signal of 200 milliwatts at 1800 rpm.

Find: The input power P_{IN} if the generator is 98% efficient.

Solution: (***Hint:*** You can think of the tachometer as a power convertor. In that case, P_{OUT} = 200 milliwatts. Then use the equation $\text{Eff}(\%) = \frac{P_{OUT}}{P_{IN}} \times 100\%$ to find the input power, P_{IN}.)

Problem 4: Given: An inertia-welder spins a rod until the stored kinetic energy in the rod is 3086 ft·lb. The rod is forced against the stationary part to be welded. When the lathe stops, the rod and stationary part are fused (welded) together. The fusion process consumes 880 calories of energy.

Find: The efficiency of the inertia-welder.

Solution: (***Hint:*** Be sure E_{IN} and E_{OUT} are in the ***same*** units before you find efficiency.)

Problem 5: Given: The wind blowing against a windmill causes a force on the windmill blades. This force develops a torque that spins the blades and blade shaft. The windmill shaft obtains its energy from this torque. The windmill shaft rotates one revolution every second. It acts as the input shaft for a generator. The generator is 85% efficient. It provides an electrical output power of 10 kilowatts.

Find: The total torque on the blades required to produce this electrical power.

Solution: (***Hint:*** Use $\text{Eff}[\%] = \frac{P_{OUT}}{P_{IN}} \times 100\%$, and $P_{IN} = T \times \omega$.)

Student Challenge

Problem 6: Given: An emergency-signal generator used by downed aviators has a 6-inch, crank-type handle that must be turned one revolution each second for proper operation. The recommended force to be applied to the handle is 12.5 pounds. The handle turns the shaft of a generator that produces 45 watts of output power.

Find:

a. The input power given to the generator by cranking the handle.

b. The efficiency of the generator.

Solution: