PREPARATORY MATH SKILLS LAB

Lab PMS 17

MATH ACTIVITY

Ratio and Proportion

MATH SKILLS LAB OBJECTIVES

When you complete this activity, you'll be able to do the following:

1. ***Define the term, "ratio."***
2. ***Explain the term, "proportion."***
3. ***Define the term, "percent."***
4. ***Change a ratio to a percent.***
5. ***Compare a ratio to a proportion.***
6. ***Solve problems that involve ratios and proportions.***

MATERIALS

For this activity, you'll need pencil and paper.

WHAT'S A RATIO?

A ratio is a quantitative comparison of objects and values. Comparisons of one object to another object, and one value to another value, are made in the following statements. Therefore, these statements describe a ratio.

1. Bill is twice as old as Jill. The ratio of Bill's age to Jill's age is 2 to 1. If Bill is 2 years old, Jill is 1 year old. If Bill is 10 years old, Jill is 5 years old. Whatever Jill's age is, Bill's age is twice as much. The ratio 2 to 1 can be written several ways. As a fraction, the ratio is written $\frac{2}{1}$. It also can be written as an indicated division ($2 \div 1$), or with a colon (2:1). (Incidentally, if Bill is twice as old as Jill is now, it can never happen again. The difference in their ages remains constant, but the ratio of their ages changes each year.)
2. Donuts are being sold by the supermarket at half price. The ratio of this special selling price to the regular price is 1 to 2. If a box of donuts is now sold for $1, its regular selling price is $2. If an individual donut is now sold for 25 cents, its regular selling price is 50 cents. The ratio of 1 to 2 can be written as the fraction, $\frac{1}{2}$. It also can be written as $1 \div 2$ or as 1:2.
3. A large driven gear has 4 times as many teeth as the 10-tooth drive gear. Here, the ratio of "the number of teeth on the driven gear to teeth on the drive gear" is 40 to 10. This ratio can be written as the fraction $\frac{40}{10}$, which can be reduced to $\frac{4}{1}$. The ratio also can be written as $40 \div 10$, or $4 \div 1$. With a colon, the ratio can be written as 40:10, or 4:1.

The first chart that follows sums up the ways in which the ratios in these examples can be written. For each of the examples, the objects were given the same units. Both Bill's and Jill's ages were given in units of years. The price of the donuts was given in units of either dollars or cents. The gears were described by number of teeth on each gear.

	In words	Fraction	Indicated division	With a colon
Bill is twice as old as Jill.	2 to 1	$\frac{2}{1}$	$2 \div 1$	2:1
They're selling donuts at half price.	1 to 2	$\frac{1}{2}$	$1 \div 2$	1:2
The driven gear has 4 times as many teeth as the drive gear.	4 to 1	$\frac{4}{1}$	$4 \div 1$	4:1

Sometimes the values to be compared don't have the same units. When this happens, the units for one of the values can usually be changed to have the same units as the other value. Let's look at an example of how to find the ratio of 15 minutes to 1 hour.

First, write the numbers and units as a fraction: $\frac{\text{15 minutes}}{\text{1 hour}}$. Next, either change the units of ***minutes*** to units of ***hours***, or change the units of ***hours*** to units of ***minutes***. For this example, we'll change ***hours*** to ***minutes***.

$$\frac{15\text{ min}}{1\,\cancel{\text{hr}} \times \left(\frac{60\text{ min}}{1\,\cancel{\text{hr}}}\right)} = \frac{15\,\cancel{\text{min}}}{60\,\cancel{\text{min}}} = \frac{1}{4} \qquad \text{(Cancel hr and min units.)}$$

Notice that when the values to be compared both have the same units, the units cancel in the ratio, leaving only a number. Also, the number is here reduced to its lowest terms.

Read the statements in the following chart. Then write the ratio described by the statement in the blanks. Write each ratio as (1) a fraction, (2) a quotient, and (3) with a colon.

Statement	Fraction	Indicated division	Colon
a. Jana is three times taller than Mark.			
b. A gallon of milk costs twice as much as a gallon of gasoline.			
c. Her brother is half as old as my brother.			

WHAT'S A PERCENT?

Now let's look at a type of ratio called **percent**. Percentage is a comparison of ***a part*** of something to ***the whole*** of the same thing. The whole of the same thing is assumed to consist of 100 equal parts.

Percent is just the comparison of a certain number to the number 100. If you had 10 red pencils and 90 yellow pencils, then 10 out of the total number of pencils (100) are red. The ratios $\frac{10}{100}$ and 10:100 are each equal to 10 percent. Since 90 of the pencils are yellow, the ratios $\frac{90}{100}$ and 90:100 are equal to 90 percent.

When writing a number as a percent, the symbol "%" usually is used in place of the word, ***percent***. This means that a value such as 10 percent usually is written as 10%.

Sometimes it's necessary to change a ratio, such as 1:6, to a percentage. For example, Figure 1 is a circle that's divided into 6 equal parts. One part is shaded. What percentage of the circle is shaded?

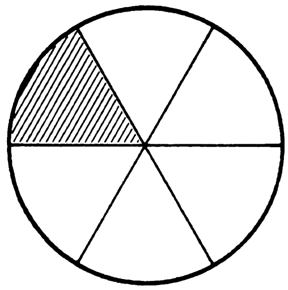


Fig. 1 Circle is divided into 6 equal parts.

Since one part is shaded, and there are six parts in the circle, you can write the ratio as $\frac{1}{6}$. To change this ratio to a percentage, divide the number on top (numerator) by the number on the bottom (denominator) to get a decimal number. Then change the decimal number to a "percent" by multiplying by 100. This is easy to do with the help of a calculator. Simply divide the number 1 by the number 6.

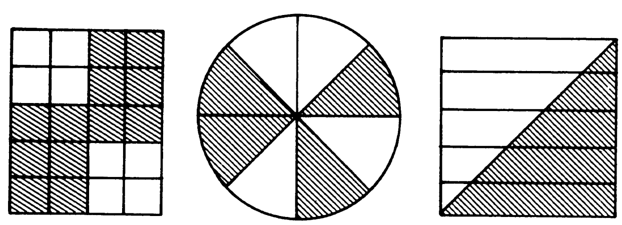
Then multiply the calculator answer (0.167) by 100%.

$$0.167 \times 100\% = 16.7\%$$

This answer tells us that 16.7% of the circle is shaded.

The following exercises will help you understand and learn how to work with a ratio.

1. What is the ratio of shaded to unshaded areas in the following figures?



a. b. c.

2. What ***percentage*** of each figure in Problem 1 is ***shaded?***
3. What ***percentage*** of each figure in Problem 1 is ***unshaded?***
4. Does adding the percentages in Problems 2 and 3 give 100% for each figure?
5. Solder is a mixture of lead and tin. "Soft" solder has 6 parts tin and 4 parts lead.
   a. How many grams of tin are in 1 kilogram of soft solder? ***(Hint:*** 6 parts + 4 parts = 10 parts = whole.)
   b. How many grams of lead are in 1 kilogram of soft solder?

WHAT'S A PROPORTION?

So far, you've learned that a ratio is simply a comparison of one value to another value. You've also learned about a type of ratio called "percent." A percentage compares one part of something to 100 equal parts of the same thing. Now let's learn about one more important type of ratio. It's called a "proportion."

Proportion is the relationship of one ratio, such as $\frac{4}{2}$, to another ratio of equal value, such as $\frac{10}{5}$. The phrase ***"of equal value"*** is important here. For the ratio, $\frac{4}{2}$, 4 divided by 2 equals 2. For the ratio, $\frac{10}{5}$, 10 divided by 5 equals 2. These ratios have equal value. Therefore, they are said to be ***proportional.***

The ratio of $\frac{4}{2}$ is also proportional to $\frac{8}{4}$, $\frac{12}{6}$, $\frac{20}{10}$—and so on. The ratio, $\frac{2}{4}$, is proportional to ratios such as $\frac{5}{10}$, $\frac{3}{6}$, $\frac{8}{16}$, because they all have equal value. If the ratios don't have equal values, they're not proportional.

"Constant of proportionality" is a term used in technology. Frequently, this term is shortened to just "constant." As you've seen, when a ratio is proportional to another ratio, the ratios have equal values. Since the values don't change, they're said to be "constant."

For example, a spring is rated by its "spring constant." A spring constant is the ratio of *force* needed to stretch the spring a certain distance to the *distance stretched* (f_1/d_1). It's also equal to the ratio of force needed to stretch the spring another distance to the new distance (f_2/d_2). These ratios are proportional. Therefore, they're constant.

Let's say that for a certain spring, a force of 10 pounds is needed to stretch the spring 2 inches, and a force of 20 pounds stretches the spring 4 inches. The ratios can be written as:

$$\left(\frac{f_1}{d_1} = \frac{f_2}{d_2}\right) \rightarrow \left(\frac{10 \text{ lb}}{2 \text{ in.}} = \frac{20 \text{ lb}}{4 \text{ in.}}\right) \rightarrow \left(5\,\frac{\text{lb}}{\text{in.}} = 5\,\frac{\text{lb}}{\text{in.}}\right)$$

The constant value of these ratios is 5 lb/in. Therefore, the spring constant is 5 lb/in. The force needed to stretch or compress a spring, or the distance a spring moves when a known amount of force is applied, can be determined if you know the spring constant.

The following exercises will help you learn how to work with proportions.

6. Which of the following ratios is proportional to $^{40}/_{5}$?
   a. $^{20}/_{10}$
   b. $^{8}/_{1}$
   c. $^{5}/_{40}$
   d. $^{80}/_{11}$

7. Which of the following ratios is proportional to $^{7}/_{63}$?
   a. $^{14}/_{126}$
   b. $^{1}/_{8}$
   c. $^{1}/_{7}$
   d. $^{9}/_{1}$

8. What's the constant of the ratios $\frac{12}{1}$ and $\frac{144}{12}$?

9. What's the constant of the ratios $\frac{1}{8}$ and $\frac{8}{64}$?

10. A force of 70 newtons compresses a spring 2 cm. A second force compresses the same spring only 1 cm. How much force is applied the second time?

    [***Hint:*** Use the proportion, $\frac{70\text{ N}}{2\text{ cm}} = \frac{x\text{ (N)}}{1\text{ cm}}$.]

